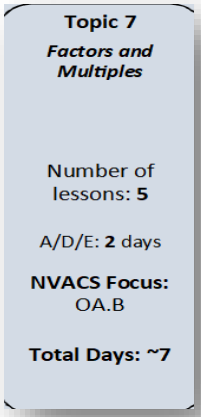
Grade 4 Topic 7: Factors and Multiples



Big Conceptual Idea: Operations and Algebraic Thinking (pp. 29-31)
Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 365A-365F), the Topic Planner (pp. 365I-365J), all 5 lessons, and the Topic Assessments (pp. 405-406A).

Mathematical Background: Read Cluster Overview (TE, pp. 365A-365F)	Topic Essential Questions: How can you use arrays or multiplication to find the factors of a number? How can you identify prime and composite numbers? How can you find multiples of a number? *Reference TE p. 365 and Answering the Topic Essential Questions (TE, pp. 403-404) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

7-1	7-2	7-3	7-4	7-5	Assessment

2 A/D/E days used strategically throughout the topic.

4th grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:
Topic 7 was moved before Topic 3 as it develops student understanding of factors and multiples by going back to accessing basic understanding of multiplication and division. The WCSD curriculum committee felt that accessing this understanding prior to developing understanding of multi-digit multiplication and division is foundational to support students' application of strategies (WCSD 4th Grade Curriculum Pacing Framework, p. 4).

The focus of this topic is on factors and multiples, as well as whether a whole number is prime or composite. Nevada Academic Content Standards (NVACS) 4.OA.B.4, state that students should begin to "gain familiarity with factors and multiples" (2010). Reys, Linquist, Lamdin, Smith, & Suydam state, "each of two numbers that are multiplied together to give a product is called a factor of that product…in looking at multiples, we usually begin with a number and generate multiples of the number" (2001, p. 288). Karen Karp states, "factors of a number can be found in pairs by thinking about multiplication…the product of any non-zero whole number and a given non-zero whole number is a multiple of both. Factors and multiples are closely related" (**enVision**math2.0 PD Video). Multiples are products of any whole number.

Confusion for students may arise between factors and multiples. According to Reys, et al., the confusion happens "when teachers ask 'What is 36 a multiple of?' Teachers are actually asking children to find the factors of 36. Or 'What is 4 a factor of?' teachers are actually asking children to find the multiples of 4" (2001, p. 288).

Children need to see and understand the relationship between factors and multiples, and be able to think or move between the two concepts. Consider using a Venn diagram to facilitate this relationship between the two concepts. For example, 4 is a factor of 8 and 8 is a multiple of 4. Using a Venn diagram is also a useful organizer for students to begin to develop an understanding of common multiples between numbers. This commonality will support students when they begin to find common denominators when adding and subtracting unlike fractions in later grades.

One essential understanding about the relationship between factors and multiples is "whole numbers have a **distinct set** of whole number factors, but a whole number has an **infinite** number of whole number multiples" (Karen Karp, **enVision**math2.0, Topic 7 PD video).

Students will use their prior knowledge of factors to determine prime whole numbers and composite whole numbers. Prime numbers are whole numbers greater than one (1) that have exactly two factors; one and itself. Reys, et al., describe exploration that focuses on primes:

- twin primes (pairs of primes that are two apart, such as 11 and 13)
- reversal primes (pairs of primes, such as 79 and 97)
- or infinitude of primes
  (2001, p. 289).

This exploration will be important in Lesson 7-4 when students determine which numbers are prime or composite. Composite numbers are whole numbers greater than 1 with two or more factors. The emphasis on **whole numbers greater than 1**, for both prime and composite numbers will be imperative for students. Students will wonder about the numbers 0 and 1 being prime or composite, but the emphasis on **whole numbers greater than one** will help students understand that **0 and 1 are neither prime nor composite**.

Focus Math Practice 8: Repeated reasoning
Focus opportunities for students to develop *Mathematical Practice 8* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 7-3. Reference the Teacher's Edition (pp. F28-F28A) and the NVACS (2010, p. 8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
factor pairs multiples generalize prime number composite	*factor*

Additional terminology that students may need support with: arrays, arrange, groups of, rows, columns, repeated factor pair, Commutative Property of Multiplication

*Consider using the additional terminology to label anchor charts used throughout this topic.

*Collaborative Team Conversations (CTC)
Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "What tools, strategies or models are students using to find all factors of a whole number? What tools, strategies or models are students using to find multiples of a whole number?

Lesson	Evidence	Look for
7-3	*Independent Practice* (digital platform or student work samples) Items 3, 4 and 5	Focus CTC around the big idea: • students recognize and generate patterns in factoring whole numbers.
7-5	*Quick Check (*digital platform*)*	Focus CTC around the big idea: • students recognize and generate patterns in finding an infinite amount of multiples.

