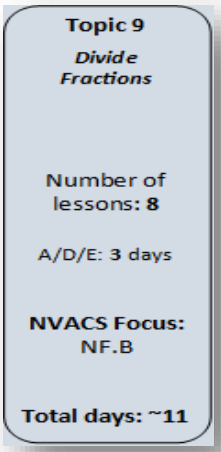
Grade 5 Topic 9: Apply Understanding of Division to Divide Fractions



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 9 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 455A-455F), the Topic Planner (pp. 523A-523C), all 8 lessons, and the Topic Assessments (pp. 581-581A).

Mathematical Background: Read Topics 8-9 Cluster Overview/Math Background (TE, pp. 455A-455F)	Topic Essential Question: How are fractions related to division? How can you divide with whole numbers and unit fractions? *Reference Answering the Topic Essential Questions (TE, pp. 579-580) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

9-1	9-2	9-3	9-4	9-5	9-6	9-7	9-8	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on Nevada Academic Content Standards (NVACS) cluster 5.NF.B; "Apply and extend previous understandings of multiplication and division to multiply and divide fractions" (2010). Topic 9 will specifically target:

- 5.NF.B.3; Interpret a fraction as division of the numerator by the denominator *(a/b) = a ÷ b*. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fractions models or equations to represent the problem (NVACS, 2010).

- 5.NF.B.7a; Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3 (NVACS, 2010)*.

- 5.NF.B.7b; Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4 (NVACS, 2010)*.

- 5.NF.B.7c; Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? (NVACS, 2010).*

Topic 9 moves students from multiplying with fractions to dividing. In 5th grade, the focus is on strengthening students' understanding of division and its connection to multiplication. It is critical that students engage in the actual modeling and/or representation of the division process. Representational models are critical tools for extending understandings related to the division of whole numbers to this new content, division of whole numbers and unit fractions (Fennell, Topic 9 PD video enVisionmath2.0).

In fifth grade, students only work with whole numbers and unit fractions. They will not be asked to divide a fraction by a fraction until sixth grade. This boundary allows students the time to build conceptual understanding before they are asked to apply abstract algorithms. At this point students do not need to learn the U.S traditional algorithm for dividing fractions. Moving students too quickly to that level of abstraction can be detrimental to their learning. Division of fractions is one of the most misunderstood and mysterious algorithms in mathematics. To avoid this mystery, we need to help students really understand when and how to divide fractions (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The importance of teaching this content for understanding rather than focusing on procedures first is reiterated by Francis Fennell, "You may have heard, ours is not to wonder why, just invert and multiply. Our challenge, and our responsibility, is to ensure that all students understand how and why these procedures work. And that is so important" (Topic 9 PD video enVisionmath2.0).

A common misconception is that division will always make numbers smaller (Karp, Bush & Dougherty, 2014). Students tackled a related misconception about multiplication in Topic 8 (see Topic 8 instructional note).

Students need to think about the roles of the dividend and the divisor in relation to context to determine if their answers make sense. They should be able to determine what is being shared and how many equal shares are needed. Modeling the context of a problem can help students decide if their answer is reasonable and understand why.

Math Practice 8: Look for and express regularity in repeated reasoning

Focus on opportunities for students to develop *Mathematical Practice 8* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 9-8. Reference the Teacher's Edition (TE, pp. F28-F28A) and the NVACS (2010, p.8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	unit fraction *dividend* *divisor* *quotient* *inverse operation*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using models to represent division situations containing whole numbers and fractions?"
"Are students using multiplication to estimate and determine if quotients are reasonable?"

Lesson	Evidence	Look for
9-3	*Homework and Practice* (student work samples) Item 13	Focus CTC around the big idea: • ability to connect visual model with a division expression. • use of multiplication to check reasonableness of a quotient.
9-3	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
9-6	*Solve and Share* (student work samples)	Focus CTC around the big idea: • student strategies and models used to represent the context. • use of multiplication to check reasonableness of a quotient.
9-6	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 579-582	Use *Scoring Guide* TE pp. 579-582A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 9-1: Fractions and Division		
5.NF.B.3 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In 4th grade students learned that a multiple of a/b is 1/b (4.NF.B) Students divided whole numbers and decimals to find quotients and remainders in previous topics. Developing the Big Idea: Students build conceptual understanding of division through use of tools, models and procedures to explore dividing two whole numbers where the quotient is a whole number and a fraction.	Solve and Share: A wide range of strategies will be used to solve this problem. Some students will partition every waffle into fourths while others will share 1 whole waffle and only partition the remaining two. Even here, students may partition by halves or fourths. Yet, the share of 1 person is always 6/4. Use students' models and thinking to analyze this pattern and discuss about how division is related to fractions. Draw out the idea that when equal/fair sharing, the total amount is partitioned by the number of shares. Students are building towards the idea that fractions and division are the same, or that $a \div b = a/b$. Students can test their strategies and conjectures using the *Look Back!* problem which eliminates one of the waffles. Visual Learning: An equal/fair sharing problem is modeled using visual representations including a number line. How do visual representations help to solve these problems? Division problems interpreted as fractions are modeled. Can students generalize this idea to create a mathematical rule? Students can test a new model/strategy using the *Convince Me!* problem.

