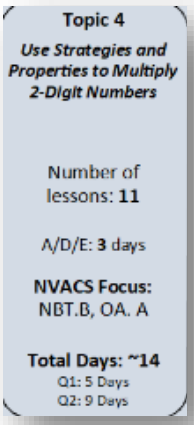
Grade 4 Topic 4: Use Strategies and Properties to Multiply by 2-Digit Numbers



Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 4 Professional Development Videos located in Pearson Realize online. Read the Teachers' Edition (TE): Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 167A-167D), all 11 lessons, and the Topic Assessments (pp. 243-248A).

Mathematical Background: Read Cluster Overview (TE, pp. 43A-43F)	Topic Essential Questions: How can you use a model to multiply? How can you use the Distributive Property to multiply? How can you use multiplication to solve problems? (TE, p. 167) *Reference TE (p. 167) and Answering the Topic Essential Questions (TE, pp. 243-244) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	4-9	4-10	4-11	Assessment

3 A/D/E days used strategically throughout the topic.

4th grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:

This topic builds from Topic 3 by using place-value understanding and estimation to multiply multi-digit whole numbers. Focus instruction on Nevada Academic Content Standard (NVACS) 4.NBT.B.5 (2010). Emphasis for standard 4.NBT.B.5 is to "multiply two two-digit whole numbers using strategies based on place value and the properties of operations" (NVACS, 2010). The problem types or structures in this topic continue to focus on equal groups and multiplicative comparison when multiplying. As stated in the Topic 3 WCSD Curriculum Guide, Van de Walle, Karp, and Bay-Williams explain that equal group problems can be repeated addition or rate problems. Repeated addition problems such as, "If three children have four apples, how many apples are there?" and rate problems, 'If there are four apples per child, how many apples would three children have?'" (Van de Walle et al., 2010, pp. 154-155). Multiplicative comparison problems are when there are two different sets. Van de Walle, et al., state, "one set consists of multiple copies of the other. An example of a multiplicative comparison problem is, 'Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?'" (2010, p. 155).

This topic focuses on the distributive, commutative and associative properties of multiplication, as well as the partial product algorithm using an area model or open array. Also included is the U.S. traditional algorithm. The U.S. traditional algorithm for multiplication is based on the Distributive Property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand. Instruction that includes the conceptual knowledge behind procedures is crucial. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Van de Walle, et al., state, "once having begun with traditional algorithms, it is extremely difficult to suggest to students that they learn other methods" (2010, p. 217). Be cautious introducing the U.S traditional algorithm without conceptual understanding.

Use of area models and the partial products algorithm rely on the distributive property and can help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These algorithms allow students to see the partial products within the algorithms and how each is created, while avoiding the errors that often occur when regrouping and recording using the U.S. traditional algorithm. These algorithms also allow students to work horizontally or vertically, and to multiply factors and add partial products in varying order. You may consider using partial products algorithm instead of the U.S. traditional algorithm, especially until students are secure with the properties, models and partial products algorithm.

Area models and the partial products algorithm can be very efficient. Using these strategies, students create separate values for each partial product without additional time or writing when compared to the U. S. standard algorithm. The area model has many advantages over the U.S. traditional algorithm, especially as students begin using two-digit multipliers (Van de Walle, et al., 2014). Students should understand that multiple strategies could be used to solve these problems with some being more appropriate than others in certain instances. Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014).

Estimation is important as students use estimation strategies to check for reasonableness to their answers. In this topic, students will also compare and round whole numbers. Rounding whole numbers is an **estimation** strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, et al., 2010, p. 241). Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Van de Walle, et al., (2010) say,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are *reasonable* for the situation or problem at hand. (p. 242)

Students should be able to use rounding flexibly and understand it conceptually, so it can be a useful estimation strategy (Van de Walle, et al., 2010).

Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups, this will make finding an estimate easier." (Van de Walle, et al., 2010, p. 247)

Karen Karp, Sarah Bush and Barbara Dougherty (2014), state the zero trick or multiply a number by ten, just add a zero to the end of the number expires because when students begin to multiple decimals in 5th grade this rule no longer applies. Be cautious when teaching the "zero trick" and consider avoiding it all together. Instead, consider connecting it to our place-value base ten system discussed in Topic 1.

