Grade 5 Topic 5: Use Models and Strategies to Divide Whole Numbers

Topic 5
Use Models and Strategies to Divide Whole Numbers

Number of lessons: **8**
A/D/E: **3** days
NVACS Focus: NBT.B
Total Days: ~11

[5th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 5 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition. (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 237A-237C), all 8 lessons and the Topic Assessments (pp. 297-298A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What is the standard procedure for division and why does it work? *Reference Answering the Topic Essential Questions (TE, p. 293-294) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

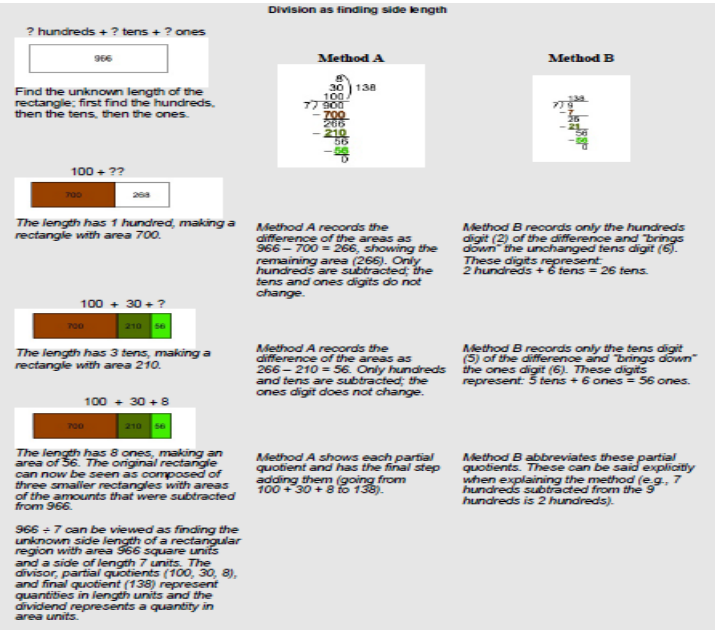
5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	Assessment

3 A/D/E days used strategically throughout the topic

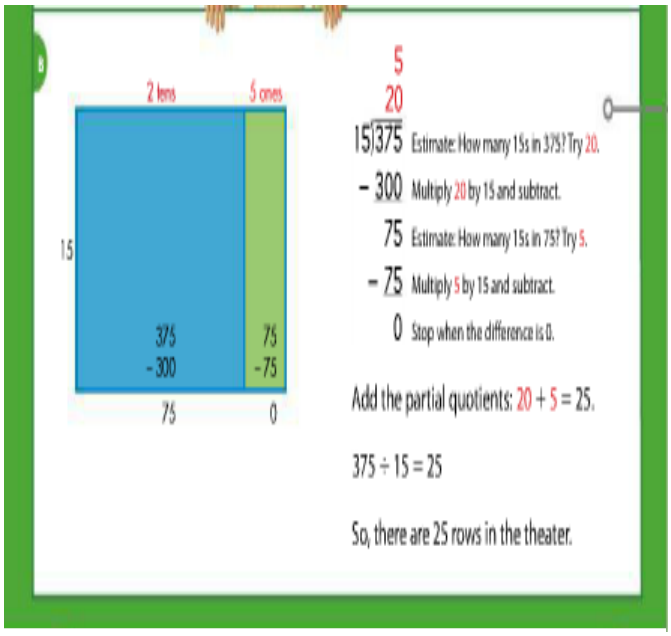
Instructional Note:

Instruction for Topic 5 is focused on Nevada Academic Content Standards (NVACS) 5.NBT.B.6 which ask students to "Find whole-number quotients of whole numbers with up to four-digit dividends and a two-digit divisor…" (2010). This standard continues with "…using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models" (NVACS, 2010, 5.NBT.B.6). This standard requires 5th graders to use multiple strategies flexibly and then explain why those strategies are appropriate. The Progression Documents for the Common Core Math Standards further explain that "Division in 5th grade involves breaking the dividend apart…applying the distributive property…and can also be viewed as finding an unknown factor…" (Common Core Standards Writing Team (CCSWT), 2015, p. 18).

This topic will rely heavily on using understandings students have built around place value and multiplication to solve division problems with whole numbers. To become procedurally fluent, students need to build deep and flexible knowledge about which strategies and procedures are appropriate and why (Pellegrino & Hilton, 2012). Thinking of **division as multiplication with a missing factor** will help students connect prior knowledge and learned strategies with new ideas. Students' experience with the area model allows them to use multiplication to solve division problems. The quotient of any division problem can be reinterpreted as a missing factor and therefore a missing side on the area model. The divisor takes the other side on the area model as the other factor and the dividend takes the place of the product. Students can begin with any underestimate, subtract the partial product created, and continue until the area model represents a true multiplication fact. Using the area model, students visually connect multiplication and division. Through practice they will naturally learn to become more efficient (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The area model connects to the partial products algorithm and builds conceptual knowledge needed to become procedurally fluent with division algorithms.