		The *Independent Practice* problems are practice rewriting division problems as fractions and vice versa. Items 18-21 provide context, which can help students make sense of division and its connection to fractions. Assess and Differentiate: The third activity in the *Intervention* Activity uses whole number Partitive division problems to help students think about the relationship between division and fractions. These problem types are an excellent jumping off point for this concept and could be used whole class, in small groups, or to formatively assess student's current understanding. Students can use the context of these equal/fair sharing problems to think about the quantity being shared and the number of shares needed.
Lesson 9-2: Fractions and Mixed Numbers as Quotients		
5.NF.B.3 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students worked with mixed numbers in Topics 7 and 8 and whole number division in Topic 5. Students explored dividing whole numbers when the quotient is a fraction in the previous lesson. Developing the Big Idea: Students continue to build conceptual understanding by dividing whole numbers when the quotient is a whole number and a fraction.	Solve and Share: Look for students using a wide range of strategies and facilitate a discussion using students thinking and explanations. Using visual models will build conceptual understanding since students can see how a whole number is partitioned to create a quotient with a fraction. Some students may solve this problem by setting up a division problem (8 ÷ 5) and then reinterpreting this as a fraction ($\frac{8}{5}$). Can students explain how this model is appropriate for this context? What does an answer of $\frac{8}{5}$. mean in this context? Visual Learning: Students will see a problem modeled with division and then reinterpreted as a fraction. Can students explain why this rule is appropriate? What does a solution with a fraction mean? Students can practice reasoning with this new content using the *Convince Me!* problem which asks for a solution range instead of an exact answer. Students will need to understand how to rename fractions as mixed numbers and fractions greater than one, in order to complete the *Independent Practice* items. Modeling this change with fraction strips will help to build understanding. Assess and Differentiate: Students may need assistance using the model provided on the *Reteach* page as it includes several steps. As an extension, consider having students create a context for questions 2-9 on the *Homework and Practice* page (SE, p. 537).
Lesson 9-3: Use Multiplication to Divide		
5.NF.B.7b 5.NF.B.7c MP.2 MP.4 MP.7	Access Prior Learning: Students have connected multiplication and division in previous grades and topics. Developing the Big Idea: Student build conceptual understanding of division with fractions by connecting to multiplication as a means to check a quotient.	Solve and Share: The context of this problem allows students to use multiplication understandings to model and solve. However, students will find that division is also used, as each sandwich is partitioned into fourths. This means that the quotient will also be partitioned into fourths. Look for students utilizing models and use their strategies to facilitate a class discussion. Draw out the idea and that when you divide a number of things (*a*) into equal parts (*b*), you get *a x b* parts. Therefore, *a ÷ (1/b) = a x b*. Use the *Look Back!* to challenge students to model this problem with an equation. Visual Learning: Students see that when dividing wholes by a fraction, more parts are created because each whole is partitioned. This understanding allows students to use multiplication to solve these types of problems. Challenge students to analyze the patterns visible between multiplication and division and create a rule for dividing a whole number by a fraction. Have students test their rule using the models provided on questions 6-9 on the *Independent Practice* page (SE, p. 541). Item 18 on the *Math Practices and Problem Solving* page can be used to formatively assess students. This item asks students to represent a context using both a multiplication and a division equation. Assess and Differentiate: The *Intervention Activity* uses fraction strips to help students visualize the connection between multiplication and division. These tools can be used to great effect during all parts of this lesson. *CTC: *Homework and Practice* (student work samples) *Item 13* *CTC: *Quick Check* (digital platform)

