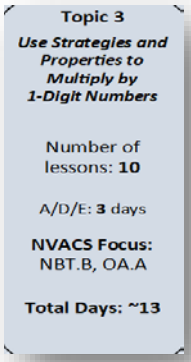
Grade 4 Topic 3: Use Strategies and Properties to Multiply by 1-Digit Numbers



Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 3 Professional Development Video (located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 91I-91J), all 10 lessons, and the Topic Assessments (pp. 165-166A).

Mathematical Background: Read Cluster Overview (TE, pp. 43A-43F)	Topic Essential Questions: How can you multiply by multiples of 10, 100 and 1,000? How can you estimate when you multiply? *Reference TE page 91 and Answering the Topic Essential Questions (TE, pp. 161-162) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9	3-10	Assessment

3 A/D/E days used strategically throughout the topic.

4th grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:

This topic focuses on using place-value understanding and estimation to multiply multi-digit whole numbers. Focus instruction on Nevada Academic Content Standards (NVACS) standards 4.NBT.B.5. Emphasis for standard 4.NBT.B.5 is to "multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations".

The problem types or structures in this topic focuses on equal groups and multiplicative comparison when multiplying. Van de Walle, Karp, Bay-Williams state that equal group problems can be repeated addition or rate problems. "Repeated addition problems can be problems such as, 'If three children have four apples, how many apples are there?" and rate problems may be, "If there are four apples per child, how many apples would three children have?'" (2010, pp. 154-155). Multiplicative comparison problems are when there are two different sets and can be more complex for developing students mathematical understanding of multiplication. Van de Walle, et al., state, "One set consists of multiple copies of the other. An example of a multiplicative comparison problem is, 'Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?'" (2010, p. 155).

The distributive, commutative and associative properties of multiplication, as well as the partial product algorithm using an area model or open array are all areas of focus in this topic. Additionally, this topic does show students the U.S. standard algorithm. The U.S. standard algorithm for multiplication is based on the distributive property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand if taught in a systematic procedural manner. Instruction that includes the conceptual knowledge behind procedures is crucial. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Van de Walle, et al., (2010) states, "once having begun with traditional algorithms, it is extremely difficult to suggest to students that they learn other methods" (p. 217).

Be cautious introducing the U.S standard algorithm without conceptual understanding, as students are introduced to multi-digit multiplication of whole numbers. Use of area models and the partial products algorithm rely on the distributive property and can be used to help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These algorithms allow students to see how the partial products are created while avoiding the errors that often occur when regrouping and recording using the U.S. standard algorithm. They also allow students to work horizontally or vertically, and to multiply factors and add partial products in varying order. You may consider using partial products algorithm instead of the U.S. standard algorithm, especially until students are secure with the understandings of how area models and partial products support the distributive property leading to more algorithmic work.

Area models and the partial products algorithm can be very efficient. Using these strategies, students create separate values for each partial product without much more additional time or writing when compared to the standard algorithm. The area model has many advantages over the standard algorithm, especially as students begin using two-digit multipliers (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Students should understand that multiple strategies can be used to solve these problems with some being more appropriate than others in certain instances. Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014).

Estimation is important as students use these strategies to check for reasonableness of their answers. In this topic students will also compare and round whole numbers. Rounding whole numbers is an estimation strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, et al., 2010, p. 241). Rounding is one

strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Van de Walle, et al. write,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are *reasonable* for the situation or problem at hand. (2010, p. 242)

Students should be able to use rounding flexibly and understand it conceptually, so it can be a useful estimation strategy.

Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups, that will make finding an estimate easier" (Van de Walle, et al., 2010, p. 247). Karen Karp, Sarah Bush and Barbara Dougherty, write in "13 Rules that Expire", that the zero trick or teaching children to multiply a number by ten by just adding a zero to the end of the number will break down or "expire" in student learning trajectories. The "zero trick" expires when students begin to multiply decimals in 5th grade and this rule no longer applies. Be cautious when teaching the "zero trick" and consider avoiding it all together. Instead, consider connecting it to our place-value base-ten system discussed in Topic 1.