Progression Documents, Numbers and Operations in Base Ten, pg. 17



enVisionmath 2.0, Lesson 5-4 Visual Learning Bridge

The NVACS do not require fifth graders to become procedurally fluent with the long division algorithm. The long division algorithm is efficient and useful once students have built the appropriate conceptual understandings. Until they have, allowing students to choose the strategy they currently understand will allow them to build the skills needed to become procedurally fluent.

Online tools from enVision 2.0 are available which allow students to manipulate base ten blocks and visually observe regrouping while exploring division strategies.

Students will encounter and work with two different division problem types in Topic 5. Quotative division, also known as measurement or "chunking" division, gives the group size and students must find the number of groups needed. An entry point to this type of division problem can be repeated subtraction or addition. Partitive division, commonly referred to as "dealing" division, gives the number of groups and not the number within each group. An entry point for this type of division problem can be dealing out the whole to individual groups one by one or in small quantities. Exposure to both types of division is important for building understanding. However, when students are using entry level strategies to make sense of division concepts, allowing time to explore one of the problem types before switching to the other can help to reinforce strategies and understandings.

Math Practice 1: Make sense of problems and persevere in solving them

Focus on opportunities for students to develop *Mathematical Practice 1* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 5-8. Reference the Teacher's Edition (TE, pp. F21-F21A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	quotient *dividend* *divisor* *remainder*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using multiplication to help them solve division problems?
"Are students using a partial quotients strategy to divide with whole numbers?"

Lesson	Evidence	Look for
5-2	*Solve and Share* (student work samples)	Focus CTC around the big idea: • reasonable estimates for the context of the situation. • use of multiplication and number sense to make a reasonable estimate.
5-1	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
5-4	*Math Practices and Problem Solving* (student work samples) Item 22	Focus CTC around the big idea: • use of a partial quotients strategy. • use of multiplication and place value understanding to explain reasonableness of solution and strategy.
5-4	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 293-298	Use *Scoring Guide* TE pp. 293-298A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 5-1: Use Patterns and Mental Math to Divide		
5.NBT.B.6 MP.2 MP.3 MP.6 MP.7 MP.8	Access Prior Learning: In 4th grade students applied division to build place value concepts and to solve word problems (4.NBT.A.1) (4.OA.A.2). Students have used place value patterns in previous topics. Developing the Big Idea: Students extend place value understandings to build conceptual knowledge of dividing by multiples of 10.	Solve and Share: The *Solve and Share* problem is an example of measurement division. This problem type allows students to use the known group size as chunks to find the unknown number of groups. Repeated subtraction can be used as an entry point strategy while repeated addition and multiplication can be used to build up to the solution. Place value patterns can be used to extend this thinking to larger numbers and help move students towards more efficient strategies. Students look for patterns that will help them to divide by multiples of 10. This problem connects to lessons in previous topics using operations and place value to discover helpful number patterns (structure of number). The *Look Back!* can be used to facilitate a discussion that uses the observed patterns to connect multiplication and division. How can students use these patterns to determine whether an answer to a division problem is reasonable? Visual Learning: The *Visual Learning Bridge* problem is an example of partitive division. This provides the number of groups and asks students to determine the group size. Repeated subtraction or building up is more difficult with this problem type. Students might choose to equal share to divide the set into groups. Other students might estimate the group size and use multiplication to check their guess. Patterns in the zeros of quotients are shown for dividing by a multiple of 10. Students will benefit from seeing the inverse multiplication and division facts together. Some students may use a multiplication strategy to solve the *Convince Me!* which can be compared to the inverse division fact. Ask, "Why are both strategies useful for solving a division problem? How does each represent the context of the problem?" Students will be able to demonstrate understanding with a small sample of problems from the ***Independent Practice***. The *Math Practice and Problem Solving* provides practice and application for students proficient in dividing by multiples of 10 (SE, p. 232). Have students explain their thinking for item 30. Assess and Differentiate: Student's proficient using place value patterns to divide by powers of 10 should move to *Homework and Practice* (SE, p. 244). *CTC: *Quick Check* (digital platform)
Lesson 5-2: Estimate Quotients with 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students used rounding and estimation in previous topics. In 4th grade, students estimated quotients with 1-digit divisors (4.OA.A.3). Developing the Big Idea: Students extend knowledge of estimating and rounding to include division with 2-digit divisors. This conceptual understanding is the foundation for building procedural skills with division algorithms.	Solve and Share: Division algorithms require students to underestimate and then adjust their estimates before finding an exact quotient. This problem allows students to practice using rounding and compatible numbers to estimate an answer to a division problem. Facilitate a discussion about which numbers and methods for estimation will produce the closest results. Students can use multiplication to check whether their estimated quotients are reasonable and strengthen their understanding of the connection between the two operations. Visual Learning: The example shown offers opportunities to discuss how to fine tune estimates and how doing so assists in solving division problems. This understanding will help students use division algorithms in later topics. Student answers to the *Guided and Independent Practice* problems will vary. Students should be able to explain how they created their estimate and whether it is an underestimate or an overestimate. Assign a small number of problems and ask students to explain their reasoning to build conceptual understanding. Math Practices and Problem Solving problem may be used formatively to check student understanding and/or to facilitate a class discussion. Assess and Differentiate: The *Reteach* page focuses on using compatible numbers. Strategies and answers to the *Homework and Practice* pg. 249 will vary. Consider asking students to explain their estimating strategies. *CTC: *Solve and Share* (student work samples