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 403-406	Use *Scoring Guide* TE pp. 403-406A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 7-1: Understand Factors		
4.OA.B.4 MP.2 MP.3 MP.5 MP.7	Access Prior Learning: In third grade (3.OA.A.1), students learned to interpret products of whole numbers as the total number of objects in *n* groups of *n* objects. Students also learned to use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, etc. (3.OA.A.3). Developing the Big Idea: Students will continue to develop their understanding of multiplication by finding factor pairs for whole numbers in the range of 1-100.	Solve & Share: Consider giving students the opportunity to use tools other than graph paper. Students may use 2-colored counters, place-value blocks or other tools that may be put into rows and columns using 24 total objects. Visual Learning: In the video, students find all the factors for 12. Consider having students use tools during the video to find the factor pairs for 12, or have students determine the factor pairs for 12 before showing the video. Guided Practice: Consider using the *Guided Practice* items 1 & 2 whole group, as factors are discussed as the dimensions of a rectangular array. These items continue to develop an understanding of area. Also, consider doing item 4 whole group as it addresses square numbers. See item 4 at the bottom of the Teacher's Edition (pp. 371-372) *-continues on next page-*

		Independent Practice/Math Practices and Problem Solving: After students work on *Quick Check* items (pink check marks), facilitate a discussion around item 15 whole class. This item continues the focus on factors and begins to develop an understanding of divisibility (when a number ends in a 0 or 5, it can divide by 5 evenly or 5 is a factor of that number). Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with students who may need extra support with finding factors of numbers. Continue to encourage the use of tools for students or use Teaching Tools 9 and/or 16.
Lesson 7-2: Factors		
4.OA.B.4 MP.1 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In the previous lesson, students learned the meaning of factors and how to find factors of a number. Developing the Big Idea: In this lesson, students will continue their work with factors and factor pairs.	Solve & Share: Consider giving students an opportunity to use multiple tools or representations to complete the *Solve & Share*. Consider removing the grid, to enable students to use tools. Suggested tools include two-colored counters, place-value blocks, grid paper or other manipulatives with 20 objects. When students find factor pairs for 20, they may have more than 3 ways. Some students may have 4 by 5 and a 5 by 4. When having students to share their answers, have students share who may have more than 3. Ask students, "Does a 4 by 5 rectangle and a 5 by 4 rectangle have the same area?" You may consider discussing the Commutative Property of Multiplication (students learned the Commutative Property in third grade). The Commutative Property of Multiplication will be discussed further in Topic 3. Look Back: In the *Look Back!*, students determine how they know if they have found all factor pairs. Consider having a whole group discussion around this idea. Visual Learning: Factor pairs and multiples are discussed in the *Visual Learning Animation*. Connect these mathematical terms to what students did in the *Solve & Share*. Convince Me: Consider using the *Convince Me!* as part of the formative assessment process to see if students understand how to find factor pairs and when they know all factor pairs have been found. If students have an understanding of this concept, consider having students complete selected *Independent Practice/Math Practices and Problem Solving* items. If students are still struggling, consider using the *Guided Practice* items with these students. Guided Practice: Consider using item 2 as a whole class discussion, as students are asked to find a factor in all even numbers. This item addresses some divisibility understanding for students (all even numbers are divisible by 2). Watch for students who invent this concept on their own. Independent Practice/Math Practices and Problem Solving: Consider using item 26 whole group, as students solve a multiplicative comparison problem with "3 times" as many. Multiplicative comparison will be further discussed in Topic 6, but this item will give students some exposure to this mathematical idea. Assess and Differentiate/Intervention Activity: If students are still struggling with finding factor pairs after two lessons on factors, consider using the *Intervention Activity* or *Reteach* page with students. Again, encourage tool use.
Lesson 7-3: Math Practices and Problem Solving- Repeated Reasoning		
4.OA.B.4 4.NBT.B.5 MP.8 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In previous lessons, students have worked with factors, factor pairs and multiples. Students have also worked with repeated reasoning in previous grades, topics and lessons. Developing the Big Idea: In this lesson, students will continue working with factors, factor pairs and multiples by making generalizations from repeated reasoning.	Note: Most *Math Practices and Problem Solving* lessons are at the end of each topic, but in this topic the *Math Practice and Problem Solving* lesson is in the middle of the topic. Students will develop a generalization when all factor pairs start repeating, all factors have been found for that number. Solve & Share: Consider removing the table with the example and non-example of a rectangular array to formally assess students' prior knowledge with arrays. Give students an opportunity to use concrete tools to arrange 24 objects into a rectangular array. Look for students who may have used multiplication or division to find all the factors and factor pairs for 24. Look for students who may have connected the commutative property to the array arrangement. Students may not recall the name of the property, but may have examples of the commutative property. Consider choosing these students to share. *-continues on next page-*