"The equal sign is one of the most important symbols in elementary arithmetic, in algebra, and in all mathematics using numbers and operations" (Van de Walle, et al., 2010, p. 258). It is important that students understand the meaning of the equal sign and not simply view it as the answer to a problem. Understanding the equal sign means "to see, understand and symbolize the relationships in our number system. When students fail to understand the equal sign, they typically have difficulty when it is encountered in algebraic expressions" (Van de Walle, et al., 2010, p. 258).

Focus Math Practice 4: Model with math

Focus on opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 3-10. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
numerical expression partial products	*Associative Property of multiplication* *Distributive property* *compensation* *Commutative Property of multiplication*

Additional terminology that students may need support with: area model, array or open arrays, estimate, rounding, factors, product, mental math, reasonableness, overestimate, underestimate

*Consider using the additional terminology to label anchor charts used throughout this topic.

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding question: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks) toward equations through use of repeated addition or multiplication using the partial products algorithm?"

Lesson	Evidence	Look for
3-2	*Solve & Share* (student work samples)	Focus CTC around the big idea: • student strategies and models. • student estimation; overestimation or underestimation related to actual answer.
3-6	*Convince Me!* (digital platform or student work samples)	Focus CTC around the big idea: • partial products allows for students to start in any place value. • connects to the distributive property. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 161-166	Use *Scoring Guide* TE pp. 161-166A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 3-1: Mental Math- Multiply Multiples of 10, 100 and 1,000		
4 NBT.B.5 MP.2 MP.4 MP.7	Access Prior Learning: In third grade, students multiplied whole numbers by multiples of 10. In Topic 1, Students also learned about the base 10 system and 10 times. Developing the Big Idea: In this lesson, students will use their previous knowledge and multiply by 10, 100 and 1,000 using mental math.	Solve & Share: Consider giving students opportunity to use multiple tools or representations to complete the *Solve & Share*. Look Back: Consider having students include the *Look Back!* question as they answer the *Solve & Share*. Look for students who make connections to the work they did in Topic 1 around the 10 times idea. Visual Learning: In the *Visual Learning Animation*, students will make connections in multiplication by using the associative property of multiplication. Consider making an anchor chart with the associative property. Add the other properties of multiplication as they are discussed in this topic. Consider pausing the animation to have an in-depth conversation regarding the property and why the "short-cut or zero trick" works. The "zero trick" works, because it connects to shifting numbers within the place-value structure. See the instructional note regarding the "zero trick". Note: Consider moving to the *Intervention Activity* after the *Visual Learning Animation*. Another Example: Consider using the *Another Example!* as it discusses place-value relationships and patterns when multiplying by 10, 100 and 1,000. Consider using *Another Example!* before the *Convince Me!*. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students as it uses concrete tools to explain multiplying by multiples of 10, 100, or 1,000. Some students need concrete or representational models before an abstract algorithm. Consider using the *Intervention Activity* before the *Convince Me!* and after *Another Example!*.
Lesson 3-2: Mental Math- Round to Estimate Products		
4.NBT.B.5 MP.2	Access Prior Learning: In previous Topics 1 and 2, students learned how to estimate by rounding whole numbers, sums and differences.	Solve & Share: Due to estimation lessons in previous topics, look for students who observed the word "about" in the *Solve & Share*. Consider sharing strategies where students did estimate to solve for the product. *-continues on next page-*

