Grade 3 Topic 15: Attributes of Two-Dimensional Shapes

Big Conceptual Idea: K-6, Geometry (pp. 13-14)

Prior to instruction, view the Topic 15 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 805A-805F), the Topic Planner (pp.805I-805J), all 4 lessons, and the Topic Assessments (pp. 841-842A).

Mathematical Background: Read Topic 15 Cluster Overview/Math Background (TE, pp. 805A-805F)	Topic Essential Question: How can two-dimensional shapes be described, analyzed, and classified? *Reference Answering the Topic Essential Question (TE, pp. 839-840) for key elements of answers to the Essential Question.*

Topic 15
Attributes of Two-Dimensional Shapes
Number of lessons: **4**
A/D/E: **7** days
NVACS Focus: G.A
Total Days: ~11

3rd Grade Curriculum Pacing Framework: Balanced Calendar

The lesson map for this Topic is as follows:

15-1	15-2	15-3	15-4	Assessment

7 A/D/E days used strategically throughout the topic includes the "enrichment" lessons.

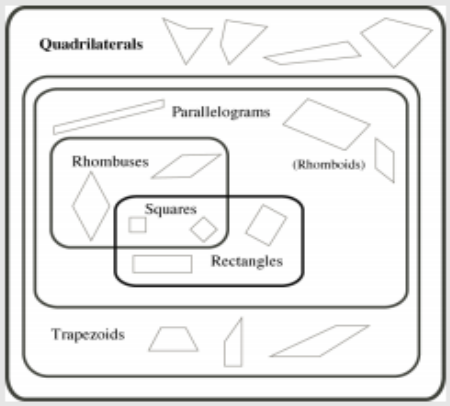
Instructional note:

Topic 15's big idea is that two-dimensional shapes can be described, analyzed, and classified based on their attributes. Students learn to analyze a variety of geometric shapes and begin to explore relationships between the shapes based on their attributes. The Geometry progression document (2014) describes attributes as, "any characteristic of a shape, including properties, and other defining features (e.g., straight sides) and non-defining features (e.g., "right-side up")" (p. 3).

Much of what makes this standard a struggle for students is the vocabulary and hierarchical inclusion of geometric shapes (any shape in a sub category is also a member of the larger category; yet a shape of a larger category may or may not be part of particular sub categories).

Geometric hierarchy is the idea that, "Shapes have many attributes that make them similar to and different from one another. You can describe and classify different groups of shapes by their attributes" (TE, p. 818). This means that a shape can fall into many different groups when being classified. For example, when classifying a square, a square meets the requirements of being a polygon, quadrilateral, rectangle, parallelogram, and a rhombus. However, a rectangle and a rhombus are not necessarily squares.

Venn diagram showing classification of quadrilaterals



Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref "T(E)"]).

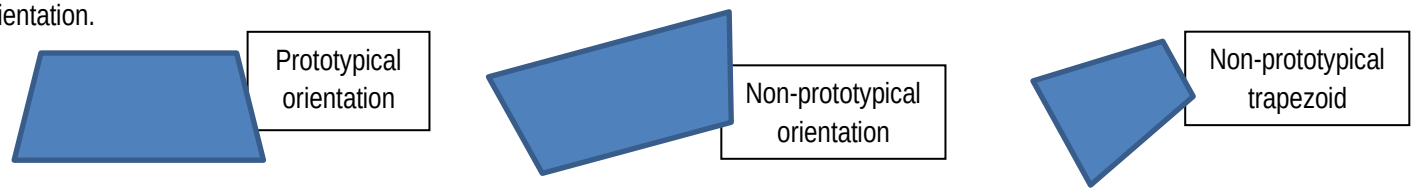
To illustrate this idea, please see the image to the right from page 18 of the K-6, Geometry progression document (2014). Additional A/D/E days allow instructional time for tasks that build understanding of the idea that shapes can be classified into a group that is part of another group. Include non-examples to help students establish limitations to categories.

Vocabulary becomes an important component of recognizing and classifying shapes by their attributes. This can be illustrated by the example of classifying a square which moves through several subcategories and also retains the attributes and names from those subcategories. For example, to be labeled a square, students have to recall that the shape must also meet all of the following criteria:

- be a polygon (and the all the attributes for a polygon)
- have four sides
- have 2 pairs of parallel sides
- all sides are the same length
- all angles are the same size

Students who have had limited experiences with working with geometric ideas in previous grades may need additional language support. Consider using the graphic organizers from Teaching Tools 24 through 28 found in the *Teacher's Resource Masters Volume 2* to support language acquisition and use. Anchor charts, cognitive content dictionaries or personal word walls in addition to writing and reasoning tasks will support students in use of the academic language needed to explore these ideas.

A common misconception that students will form is that a shape is only that shape when presented in its prototypical orientation. For example, the first trapezoid below is shown in its prototypical orientation, while the second trapezoid is shown a non-prototypical orientation.



To avoid this misconception, provide many opportunities for students to reason with polygons in multiple orientations and name polygons based on evidence of the attributes stated in the shape's definition.

