▶ Grade 3 Topic 4: Use Multiplication to Divide-Division Facts

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 4 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 105A-105F), the Topic Planner (pp.165I-165D), all 9 lessons, and the Topic Performance Assessment (pp. 233-234A).

Mathematical	Topic Essential Question:
Background:	How can unknown division facts be found using known multiplication
Read Topic 4 Cluster	facts?
Overview/Math	
Background (TE, pp.	Reference Answering the Topic Essential Question (TE, pp. 229-230) for key elements
105A-105F)	of answers to the Essential Question.

The lesson map for this topic is as follows:

		r j		,					
4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	4-9	Assessment
3 A/D/E days used strategically throughout the topic									

Instructional note:

This topic focuses on the inverse relationship of multiplication and division and using multiplication to solve division problems. These understandings meet the 2010 Nevada Academic Content Standards (NVACS) 3.OA.B6, "Understand division as an unknown-factor problem. *For example, find 32* \div 8 by finding the number that makes 32 when multiplied by 8." Students that understand this inverse relationship realize that they already know division facts because they know the multiplication facts.

Although students explored the concept of division in Topic 1 as fair sharing and repeated subtraction, they have not yet fully explored the relationship between multiplication and division. Students are familiar with using models such as arrays and bar diagrams to represent multiplication situations. Now they will be asked to use these models to explore division situations. For example, given a visual model for the problem $24 \div 6$, "Where is the 24 represented in the model? Where is the 6 represented in the model? Where would the unknown be represented in the model?" Facilitate discussions helping students draw connections between the models, multiplication and division equations, and the inverse relationship between multiplication and division.

As a reminder from Topic 1, there are 2 different types of division problems:

Partitive (dealing or fair sharing): Number of groups are known; the size of each group is unknown

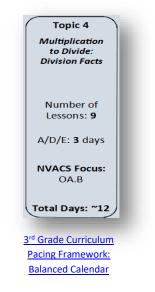
Measurement (chunking): Size of the group is known; the number of groups are unknown

Focus Math Practice 1: Make sense of problems and persevere

Focus on opportunities for students to develop Mathematical Practice 1 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 4-9. Reference the Teacher's Edition (TE, pp. F21 - F21A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.		
New Academic Vocabulary:	Review Academic Vocabulary:	
(First time explicitly taught)	(Vocabulary explicitly taught in prior grades or topics)	
dividend	even	
divisor	odd	
fact family	multiple	
quotient	factors	
	division	
	multiplication	

Additional terminology that students may need support with: related fact, inverse relationship, opposite



*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students able to use the relationship between multiplication and division to find unknown facts?"

Lesson	Evidence	Look for
4-1	<i>Quick Check</i> (digital platform)	 Focus CTC around data analysis and collection of student workspace (scratch paper). students understand that multiplication and division are inverse operations. students use the inverse operation to determine fact families.
4-7	Solve & Share (student work samples)	 Printable version available under "Teacher Resources". Focus CTC around the big idea: students understand that multiplication and division are inverse operations. students use the inverse operations and known facts to determine fact families and their equations.

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 229-234A
Assessments (summative)	SE pp. 229-234	

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 4-1: R	Relate Multiplication to Division	
3.OA.B.6 3.OA.A.3	Access Prior Learning: In the previous topics 1-3, students learned the meaning of multiplication and multiplication facts as well as strategies for	Topic Opener: Introduce the <i>Topic Essential Question</i> , "How can unknown division facts be found using known multiplication facts?" (TE, p. 165). Reviewing the anchor chart from Topic 3 and adding new ideas from this topic will help students to see the development of concepts and make connections.
MP.1	solving unknown multiplication	You might also consider having students complete the Daview What You Knownfor to
MP.2	facts.	You might also consider having students complete the <i>Review What You Know</i> prior to beginning instruction on Topic 4 so that you can respond to student instructional needs using
MP.4		the Item Analysis for Diagnosis and Intervention (TE, p. 166-168).
MP.5 MP.7	Beginning of the Big Idea: Students are <i>beginning</i> to understand that multiplication and	Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson.
MP.8	understand that multiplication and division have an inverse relationship and the use of multiplication facts can help them divide numbers.	Solve & Share: During the <i>Solve & Share</i> , consider asking students, "What do we know in the model (e.g. number of rows and columns)?" "What are we solving for (total number in the array)?" "What operations could we use?"
		If possible, consider having a student that has noticed the multiplication and division equations use the same numbers share their solution and reasoning. Students might identify that this shows they are related facts or fact families. The <i>Visual Learning Animation</i> will explain the idea of fact family's and the inverse relationships they show.
		During the whole group discussion of student solution strategies and reasoning is a good time to introduce vocabulary. Connect students' informal language to formal mathematical terms.
		Visual Learning: Consider pausing the video after it asks, "How does this array show multiplication?" so that you can discuss this whole group. Depending upon the ideas shared during your classroom discussion of the <i>Solve & Share</i> you might also consider pausing the video after it asks, "How does this array show division?"
		The <i>Visual Learning Animation</i> will ask students, "What is the same about all of the equations?" and, "What is different?" after displaying a fact family. You might consider asking the same questions when doing the <i>Try It!</i> to begin to develop readiness for ideas regarding fact families and the inverse relationships they model.
		-continues on next page-