	Developing the Big Idea: In this lesson, students will estimate products of multi-digit whole numbers to check if they are reasonable.	Look Back: Consider facilitating a discussion around the *Look Back!*, as students begin to think about estimation as an underestimate (less than the actual number) or overestimate (greater than the actual number) based on how they chose to estimate. Convince Me: Rounding is used to estimate each class' money earned for items sold. Remember, rounding is an estimation strategy. Consider adding *underestimate* and *overestimate* to your estimation anchor chart after discussion around the *Convince Me!*. Another Example: In the *Another Example!*, students discuss how an estimate is reasonable. Consider using this example with all students, so students continue to reason through estimation. Students should use estimation throughout the lessons in this topic. Independent Practice/Math Practices and Problem Solving: Items 12 and 13 deal with multiplicative comparison problems. Consider guiding students in a whole or small group discussion if assigning these items. Assess and Differentiate/Intervention Activity: If students are still struggling with estimation, consider using the *Intervention Activity* to scaffold understanding. The *Intervention Activity* uses a number line to help students round numbers based on their proximity to landmarks. *CTC: ***Solve & Share*** (student work samples)
Lesson 3-3: The Distributive Property		
4.NBT.B.5 MP.1 MP.2 MP.4 MP.7	Access Prior Learning: In third grade, students learned the distributive property and used the property to understand strategies for remembering basic facts. Developing the Big Idea: In this lesson, students will review the distributive property and use it to find products for multi-digit whole numbers.	Note: Consider reading the instructional note for item 12 before teaching the lesson (TE, pp. 109-110). Solve & Share: Students learned to determine area in third grade. Students will use this knowledge of area to answer the *Solve & Share* looking for the unshaded area. The *Solve & Share* begins to have students think about the distributive property with subtraction. Look Back: Consider using the *Look Back!* after the *Solve & Share*. Watch for students who do not see equality in the two equations. This may be a place to consider discussing the true meaning of the equal sign. The equal sign does not mean "the answer to", it means "the same as" or "equivalent to". Have students think about the equal sign as a balance. See instructional note for more information regarding the equal sign. Visual Learning: Mathematical language of numerical expression and distributive property is discussed in the *Visual Learning Animation*. Consider having students estimate the product before solving the expression. In the *Visual Learning Animation*, students use place-value understanding to solve the expression. Consider having a discussion around the importance of place-value while multiplying, as well as how an area model or open array supports place-value. Convince Me: Refer to the note for the *Convince Me!* in the TE (pg. 108) to guide the conversation. Another Example: Consider using the *Another Example!* with the whole class. The *Another Example!* discusses the distributive property with addition and the distributive property of subtraction. Consider using this as part of a classroom discussion to elicit student invented strategies. Students may invent the distributive property with addition or subtraction. If students do not come up with one or the other, consider using a past student's name with the "invented strategy". Guided Practice: Consider using item 1 with the whole class as the area model and distributive property are used simultaneously. Consider having students figure out the equation. For example, students are finding the product for 4 X 13 and there are two ways to find the product. Ask students to find other ways to determine the product. For example, students may say, "4 x (5 + 8) = (4 X 5) + (4 X 8)". Make an anchor chart with student responses. Independent Practice/Math Practices and Problem Solving: For items 3-10, consider having students estimate before using the distributive property to solve in order to check the reasonableness of their answer. *-continues on next page-*

		Many of the problems in the *Math Practices and Problem Solving* ask students to use the distributive property with addition or subtraction. Consider using item 12 with students before independent work. Refer to the Coherence Note in the TE (pg. 109-110) for more information.
Lesson 3-4: Mental Math Strategies for Multiplication		
4.NBT.B.5 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In the third grade, students used the distributive property as a strategy for finding basic facts. Students also used the Associative and Commutative Property to multiply. In the previous lessons, students have used the distributive property and mental math to multiply numbers by 10, 100, 1000. Developing the Big Idea: In this lesson, students will use the distributive, associative and commutative properties of multiplication to find products for multi-digit whole numbers mentally.	Solve & Share: Consider using the *Solve & Share* as a Number Talk. A Number Talk is where students solve each of the problems mentally and explain their thinking. Record students' responses on the board or chart paper. Students may use the Associative or Commutative Property of Multiplication to solve the problem. For example, the problem 25 x 9 x 4, students may solve the problem by using the commutative property and compatible numbers. Students may multiply 25 and 4 to get 100 and then multiply 100 and 9 to get 900. Look Back: Consider having students work on the *Look Back!* on their own, then discuss different ways students may have solved the problem whole group. Visual Learning: The *Visual Learning Animation* discusses the commutative and associative property of multiplication. Consider using the students' responses from the *Solve & Share* to reiterate these properties and compare to the animation. Another Example: The focus of the *Another Example!* is compensation. Compensation is an estimation strategy. Students use the information from the *Visual Learning* to solve the multiplication problem. Consider using this as a whole class Number Talk to see if a student uses compensation to solve the problem. If a student does not use compensation, consider using a student's name from a previous year and share the compensation estimation strategy with your class. Independent Practice/Math Practices and Problem Solving: Item 13 is a multi-step addition and subtraction problem, with the start unknown. Consider using this item to review Topic 2 or for summative grading purposes.
Lesson 3-5: Arrays and Partial Products		
4.OA.B.4 MP.8 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In third grade, students used place-value blocks and other tools to make arrays. Developing the Big Idea: In this lesson, students will use place-value blocks and other tools or models to find partial products to multiply a 1-digit whole number by a 3-digit whole number.	Solve & Share: Consider having students estimate before solving the problem. Consider sharing students' strategies who may have used place-value blocks and the partial product algorithm to solve the problem. Both of these student strategies will be discussed in the *Visual Learning Animation*. Visual Learning: The partial products algorithm is discussed in the animation. The partial product algorithm uses place-value understanding to multiply multi-digit whole numbers. Connect the distributive property from previous lessons to the partial product algorithm. Also, note the place-value blocks are a representation of an array. Place-value blocks are used as an array throughout the lesson whether concrete or representational. Convince Me: Ask students to connect the partial products algorithm to place-value blocks (concrete). Have students think about how the place-value blocks can be shown representationally. Having students draw the place-value blocks will connect to the *Another Example!*, which has students draw a model. Another Example: Consider using the *Another Example!* as students use representations to 1,000 as well as partial products to solve the problem. Guided Practice: Consider giving students opportunities to use tools or other representations as they solve items 1-2 using the partial product algorithm to compare. Independent Practice/Math Practice and Problem Solving Consider having students complete item 8 and then discuss whole class. In item 8, students decide what multiplication equation is represented by the place-value blocks. Remind them of the problem in the *Visual Learning Animation* to help guide them with the problem. For item 9, consider having students estimate and discuss overestimation and under-estimation.