Lesson 5-3: Use Models to Divide with 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.4 MP.5 MP.6	Access Prior Learning Students used an area model in 4th grade (4.NBT.B.5) and in Topic 3. Developing the Big Idea Students extend knowledge of the area model to include using two-digit divisors.	Solve and Share: A grid is provided to scaffold student use of the area model to solve a division problem. Look for students using the distributive property to break apart the two-digit divisor and model this problem. Use their work/ideas to facilitate a discussion drawing connections between using an area model to solve multiplication and division problems. Visual Learning: An area model is demonstrated to solve a division problem. The *Convince Me!* introduces a variable. Ask students to compare an area model used for multiplication versus one for division. Where are the factors and the product? Where is the quotient, divisor and dividend found? Provide students with a blank sheet of paper to enable them to show their strategies and reasoning when working on the *Independent Practice* page. Caution: Using a grid here may create misconceptions since the larger numbers in the problems will not match the number of squares available. If you choose to use a grid, make sure to discuss this choice and the potential challenges. Assess and Differentiate: The *reteach* page models using base ten blocks to solve a partitive (fair/equal share) division problem. This is a different understanding than the area model practiced in the beginning of the lesson.
Lesson 5-4: Use Partial Quotients to Divide		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning Students used an area model to solve a division problem in lesson 5-3. Developing the Big Idea Students connect the area model to partial quotients to build conceptual understanding of division and procedural skill.	Solve and Share: Look for students using different strategies to solve this problem. Facilitate a discussion to draw out ideas about how underestimates are used to create partial quotients. The *Look Back!* can be used to remind students that estimation and multiplication should be used while working with division problems. Visual Learning: An area model is demonstrated alongside the partial quotients strategy. Connect the partial quotients strategy to student ideas used during the *Solve and Share*. Partial quotients can be interpreted as underestimates of the answer to a division problem. How can we create more accurate underestimates while finding partial quotients? What similarities and differences can be found between partial quotients and other strategies? Consider using the *Convince Me!* to give students an opportunity to analyze the mathematical thinking of others (MP.3). Assess and Differentiate: The *Intervention Activity* reminds students they should use estimation and multiplication to solve division problems. The *Reteach* page reviews division vocabulary and models using multiplication to create partial quotients. *CTC: *Math Practices and Problem Solving* (student work samples) Item 22 *CTC: *Quick Check* (digital platform)
Lesson 5-5: Divide by Multiples of 10		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: Students have solved division problems using various strategies in previous lessons. Developing the Big Idea: The standard algorithm for division is introduced using multiples of 10 as the divisor. Instructional Note: The remainder created by the Solve and Share (money) and the Visual Learning Bridge (students) must be handled differently and change the quotient. Consider using these differences to facilitate a discussion about how context	Solve and Share: Encourage students to use various strategies such as repeated subtraction, skip counting, area models and partial quotients. Consider asking students to display their strategies and facilitate a discussion focused on connections between the different methods. Seeing the connection between an entry-level strategy such as repeated subtraction and a more efficient algorithm such as partial quotients will help to push students towards more sophisticated strategies and algorithms. The remainder in the *Solve and Share* problem is ignored due to the context of the problem. However, the *Visual Learning Bridge* problem will have a remainder requiring students to create one more group (all students need to go on the field trip). Consider facilitating a discussion about how context can change the solution to a division problem. Visual Learning: The U.S. traditional algorithm is modeled. Draw connections between this algorithm and other strategies students are currently using and practicing. *-continues on next page-*