Focus Math Practice 1: Make sense and persevere
Focus opportunities for students to develop *Mathematical Practice 1* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 4-11. Reference the Teacher's Edition (pp. F21-F21A) and the NVACS (2010, p. 6).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
compatible numbers	*Distributive Property* *Commutative Property of Multiplication* *Associative Property of Multiplication* *area model* *partial product algorithm*

Additional terminology that students may need support with: array or open array, estimate, rounding, factors, product, partial product, mental math, variable and bar diagram

*Consider using the additional terminology to label anchor charts used throughout this topic.

*Collaborative Team Conversations (CTC)
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks or arrays) towards a more sophisticated understanding (partial product algorithm; area model)?"
"Are students making connections between Topic 3 and Topic 4?"

Lesson	Evidence	Look for
4-5	*Intervention Activity* (student work samples)	Focus CTC around the big idea: • student strategies and models. • student connection between Topic 3 and Topic 4.
4-9	*Solve & Share* (student work samples)	Focus CTC around the big idea: • student strategies and models connected to Topic 3. • difference in estimation strategies between rounding and compatible numbers; how the estimate leads to overestimation and underestimation.

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 243-248	Use *Scoring Guide* TE pp. 243-248A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 4-1: Mental Math- Multiply by Multiples of 10		
4 NBT.B.5 4.OA.A.3 MP.2 MP.7	Access Prior Learning: In Lesson 3-1, students multiplied whole numbers by multiples of 10, 100, and 1,000. In Topic 1, students also learned about the base-10 system and 10 times. Developing the Big Idea: In this lesson, students will use their previous knowledge to multiply two multiples of 10.	Solve & Share: Consider giving students the opportunity to use multiple tools or representations to complete the *Solve & Share*. Visual Learning: In the *Visual Learning Animation*, students will make connections in multiplication by using the associative property of multiplication. Consider making an anchor chart with the commutative and associative properties for multiplication. As in Topic 3, consider pausing animation to have an in-depth conversation regarding the property and why the "short-cut or zero trick" works. See the instructional note regarding the "zero trick". Convince Me: In the *Convince Me!*, students are asked to solve a missing factor multiplication. Consider facilitating a discussion around finding missing factors. Ask students, "What related fact could help us solve this problem?" Another Example: Consider using the *Another Example!* with all students as it asks students how the associative and commutative property of multiplication help with the patterns of zero. Facilitate a discussion around place-value understanding. Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition (SE). Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Assess and Differentiate/Intervention Activity: Be cautious when doing the *Intervention Activity* with students as it addresses the "zero trick". Instead, consider using place-value blocks to help students understand multiplying by two multiples of 10.
Lesson 4-2: Use Models to Multiply 2-Digit Numbers by Multiples of 10		
4.NBT.B.5 MP.1 MP.2 MP.4 MP.5	Access Prior Learning: In lesson 3-5, students used an array to find partial products. This approach used place-value understanding and the distributive property. Developing the Big Idea: In this lesson, students will use place-value blocks, area models, arrays and partial products to multiply a 2-digit number by a multiple of 10.	Look Back: Consider facilitating a discussion around the *Look Back!* as it uses place-value understanding from Topic 1 and addresses our base-10 system of 10 times as many. Visual Learning: Students continue using an area model or open array to multiply 2-digit by 2-digits whole numbers. In the *Visual Learning Animation,* both an area model and open array are shown. Consider having students use both models and connect to Topic 3. Independent Practice/Math Practices and Problem Solving: Consider using items 13 and 14 as a review for Topic 2. You may also consider using these items as a grade. For item 17, consider having students estimate before solving and then facilitate a discussion around over/under estimate in regards to money. Assess and Differentiate/Intervention Activity: Consider facilitating a discussion around the *Intervention Activity* as it gives all students access to multiplying 2-digit whole numbers by 10 using place-value blocks.
Lesson 4-3: Estimate- Rounding		
4.NBT.B.5 4.OA.A.3 MP.2 MP.3	Access Prior Learning: In previous topics, students used rounding as an estimation strategy to find sums, differences or products. Developing the Big Idea: In this lesson, students will use rounding to estimate the product of two 2-digit whole numbers.	Solve & Share: Consider having students share the different factors they used to solve for a product close to 1,400. Facilitate a conversation around which combination of numbers were an underestimate of 1,400, and which combinations were an overestimate to 1,400. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students. Consider using a number line as a tool to check for reasonableness.