Lesson 3-6: Use Partial Products to Multiply By 1-Digit Numbers		
4.NBT.B.5 MP.4 MP.5 MP.7 MP.8	Access Prior Learning: In the previous lesson, students used an array of place-value blocks to find partial products to multiply a 1-digit whole number by a 3- or 4-digit whole number. Developing the Big Idea: In this lesson, students will continue using the partial product algorithm to multiply.	Note: This lesson is similar to Lesson 3-5. Students continue developing their understanding of the partial product algorithm. Solve & Share: Consider having students estimate before solving the problem. Also, consider prompting students to use more than one strategy to answer the *Solve & Share* problem. Convince Me: Read the notes at the bottom of the Teacher's Edition (p. 126) before teaching the lesson. Consider using the *Convince Me!* with all students as they develop an understanding that they can start with any place-value position when using the partial products algorithm. Independent Practice/Math Practices and Problem Solving: For items 4-7, consider having students estimate before finding the actual answer in order to check the reasonableness of their answer. Item 16 is a *Quick Check* item and a multiplicative comparison problem. Consider using this formatively to assess student understanding, with an emphasis on context within the student's explanation. Assess and Differentiate/Intervention Activity: Consider using the questions to facilitate a discussion around the *Intervention Activity* as the activity has students thinking about generalizing place-value understanding when using the partial product algorithm. Consider giving students an opportunity to use place-value blocks or other representations as they solve using the partial product algorithm. *CTC: *Convince Me!* (digital platform or student work samples)
Lesson 3-7: Multiply 2- and 3-Digit Numbers by 1-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In the previous lessons, students learned and applied the partial product algorithm. Beginning of the Big Idea: In this lesson, students will begin to learn the U.S. standard algorithm for multiplication.	Solve & Share: Consider removing "How can you use paper and pencil to find how many books were ordered?" Ask students to estimate and solve the problem any way they choose, even if it is using place-value blocks. Visual Learning: In the *Visual Learning Animation*, students are shown the U.S. traditional algorithm for multiplication. Consider showing or not showing students the algorithm. Do not expect students to be secure in the U.S. traditional algorithm this year, as students will have multiple opportunities in the future to become secure. Consider having students compare the partial product algorithm and the U.S. traditional algorithm. See the *Prevent Misconceptions* for how and where students struggle with U.S. traditional algorithm (TE, p. 132). Convince Me: Consider encouraging students to use multiple strategies or models to solve this problem. Consider recording strategies on an anchor chart to show the comparison. Continue having students estimate to check for reasonableness. Another Example: In the *Another Example!*, students continue comparing the partial products algorithm to the U.S. traditional algorithm. Consider recording strategies on an anchor chart to show the comparison. Continue having students estimate to check for reasonableness. Independent Practice/Math Practices and Problem Solving: Items 15-18 are multiplicative comparison problems. Consider discussing the context of the problems with students and encouraging use of the bar diagram. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students as it brings in the concrete tools to explain the U.S. traditional algorithm more in-depth by using place-value understanding and names to explain the algorithm. It also demonstrates how regrouping works.
Lesson 3-8: Multiply 4-Digit by 1-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.2 MP.4	Access Prior Learning: In the previous lesson, students continued multiplying numbers by using partial products algorithm and the U.S. standard algorithm.	Solve & Share: Consider accepting multiple strategies or models students used to solve problem. Consider having students do a Gallery Walk (ELL Toolkit, p. 22) as well as critique the reasoning of others. *-continues on next page-*