Lesson 9-4: Divide Whole Numbers by Unit Fractions		
5.NF.B.7b 5.NF.B.7c MP.1 MP.2 MP.4 MP.5	Access Prior Learning: Students divided a whole number by a fraction in the previous lesson. Developing the Big Idea: Students use models and procedures to build a conceptual understanding of division with fractions by connecting to multiplication.	Solve and Share: A grouping problem is given with a context that leads to using circle fractions. Consider providing Teaching Tool 14 (circle fraction models) to encourage use of a visual model to solve this problem. Can the circle models represent a division and multiplication equation? How does the division equation also represent the context of this problem? Visual Learning: A Measurement division problem is modeled with circle fractions and a number line. This type of problem helps students see how fractional chunks can be subtracted or added up to form a larger total. Students see why the quotient becomes larger than the dividend or divisor. Can this problem be solved with multiplication? Students can test their ideas using the *Convince Me!* problem. Use of a number line or visual model is encouraged for solving the *Guided and Independent Practice problems.*
Lesson 9-5: Divide Unit Fractions by Non-Zero Whole Numbers		
5.NF.B.7a 5.NF.B.7c MP.2 MP.3 MP.4 MP.5 MP.8	Access Prior Learning: Students have learned about the meanings of division as well as how fractions can partition wholes in previous grades and topics. Developing the Big Idea: Students use models to build conceptual understanding of dividing a unit fraction by a whole number.	Solve and Share: Ask students to estimate before solving. If 4 people are sharing a half, will the quotient be larger or smaller than $\frac{1}{2}$? Students can use concrete tools and visual models to represent the context of this problem. Facilitate a discussion to compare students' strategies and solutions. What happens to the quotient when a unit fraction is partitioned into equal shares? Can this problem be represented using a multiplication fact? Visual Learning: Dividing a unit fraction by a whole number is shown using an area model (pan of cornbread) and a number line. Help students connect these strategies to using multiplication. The *Convince Me*! can be used to further that discussion. Consider assigning a small number of items from the *Independent Practice* to enable students have time to try various models. Encourage students to connect their models to a related multiplication fact. Assess and Differentiate: Partitioning a rectangular area model is show on *Another Look!* Students should explore using this model, as well as, the number line to solve the *Homework and Practice* problems.
Lesson 9-6: Divide Whole Numbers and Unit Fractions		
5.NF.B.7a 5.NF.B.7b 5.NF.B.7c MP.1 MP.2 MP.4 MP.8	Access Prior Learning: Students divided unit fractions and whole numbers in previous lessons. Developing the Big Idea: Students connect procedures to models and drawings to strengthen conceptual knowledge and build procedural skill with dividing unit fractions and whole numbers.	Solve and Share: Look for students using visual models and numerical expressions to represent this problem. Facilitate a discussion to help students connect the different representations. Are students using representations to make sense of the problem before calculating? How can a numerical expression and a visual model represent the same context mathematically? Visual Learning: The *Visual Learning Bridge* demonstrates use of an area model and a number line. How is this similar or different to strategies shared by students during the *Solve and Share*? Help students to make connections between the models and the numerical expression representing the problem. Can students generalize about the size of a quotient based on whether the dividend is whole number or a fraction? Assess and Differentiate: On the *Homework and Practice* pages, students practice using models to solve problems with unit fractions divided by a whole number and a whole number divided by a unit fraction. *CTC: ***Solve and Share*** (student work samples) *CTC: ***Quick Check*** (digital platform)
Lesson 9-7: Solve Problems Using Division		
5.NF.B.7c MP.1 MP.2 MP.4 MP.6	Access Prior Learning: Students divided unit fractions and whole numbers in previous lessons. Securing the Big Idea: Students apply knowledge of dividing unit fractions and whole numbers to solve word problems.	Solve and Share: Students should work to model and create a plan before solving this multi-step problem. Share student strategies and explanations to draw out ideas about how good problem solvers make sense of a problem. Visual Learning: Students might need additional time and strategic questioning support to help them make sense of this problem. Are students able to create a model of their own that matches the context of this problem? *-continues on next page-*

		The remainder is reported as R6 in the Visual Learning Bridge. Students in 5th grade will understand that a remainder is actually a fractional piece. Can they write this remainder correctly as a fractional piece? Watch for students who represent all fractions as tenths. They might have misconceptions of fractional pieces due to over generalizing the base-ten system. Assess and Differentiate: The *Reteach* page breaks a word problem into smaller pieces. It may be difficult for students to complete this independently because the context of the problem can be lost. Consider skipping the reteach section and instead assigning a single (or small number) of *Homework and Practice* items so that students have time to explain their strategies and solutions. Writing their thinking and sharing can help to strengthen mathematical reasoning.
Lesson 9-8: Math Practices and Problem Solving- Repeated Reasoning		
5.NF.B.7a 5.NF.B.7b 5.NF.B.7c MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning: Students have practiced the thinking habits of MP.8 in previous topics. Students have divided unit fractions and whole numbers in previous lessons. Securing the Big Idea: Students apply conceptual understanding of division and multiplication to analyze equations and generalize a mathematical rule.	Solve and Share: Students look for patterns in the computations when dividing a unit fraction by a whole number and then dividing the whole number by the same unit fraction. Have students create and test mathematical conjectures leading to a rule, using division and multiplication understandings. What do students discover about the relationship between these computations? Can these ideas also be connected to multiplication? Visual Learning: The discussion started in the *Solve and Share* is carried into the *Visual Learning Bridge*. Hold off showing the last part of the video until students have had a chance to make their own generalizations about the patterns. The relationship between dividing and multiplying whole numbers and unit fractions is explained and shown numerically. The *Guided Practice, Independent Practice*, and *Math Practices* and *Problem Solving* pages ask students to apply the relationships modeled in the *Visual Learning Bridge* and discovered in the *Solve and Share*. Assess and Differentiate: *Another Look!* shows equations with division and multiplication facts. Students are asked to make a generalization about each set. Consider using this table earlier in the lesson to help students connect multiplication to division as part of their generalizations.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Common Core State Standards Writing Team. (2015). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Numbers in Operations Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Karp, K., Bush, S., & Dougherty, B. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8.* (Vol. 2). New York, NY: Pearson.