		Visual Learning: The *Visual Learning Animation* has students making generalizations related to factor pairs and factors. Consider making an anchor chart with ways students can generalize repeated reasoning. Convince Me: The *Convince Me!* has students use a diagram to justify that all factor pairs have been found for the number 24. Consider having students come up with their own way to ensure all factor pairs have been found for 24. Students can generalize all factor pairs have been found when they begin to see a reversal in the pairs. Consider using the *Convince Me!* formatively to assess student understanding. If students seem to have an understanding of factors and factor pairs, have students begin the *Guided Practice* and *Independent Practice/Math Practice and Problem Solving* on their own. *CTC: *Independent Practice items 3-5* (digital platform or student work samples)
Lesson 7-4: Prime and Composite Numbers		
4.OA.B.4 MP.8 MP.2 MP.3 MP.7	Access Prior Learning: Students worked with factors and factor pairs in lessons 7-1, 7-2 and 7-3. Beginning the Big Idea: In this lesson, students will use their understanding of factors and factor pairs to decide which numbers are prime or composite based on how many factors or factor pairs make up the number.	Solve & Share: The idea of the *Solve & Share* is for students to use each color individually. For example, students will use the red color tiles to make arrays of 2 x 1, 1 x 2. Consider using an anchor chart as students share all the possible arrays (factors and factor pairs) and label the different arrays. In the example, the red color tiles are labeled 2 x 1, 1 x 2. Support students in connecting the reasoning that some of the colored tiles have only two arrays, while some colored tiles have multiple arrays. This begins to support the mathematical concept of prime and composite whole numbers. Visual Learning: *Prevent Misconceptions* addresses the numbers 0 and 1 (TE, p. 388). Zero (0) and 1 are neither prime nor composite. You may consider having students determine if these numbers are prime or composite after the discussion around the *Visual Learning* has taken place. Students need to understand that a prime number is a whole number greater than 1 that has exactly two factors, 1 and itself. A composite number is a whole number greater than 1 that has more than 2 factors. This discussion will help students on item 25 of *Math Practices and Problem Solving*, which is a *Quick Check* item. Convince Me: Consider using the *Convince Me!* formatively to determine if students have developed an understanding of prime and composite numbers. Do students understand that prime and composite numbers are different? Are they able to explain why a number cannot be prime nor composite? Independent Practice/Math Practices and Problem Solving: Items 20 and 21 in *Math Practices and Problem Solving* ask students to use the graph to solve these items. As a consideration, ask students to solve and then discuss as a whole group how to use information in a graph to solve problems. Consider using item 17 of the *Homework & Practice* pages during class instruction. You may consider having students work in small groups or partners on this activity. Prompt and support students to discover the patterns on the Number Chart for prime and composite numbers. Discuss whole group after students have had some opportunity to discover any patterns. Consider providing students a pre-printed number chart to 100 if time is a concern. Assess and Differentiate/Intervention Activity: Consider using the *On-Level* and *Advanced Center Games* with all students. This game may be played multiple times, as there is a range of numbers that may be used with each of the criteria given on the game board.
Lesson 7-5: Multiples		
4.OA.B.4 MP.8 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: Students learned about factors and factor pairs in lessons 7-1, 7-2 and 7-3. In third grade, students also learned a product is the solution to a multiplication problem. Developing the Big Idea: Students will continue to build their understanding of factors and the	Solve & Share: Consider removing the table on the *Solve & Share* to enable students to derive strategies that reflect higher-level thinking or increased cognitive demand. Watch for students who may see a pattern and the relationship between golf balls per player and golf balls in play as they work through the problem. Students construct the understanding that the relationship is a multiplication problem. Consider supporting students, they only derive 9 x 1, with guiding questions such as, "How many golf balls would be in play if each player had 3 golf balls?" *-continues on next page-*

	relationship between a factor and a multiple or product.	Visual Learning: Students will "extend the idea of decomposition to multiplication and learn to use the term "multiple" (CCSWT, 2011). Convince Me: The *Convince Me!* addresses the context of a problem. Students need to develop the understanding of when it is not possible to go to the next multiple, as that would be greater than 60. Students might want to continue with finding many multiples of 8; if so, address the context of the problem. This will develop students' problem solving skills. Guided Practice: Items 1 and 2 continues the work with the Ferris Wheel problem. These items may help students determine why a multiple does not work for a particular number. Students may use their understanding of factors to support their work with multiples. Independent Practice/Math Practice and Problem Solving: Item 29 uses a Venn diagram to organize common multiples for numbers. Consider using this item whole group. By doing this problem together, students begin to see numbers have common multiples and this knowledge will support them when they begin to find common denominators when adding and subtracting fractions with unlike denominators. Assess and Differentiate/Intervention Activity: *Reteach* and *Homework & Practice* uses a multiplication table to find multiples. What is the purpose of the multiplication table? Is it a tool? Or used procedurally? As students find multiples, have students pay close attention to the arrays that are formed. For example, 49 is a multiple of 7. A 7 by 7 array has 49 squared units. Emphasize the use of the table as a tool and avoid making it into a procedural process for students. *CTC: *Quick Check* (digital platform)

References

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Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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