	changes the way remainders and answers are reported. *See instructional note for lesson 5-6.*	The long division algorithm is shown again in the *Guided Practice* items 5-7 (SE, p. 265). Facilitate making connections between partial quotients and the algorithm shown. The focus of the standards in 5th grade are to "use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models" Consider asking students not ready for the long division algorithm to move to items 8-13 to continue practicing and working with other strategies or algorithms. Assess and Differentiate: Students practice placing the first digit using the long division algorithm during the *Intervention Activity.* This activity could be connected to estimation as a tool for determining a reasonable answer to a division problem. The U.S. traditional algorithm for division is shown in the steps on the *Reteach* page.
Lesson 5-6: Use Estimation to Place the First Digit of the Quotient		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.7	Access Prior Learning: Students have used estimation and partial quotients to solve division problems in previous lessons. Developing the Big Idea: Students use estimation to think about the magnitude of quotients created by different sized dividends and divisors. Instructional Note: Consider using this lesson before 5-5. This lesson requires students to use number sense, reasoning and estimation with powers of 10 to determine the magnitude of a quotient. This could help students become more proficient with all division algorithms.	Solve and Share: Estimation strategies such as rounding and compatible numbers will help students to think about the size of quotients for the different division problems. Use strategic questioning to assist students in noticing patterns in the magnitude of quotients and connect this to place value and multiplication understandings. Visual Learning: The *Visual Learning* models using powers of 10 to determine a range for the quotient. Orchestrate a whole class discussion employing additional experiences, explorations and explanations to bridge the connection between determining the range of a quotient and placing the first digit using the algorithm. Consider solving the same problem with an area model and the partial products or partial quotients algorithms to maintain focus to the requirements of the standard. The intent of the standards is to provide students experiences with "use(ing) strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models". What other strategies or algorithms might we use to solve this problem? How do these connect with what was shown? Help students to notice that all division strategies are more efficient when we use the largest possible underestimate for each place value. This helps to determine the size of the quotient and the first digit. Assess and Differentiate: The connection between multiplying by powers of 10 and determining the first digit in a quotient are made explicit in the *Intervention Activity* and the *Reteach* page. Consider focusing on other strategies or algorithms to meet the intent of the standard and build strong conceptual understanding.
Lesson 5-7: Divide by 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.4	Access Prior Learning: Students have used estimation and a variety of strategies to solve division problems in previous lessons. Securing the Big Idea: Students will practice solving division problems to build procedural skill in dividing multi-digit numbers by two-digit divisors.	Solve and Share: Encourage students to solve this problem using more than one strategy. Facilitate a discussion to make connections between the strategies. Which are more efficient? Do some strategies work better for this problem? What would happen if there were a remainder for this problem? Visual Learning: The U.S. traditional algorithm is modeled again. In the *Convince Me*! students are asked to explain how they can determine that the original estimate of 40 is too high. Can students use partial quotients or an area model to justify their thinking? Why can't we use an overestimate to solve a division problem? Assess and Differentiate: In the ***Intervention Activity*** and ***Reteach*** page, students use estimation with powers of 10 and multiplication to think about the size of an appropriate quotient given different division problems. The ***Homework and Practice*** provides application and practice for students demonstrating conceptual understanding and procedural fluency (SE, p. 280). Item 18 asks students to explain their reasoning and item 19 asks students to create a division context.
Lesson 5-8: Math Practices and Problem Solving- Make Sense and Persevere		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students have solved division problems and used MP.1 in previous lessons. Securing the Big Idea: Students will apply knowledge of division to a real world context.	Solve and Share: Encourage students to analyze the context and information to make sense of this question before answering. Look for students modeling the problem and creating a plan. Connect student ideas and work to the Thinking Habits (SE, p. 281). *-continues on next page-*

MP.6 MP.7		Visual Learning: Thinking habits of MP.1 are shown in the *Visual Learning Bridge*. How do these habits connect to the ideas shared during the *Solve and Share*? How do these habits help us to become better mathematicians? *Independent Practice*: Items 6-10 are multi-step problems and push students to think deeply applying what they have learned (SE, p. 284). Consider asking students to explain their thinking for a single item. How did students make sense of the problem before answering these questions? Assess and Differentiate: The *Intervention Activity*, *Reteach* page, and *Homework and Practice* pages use a bar diagram to model division problems. Students have previous experience with this model. How can this model help make sense of new problems? How might this support explaining our own thinking or help us understand the reasoning of others?

References

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