Lesson 4-2: U 3.OA.B.6 3.OA.A.3 MP.1 MP.2 MP.3 MP.4 MP.6 MP.7	Ise Multiplication to Divide with 2 Access Prior Learning: In lesson 4-1, students began to develop understanding of the inverse relationship between multiplication and division and the resulting fact families. Beginning of the Big Idea: Students <i>begin</i> to understand that the inverse relationship between multiplication and division can be used to solve for division with a divisor of 2 through 5. Students are also <i>beginning</i> to develop the understanding that every multiplication fact has a related division fact because of their inverse relationship.	 Independent Practice/Math Practices and Problem Solving: Consider assigning item 18 as this problem provides a formative assessment opportunity to check for students' understanding of fact families. Students will often think that any 3 numbers can be put together to make a fact family. Lesson 4-2 will revisit ideas involved with fact families should students demonstrate that they are still struggling. Assess and Differentiate: If time permits, teach students how to play <i>Teamwork</i> (TE, p. 173A). Before assigning any students the <i>Advanced</i> level activity consider asking students to play the <i>On-Level</i> with the modification that students are to create the array and show the related division fact. All students should have the opportunity to play this game. *CTC: <i>Quick Check</i> (digital platform) 2, 3, 4, and 5 Solve & Share: Since students are just beginning to develop an understanding that we can use multiplication to solve for division, consider asking what strategies and tools they might use to solve this to ensure that all students have an entry point to this problem. Visual Learning: Consider pausing the <i>Visual Learning Animation</i> after they ask, "Why are you able to use multiplication to help you divide?" (00:21) to get student responses. This will provide you with formative assessment data about whether students are understanding the inverse relationship between these two operations. The <i>Visual Learning Animation</i> asks the question, "What is the division sentence?" for the first problem while showing the division sentence. You may wish to pause the video after they display the multiplication so they can have more opportunity to reason with division as an unknown factor problem while receiving immediate feedback through the video. Also consider pausing the video after they introduce Dee's sticker problem (01:33) because the problems allow students to use edualing in
		Students have mostly worked with Partitive division (fair share) types so far. See the <i>Instructional Note</i> for more information regarding Measurement division. Assess and Differentiate: If time permits, you may consider having students play the <i>Teamwork</i> game from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the <i>Teamwork</i> game from lesson 4-1 before playing the <i>Teamwork</i> game provided in this lesson (TE, p. 179A). Child-watch to identify students who need additional support and pull them into a small group to de the <i>Instruction Article</i> (TE, p. 1704).
Lesson 4-3.1	Ise Multiplication to Divide with 6	do the Intervention Activity (TE, p.179A).
L033011 - 3. U	Access Prior Learning:	Solve & Share:
3.OA.B.6 3.OA.A.3 MP.1 MP.2 MP.4	Access Prior Learning: Students have developed the understanding that inverse relationship between multiplication and division means we can use multiplication to solve for division facts. In previous lessons in this topic, they have used this understanding to solve for division facts that have a divisor of 2 through 5. In Topic 3, Grade 3 students developed strategies for solving multiplication facts with 6 and 7 as a factor.	Solve & Share: To assess students' readiness to use known multiplication facts with 6s or 7s as factors to solve corresponding division facts, consider asking students, "How would you explain how to solve 7 x 3 to a friend that didn't know how?" This question can activate prior learning that students will be extending to new learning. By making this question a journal response that you collect, it can also provide meaningful formative assessment data on what phase students are working in to solve for 7's facts. Students that use repeated addition or skip counting are still in Phase 1, students that used a derived fact (e.g. (2 x 7) + 7) are in Phase 2, while students that just know it and would tell them it's 21 are in Phase 3. While Phase 3 is not expected at this time, all students progress at different times and speeds. Look Back: Assigning the <i>Look Back!</i> will be beneficial to students still struggling to understand the inverse relationship between multiplication and division. <i>-continues on next page-</i>