Lesson 4-4: Estimate- Use Compatible Numbers		
4.NBT.B.5 MP.2 MP.3	Access Prior Learning: In third grade, students used compatible numbers to estimate sums and differences. In the previous lesson, students estimated products of two 2-digit numbers. Beginning of the Big Idea: In this lesson, students will use number sense and compatible numbers to estimate products.	Look Back: Consider facilitating a discussion around the *Look Back!* as students are sharing their responses to the *Solve & Share*. You may also have students answer the *Look Back!* as they work on the *Solve & Share* and then facilitate a discussion. The *Look Back!* has students thinking about their number choices when estimating the product of 1,600. Visual Learning: The *Visual Learning Animation* discusses compatible numbers. Consider adding compatible numbers to an estimation anchor chart to show students there are multiple ways to estimate other than rounding. Compare compatible numbers to rounding. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students as students make connections to compatible numbers and money. You may consider using this as Number String or Number Talk to see if students make connections to estimation and money on their own.
Lesson 4-5: Arrays and Partial Products		
4.NBT.B.5 4.OA.A.3 MP.4 MP.7	Access Prior Learning: In Lesson 3-5, students used an array to find partial products and added the partial products to find the final product. Developing the Big Idea: In this lesson, students will continue using arrays and the partial products algorithm to multiply two 2-digit whole numbers.	Look Back: Consider facilitating a discussion around the *Look Back!*. The *Look Back!* brings in the concept of area through the context of the problem. Here is an opportunity to discuss area with students. Some may connect area from third grade to this problem. Students will have more opportunities to work with area in other topics. Visual Learning: Consider having students estimate before solving problems in the *Visual Learning Animation*. Convince Me: Consider facilitating a discussion with the whole class around the *Convince Me!*. The area model is given, students need to figure out the multiples and product from the area model. This problem may support students in future topics regarding area, as they will need to find the dimensions when they know the product. Guided Practice: Consider facilitating a discussion around item 3 as it asks students to check for reasonableness. Assess and Differentiate/Intervention Activity: Consider facilitating a discussion around the *Intervention Activity* with all students, because students are expanding from a one-digit by two-digit to a two-digit by two-digit. Students can make connections from Topic 3 to Topic 4. *CTC: ***Intervention Activity*** (student work samples)
Lesson 4-6: Multiply Using the Distributive Property		
4.NBT.B.5 MP.4 MP.7	Access Prior Learning: In third grade, students found the area of a rectangle. In previous lessons, students used an area model or open array to find the partial products. Developing the Big Idea: In this lesson, students will use the area model (open array) to extend their understanding of the Distributive Property and find the relationship between the two.	Solve & Share: The *Solve & Share* is another opportunity for students to see an area problem. Students should make connections to their prior learning of area and multiplication using an area model. Look Back: Consider using the *Look Back!* to facilitate a discussion around the partial products algorithm, from Topic 3. A student may have used the partial products algorithm to answer the *Solve & Share*, use this student's response to connect to the *Visual Learning Animation*. Convince Me: Consider using the *Convince Me!* as a whole class discussion, so students can see there are multiple ways to use the distributive property. List some of the ways students may have solved the problem from the *Visual Learning*. Independent Practice/Math Practices and Problem Solving: Consider using item 13 in a class discussion because it focuses on over/under estimation. Assess and Differentiate/Intervention Activity: Consider using place-value blocks or representations to support students who need scaffolding with distributive property.