MP.8	Beginning of the Big Idea: In this lesson, students will continue to use algorithms to multiply whole numbers.	Visual Learning: Consider not showing the *Visual Learning Animation* as it shows the U.S. traditional algorithm as a Step-by-Step procedure. Continue to give students the opportunity to use the partial product algorithm as it addresses multiplication through place-value understanding. Convince Me: Consider facilitating a discussion around the *Convince Me!* as it reinforces estimation. Independent Practice/Math Practice and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider using item 26 with whole class, as it asks students to use a bar diagram to solve the problem. This may help support students who are struggling with multiplication and an algorithm. Students see multiplication as repeated addition. Items 18, 23, and 25 involve multi-digit subtraction, so consider using this time for review or for a grade. Assess and Differentiate/Intervention Activity: Consider using the partial product algorithm or arrays to provide additional support for students instead of color-coding place-value in the U.S. traditional algorithm.
Lesson 3-9: Multiply By 1-Digit Numbers		
4.NBT.B.5 4.OA.A.3 MP.2 MP.3 MP.6 MP.8	Access Prior Learning: In Lessons 3-7 and 3-8, students compared the partial product algorithm to the U.S standard algorithm. Developing the Big Idea: In this lesson, students will continue using the algorithms to multiply whole numbers.	Note: Consider having students continue to estimate to check for reasonableness as they progress through the lesson. Solve & Share: Consider sharing and comparing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. Look Back: Consider using the *Look Back!* as students may use their understanding of doubling to answer the problem. Share a student's response who may have used doubling or introduce doubling as a past student's strategy. Visual Learning: Estimation is an important understanding to be developed. Ask students to use estimation as an ongoing strategy to support building place-value understanding. Consider giving students the opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation*. Consider using the first two problems as a Number Talk and have students share how they may have solved the problems mentally. Independent Practice/Math Practice and Problem Solving: Item 31 is a *Quick Check* item and a multi-step multiplicative comparison problem. Consider asking students to explain or model their thinking to assess conceptual understanding of the problem. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students, as this activity emphasizes place-value understanding when multiplying and regrouping.
Lesson 3-10: Math Practices and Problem Solving- Model With Math		
4.NBT.B.5 MP.4 MP.1 MP.2 MP.5 MP.6	Access Prior Learning: In third grade, students modeled and solved addition, subtraction and basic fact multiplication problems by drawing bar diagrams and writing equations. Developing the Big Idea: Students will continue to model with math using bar diagrams as they multiply multi-digit whole numbers.	Solve & Share: The *Solve & Share* is a multiplicative comparison problem. Consider using a bar diagram, but ask students to solve the problem in a different way. For more information regarding multiplication and division problem types, please see the Teacher's Edition (pp. F35-F36) or the Nevada Academic Content Standards (2010, p. 89). Convince Me: Consider facilitating a discussion around the *Convince Me!* as students use derived facts (known facts) to help them estimate and decide if their answer makes sense. Guided Practice: *Guided Practice* items 1-3 are multiplicative comparison problems and are different then the *Visual Learning Animation*, which is a grouping problem. Consider using the *Guided Practice* whole group and compare item 1 to the *Visual Learning* to see if students can see the difference between the two problem types.

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	A > 1 Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	A < 1 Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

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