	Developing the Big Idea: Students <i>begin</i> to understand using known multiplication facts to solve for a corresponding division fact by dividing by 6 or 7.	 Visual Learning: Consider pausing the animation at 32 seconds after it asks, "What operation should be used to solve this problem?" Accept both multiplication and division solutions with justifications. Students that respond with multiplication should also identify the inverse relationship with division. This is an opportunity to reinforce that multiplication situations <i>are joining equal sized groups to find a total amount</i>, division situations <i>are separating a total amount into equal sized groups</i>, and to clarify any misconceptions about the two operations. Convince Me: It may be beneficial to assign the <i>Convince Me!</i> to continue to support conceptual development of the inverse relationship between multiplication and division. Assess and Differentiate: If time permits, you may consider replacing the <i>Problem-Solving Reading Mat</i> with the game <i>Teamwork</i> from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the <i>Teamwork</i> game from lesson 4-1. Additional game options include the game <i>Teamwork</i> from lesson 4-2 (TE, p. 179A), or the <i>Fluency Practice Activity</i> (TE, p. 223). Child-watch to identify students who need additional support and pull them into a small group to
		do the Intervention Activity (TE, p. 185A).
Lesson 4-4: L	Ise Multiplication to Divide with 8	
3.OA.B.6 3.OA.A.3	Access Prior Learning: Students have developed the understanding of the inverse relationship between multiplication	Solve & Share: The Visual Learning Animation uses a bar diagram to represent the division problem. If possible, ask a student that uses a bar diagram to accurately model the situation to share their solution strategy and reasoning during the Solve & Share.
MP.2 MP.3	and division. In previous lessons in this topic, they have used this understanding to solve for division	Independent Practice/Math Practices and Problem Solving: Consider assigning item 21 as it reviews place-value concepts.
MP.4 MP.7	facts that have a divisor of 2 through 7.	Assess and Differentiate: If time permits, teach students that have already had the opportunity to play the modified version of <i>Teamwork</i> from lesson 4-1 (TE, p. 173A) to play <i>Display the Digits</i> (TE, p. 191A).
	Developing the Big Idea: Students further <i>develop</i> their understanding of using known multiplication facts to solve for a corresponding division fact by dividing by 8 or 9.	Child-watch to identify students who need additional support and pull them into a small group to do the <i>Intervention Activity</i> (TE, p. 191A).
Lesson 4-5: M	Iultiplication Patterns: Even and	Odd Numbers
3.OA.D.9 3.OA.A.3 MP.1 MP.2	Access Prior Learning: In Grade 2, students learned about patterns with even and odd numbers. Developing the Big Idea: Students further <i>develop</i> their understanding of patterns for even	Solve & Share: While this lesson's focus is on finding multiplication patterns for even and odd factors, the <i>Solve & Share</i> provides the opportunity to address a common division misconception that numbers can only be divided if they can be separated into equal groups. Ask students how they figured out that 15 would not work. Watch for students that say they could not divide because 2 can't go into 15. In these cases, consider asking students to make groups of 2 from 15 counters to show that we can divide, but sometimes we will have pieces that don't fit into an equal sized group. Moving beyond this explanation goes into 4 th grade standards with remainders.
MP.3 MP.4 MP.7 MP.8	and odd numbers by generalizing that all even numbers are multiples of 2. Students will also continue to <i>develop</i> their understanding of the inverse relationship between	Also, watch for students that are able to identify that cars, boats and books can be packaged and ask how they figured it out. Students that have an understanding of even and odd numbers will be able to explain that these numbers can be paired with nothing left over. To connect to today's lesson, ask which factor means that we have groups of pairs (or doubles) (e.g. 2)2 Students that give the response that they knew because the numbers end with 6 and 8
	multiplication and division.	 (e.g. 2)? Students that give the response that they knew because the numbers end with 6 and 8 and those are even numbers may be working with a memorized rule as opposed to understanding. For these students, consider providing them with counters and asking them to prove their reasoning with a direct model using counters. Independent Practice/Math Practices and Problem Solving: If students still seem to be struggling, consider assigning item 9 <i>Critique Reasoning</i> as partner or group work and then discussing as a whole group.
		-continues on next page-