Lesson 4-7: Use Partial Products to Multiply by 2-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In the previous lessons, students learned and applied the partial products algorithm to multiply two 2-digit whole numbers. Developing the Big Idea: In this lesson, students will continue using the partial product algorithm to multiply multi-digit whole numbers.	Solve & Share: Consider removing the grid as to facilitate multiple student strategies. Look Back: Consider using the *Look Back!* to facilitate a discussion around estimation. Students should be estimating to check for reasonableness throughout topics and lessons. Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate.
Lesson 4-8: Multiply 2-Digit by 2-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.2 MP.7	Access Prior Learning: In Topic 3, students were exposed to the standard algorithm. Developing the Big Idea: In this lesson, students will use algorithms to multiply two 2-digit whole numbers.	Solve & Share: The *Solve & Share* is a multiplicative comparison problem. Note: this is the only multiplicative comparison problem in Topic 4. Consider this an opportunity to reintroduce the bar diagram if students did not use one. See Table 3, problem type is a compare, larger unknown. Look Back: Consider having students solve the *Look Back!* and facilitate a discussion regarding place-value understanding within the base-10 system when we multiply. Visual Learning: In the *Visual Learning Animation*, students are shown the U.S traditional algorithm. As stated in Topic 3, students do not need to be secure using the U.S. traditional algorithm. Consider showing students the strategy, but do not "expect" them to use it all the time. Give students multiple opportunities to continue using an area model, open array and the partial product algorithm to multiply. Convince Me: Again, consider using the *Convince Me!* to facilitate a discussion regarding our base-10 system and 10 times as many instead of focusing the discussion on the "zero trick". Independent Practice/Math Practice and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate.
Lesson 4-9: Continue to Multiply By 2-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.1 MP.3 MP.4 MP.7	Access Prior Learning: In Lesson 4-8, students compared the partial product algorithm to the U.S standard algorithm. Developing the Big Idea: In this lesson, students will continue using the algorithms to multiply whole numbers.	Solve & Share: Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. Look Back: Consider having students answer the question in the *Look Back!*, and then facilitate a discussion around reasonableness by using estimation. Consider having students estimate throughout the lesson. Visual Learning: Consider having students continue estimating, and giving students the opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation*. Convince Me: Consider using the *Convince Me!* to facilitate a discussion or used formatively to assess student understanding as students have had multiple opportunities to discuss place-value and the partial products algorithm. Independent Practice/Math Practice and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Assess and Differentiate/Intervention Activity: Consider giving students the opportunity to work on the partial products algorithm, area model or array, and not have students use the U.S. traditional algorithm. *CTC: *Solve & Share* (student work samples)

Lesson 4-10: Continue to Multiply by 2-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.1 MP.6 MP.7 MP.8	Access Prior Learning: In previous lessons and topics, students compared the partial product algorithm to the traditional algorithm. Developing the Big Idea: In the lesson, students will continue to use algorithms to multiply whole numbers.	Solve & Share: Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. Look Back: Consider facilitating a discussion around the question as it asks students to think about an estimation strategy they may use or had used to solve the problem. Students may use compatible numbers and/or rounding to estimate. Visual Learning: Consider using the *Visual Learning Animation* area problem, but do not expect students to use the U.S. traditional algorithm. Consider accepting area models, arrays and the partial product algorithm. The *Visual Learning* problem is multi-step, as students will multiply and then subtract the two problems. Consider using this problem as a Gallery Walk, so students can critique the reasoning of others. Guided Practice: Item 1 is a missing factor multiplication problem within the U.S. traditional algorithm, consider giving on-level or above-level students this problem during independent practice, instead of doing whole class. Independent Practice/Math Practice and Problem Solving: Consider using item 23 to have students practice estimation strategies. Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider encouraging students to use multiple strategies to solve the problems. Assess and Differentiate/Intervention Activity: Consider giving students the opportunity to work on the partial products algorithm, area model or array, and not have students use the U.S. traditional algorithm.
Lesson 4-11: Math Practices and Problem Solving- Make Sense and Persevere		
4.NBT.B.5 4.OA.A.3 MP.1 MP.2 MP.4 MP.6 MP.7	Access Prior Learning: In previous lessons, students used multiple strategies to multiply two 2-digit whole numbers. Developing the Big Idea: In this lesson, students will continue to build fluency in multiplication using multiple strategies.	Solve & Share: Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. Look Back: Consider having students work on the *Look Back!* as they work on the *Solve & Share*. Visual Learning: Students are asked to use a bar diagram and write an equation with a variable. Consider tying the *Convince Me!* to the *Visual Learning Animation* or discussion, as students are asked to solve the problem using different strategies. To make the connection, accept multiple strategies, but emphasize the distributive property. Assess and Differentiate/Intervention Activity: Consider using the problem given for the *Intervention Activity*, but do not expect students to use the U.S. traditional algorithm. Consider accepting strategies like the area model, open array or partial product algorithm. Some students may still need the support of place-value blocks, so help those students move from concrete to representational by drawing the models.

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	A > 1 Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	A < 1 Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30(1)*, 3-19.

Karp, K., Bush, S., & Doughtery, B. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

Van De Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Pearson.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8* (2nd ed.). New York, NY: Pearson.

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