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		Assess and Differentiate: If time permits, you may consider replacing the <i>Math and Science Activity</i> with the game <i>Teamwork</i> from lesson 4-1 (TE, p. 173A). It is recommended that all students have the opportunity to play the modified version of the <i>Teamwork</i> game from lesson 4-1. Additional game options include the game <i>Teamwork</i> from lesson 4-2 (TE, p. 179A), <i>Display the Digits</i> (TE p. 191A), or the <i>Fluency Practice Activity</i> (TE, p. 223).
		Child-watch to identify students who need additional support and pull them into a small group to do the <i>Intervention Activity</i> (TE, p. 191A).
Lesson 4-6: D	ivision Involving 0 and 1	
3.OA.B.5 3.OA.B.6 3.OA.A.3 MP.2 MP.3 MP.6 MP.7	Access Prior Learning: In Topic 2, Grade 3 students developed an understanding of the Zero Property and Identity (One) Property of Multiplication. Students have also been <i>developing</i> an understanding of the inverse relationship between multiplication and division. Developing the Big Idea: Students further <i>develop</i> their understanding of the inverse relationship between multiplication and division to explain the division properties and patterns for division with 0 and 1.	 Instructional note: Students use their understanding of the Zero Property of Multiplication to explain why we cannot have 0 as a divisor. Given the problem 0 ÷ 8, which means if we start with 0 and divide it into 8 equal groups you will have 0 in each group. The <i>Visual Learning Bridge</i> (TE, p.200, Box D) explains why 0 cannot be used as a divisor. Solve & Share: To assess students' readiness to apply the Zero Property and the Identity Property of Multiplication to division ask students to provide an example of each property. As students share their strategies and reasoning for the <i>Solve & Share</i>, consider creating a class poster of student conjectures for division with 0 and 1 (these conjectures are stated in <i>Transition to the Visual Learning Bridge</i> (TE, p. 199). Use the poster to confirm, correct or clarify ideas presented in the <i>Visual Learning Animation</i>. The <i>Solve & Share</i> does not provide a prompt for dividing by 0. Consider asking students why there isn't a problem where 5 is divided by 0 and using the inverse operation reasoning to explain the last conjecture stated, "0 cannot be a divisor" (TE, p. 199). Look Back: Consider assigning the <i>Look Back!</i> as it asks students to use multiplication to justify the conjectures for division with 0.
		Assess and Differentiate: If time permits, teach students that have already had the opportunity to play the modified version of <i>Teamwork</i> from lesson 4-1 (TE, p. 173A) to play <i>Think Together</i> (TE, p. 203A). Child-watch to identify students who need additional support and pull them into a small group to do the <i>Intervention Activity</i> (TE, p.203A).
Lesson 4-7: P	ractice Multiplication and Division	
3.0A.B.6 3.0A.A.3 3.0A.A.4	Access Prior Learning: In previous lessons in Topic 4, students have used the inverse relationship between multiplication and division to solve division problems with the related	Solve & Share: When asking students, "What do you need to find (TE, p. 205)?" make sure that students realize they first need to find the number of people in each tour group and then how much 1 tour group will pay in entrance fees. Consider using Teaching Tool 1 Problem Solving Record Sheet (found in the back of the <i>Teacher Resource Volume 2</i> book) to help students organize their thinking as they work through this 2-step problem.
MP.1 MP.2 MP.3 MP.4	 multiplication fact. Developing the Big Idea: In this lesson, students are developing their understanding of the inverse relationship between multiplication and division and using a related multiplication fact to divide. Students are also developing understanding of using patterns and known facts to find unknown multiplication facts. 	 Independent Practice/Math Practices and Problem Solving: Items 14-16 show division as the image to the right. Students have not seen division written this way and do not need to solve in this form, but it is good to expose them to conventions for division. Consider discussing this as another way to write a division problem. Assess & Differentiate: If time permits, teach students that have already had the opportunity to play the modified version of <i>Teamwork</i> from lesson 4-1 (TE, p. 173A) to play <i>Tic Tac Toe</i> (TE, p. 209A). Child-watch to identify students who need additional support and pull them into a small group to do the <i>Intervention Activity</i> (TE, p.209A). *CTC: Solve & Share (student work samples)

Lesson 1-8.	Solve Multiplication and Division	Faultions
LC33011 4-0. C	Access Prior Learning:	Solve & Share:
3.OA.A.4 3.OA.A.3 MP.1 MP.2 MP.3 MP.6 MP.8	Access Filor Learning. In previous lessons, students have been developing reasoning strategies for solving multiplication and division facts. Developing the Big Idea: In this lesson, students further <i>develop</i> their understanding of strategies for solving multiplication and division facts to solve for an unknown value in an equation.	To assess student readiness, consider asking students what the equal sign means prior to introducing the <i>Solve & Share</i> . Clarify misconceptions that the equal sign is a symbol for writing where the answer goes by informing students that the equal sign communicates a relationship of equivalence or that what is on one side of the equal sign is the same as what's on the other side. Consider also writing the equation $10 = 7 + 3$ on the board and asking if this equation is correct and clarify ideas as needed. Finally consider writing $6 + 4 = 7 + 3$ on the board and again ask if it is correct and clarify as needed. Visual Learning: Consider pausing the <i>Visual Learning</i> Animation after they ask, "What makes these equations different from other equations you have seen before?" and discuss as a whole group. If these are similar to any students' solutions to the <i>Solve & Share</i> consider asking, "How are these similar to's equations that she/he used to solve the <i>Solve & Share</i> ?"
		Independent Practice/Math Practices and Problem Solving: For item 17 <i>Common Core Assessment</i> accept responses where students write $18 \div 6 = ?$ for Part A.
		Assess & Differentiate: If time permits, you may consider replacing the <i>Math and Science Activity</i> (TE, p. 215A) with the modified version of "Teamwork" from lesson 4-1 (TE, p. 173A) as it is recommended that all students have the opportunity to play this game. Additional game options include the game <i>Teamwork</i> from lesson 4-2 (TE, p. 179A), <i>Display the Digits</i> (TE, p. 191A), <i>Tic Tac Toe</i> (TE, p. 209A), or the <i>Fluency Practice Activity</i> (TE, p. 223).
		Child-watch to identify students who need additional support and place them into a small group to do the <i>Intervention Activity</i> (TE, p.215A).
Lesson 4-9: N	Math Practices and Problem Solv	ing- Make Sense and Persevere
3.OA.A.3 3.OA.D.8 MP.1	Access Prior Learning: In previous lessons, students have solved 2-step questions that involved addition, subtraction, and multiplication by thinking of the hidden question.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 1. Refer to the <i>Math Practices and Problem Solving Handbook</i> (TE, pp. F21-F21A, p. F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F21). Solve & Share:
MP.2 MP.3 MP.6 MP.8	Developing the Big Idea: Students are further <i>developing</i> strategies for making sense of	Consider reintroducing MP. 1 Thinking Habits (SE, p. F21) before introducing the <i>Solve & Share</i> . Also consider using the time where students are working on the <i>Solve & Share</i> as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the <i>Math Practices and Problem Solving Handbook</i> (p. F21A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice.
IVIF .O	problems and persevering when they get stuck.	Watch for students that try to solve the problem by using keyword strategies rather than reasoning with the context. Watch the <i>Listen and Look For</i> video prior to the lesson. Use the video to identify examples of how students might approach this problem and ways to redirect.
		Convince Me: Consider assigning the <i>Convince Me!</i> as it offers another opportunity to work with MP.1 and assess for behaviors attributed to this math practice. It also offers an opportunity to formatively assess students' understanding of the inverse relationship between multiplication division and how mathematicians can use inverse operations to check work.
		Assess & Differentiate: If time permits, you may consider replacing the <i>Problem Solving Reading Mat</i> (TE, p. 221A) with the modified version of <i>Teamwork</i> from lesson 4-1 (TE, p. 173A) as it is recommended that all students have the opportunity to play this game. Additional game options include the game <i>Teamwork</i> from lesson 4-2 (TE, p. 179A), <i>Display the Digits</i> (TE, p. 191A), <i>Tic Tac Toe</i> (TE, p. 209A), or the <i>Fluency Practice Activity</i> (TE, p. 223).
		Child-watch to identify students who need additional support and pull them into a small group to do the <i>Intervention Activity</i> (TE, p.221A).

References

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