There are two different definitions for a trapezoid, an inclusive and **exclusive** definition. Per the Geometry progression document (2014) the inclusive definition of a trapezoid states that, "a trapezoid is a quadrilateral with **at least one pair** of parallel sides" (p.3). Therefore, in the inclusive definition, a trapezoid would fit into the sub-category of parallelograms. In the Geometry progression document (2014) the exclusive definition of a trapezoid states that, "a trapezoid is a quadrilateral with **exactly one pair** of parallel sides" (p.3). Therefore, in the **exclusive definition**, a trapezoid would not fit into the sub-category of parallelograms. There is no harm in students being made aware of the two different definitions. However, **enVision**math2.0 and WCSD use the exclusive definition (K-Algebra 1).

The WCSD 3rd Grade Pacing Framework allows an additional 7 days of lessons to explore geometric ideas and build a solid understanding that will be necessary for students as they progress through the grade levels. To assist in building strong conceptual development and support learner responsive instruction, additional "Enrichment" lessons are included throughout the topic. More ideas are found at the conclusion of this document.

Focus Math Practice 6: Attend to precision

Focus on opportunities for students to develop Mathematical Practice 6 behaviors throughout the entire topic, as this is the focus of the Math Practices and Problem Solving lesson 15-4. Resources to support students' development of MP. 6 include the Teacher's Edition (pp. F26 - F26A) and the Nevada Academic Content Standards for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, students need to be familiar with the attributes of two-dimensional figures and be able to identify shared attributes among grouped two-dimensional figures. Develop thinking habits that allow students to engage in these types of problems through encouraging discussions, deliberations and debates while having students work with these ideas in triads (groups of 3).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
polygon side quadrilateral angle vertex trapezoid parallel sides parallelogram rectangle right angle rhombus square convex concave	*circle* *hexagon* *pentagon* *triangle* *rhombus* *rectangle*

Additional terminology that students may need support with: attributes, alike, different, square angle

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to classify and analyze quadrilaterals based on their attributes?"

Lesson	Evidence	Look for
15-3	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • students are able to analyze and compare quadrilaterals based on their attributes.
15-4	*Convince me!* (student work samples)	Focus CTC around the big idea: • construct shapes within given parameters.

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 839-842	Use *Scoring Guide* TE pp. 839-842A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Optional Enrichment		
3.G.A.1 MP.1 MP.3 MP.6 MP.7	Access Prior Learning: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten. Developing the Big Idea: Students further *develop* geometric concepts by identifying common attributes amongst shapes (both defining and non-defining).	Materials: Attribute Blocks (these may be checked out from the UNR LRC) Shape clues (attributes of various shapes that may fall in various subcategories). Opener: Students work in triads to create an attribute train. One player starts by picking a block. The next player must pick a block that is different in only one way. The difference can be shape, size, thickness or color. The triad keeps taking turns building their train and checking that each car is different in only one way. Once all blocks have been placed rotate the triads so that one triad checks another triad's train for accuracy. Play again, this time choose blocks that change ***in all but one*** attribute. That means that cars that touch must have only one attribute in common. Whole Group Discussion: After the second version of the game has been played, choose one group's work to discuss whole group. Consider using a fishbowl strategy to explore the train. Highlight attributes of shapes. Extend attribute thinking by giving students clues (attributes of a shape) and having them draw the shape (2nd grade standard). Example: I have four equal sides and four equal corners (this could be a square or a rhombus). Purposely choose attributes that could be used to make shapes that may fit more than one category. See "Polygons on the Geoboard" at the end of this guide for clue ideas. Encourage children to visualize or mentally construct the shape as they are drawing the shape. Create an anchor chart that lists the attributes of triangles, quadrilaterals, pentagons and hexagons (2nd grade standard). Add on to this chart throughout the topic starting with 15-1. Note: "Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes" (1st grade NVAC, 1.G.A.1).
Lesson 15-1: Describe Quadrilaterals		
3.G.A.1 MP.1 MP.3 MP.4 MP.6 MP.7 MP.8	Access Prior Learning: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten. Developing the Big Idea: Students further *develop* geometric concepts by describing and classify	Topic Opener: Introduce the *Topic Essential Question*, "How can two-dimensional shapes be described, analyzed, and classified?" (TE p. 805). Consider making an anchor chart highlighting key ideas so that students can see the conceptual development and connections throughout the topic. Have students complete the *Review What You Know* prior to beginning instruction on Topic 15 so that you can respond to students' instructional needs. Some students may benefit from additional support using the *Item Analysis for Diagnosis and Intervention* prior to beginning the topic (TE, p. 806). Consider introducing vocabulary as students encounter terminology in the lessons rather than introducing all terms at the beginning of the lesson (avoid front loading). *-continues on next page-*

	different quadrilaterals by their sides and angles.	Solve & Share: Consider providing copies of various quadrilaterals (Teach Tool 21). After introducing the *Solve & Share*, discuss the questions provided in the section *Build Understanding* to help students develop a possible strategy for solving (TE, p. 811). Consider adding non-prototypical shapes. For more details, see the *Instructional note* at the beginning of this topic. Look Back!: After discussing students' solution methods and reasoning, discuss the *Look Back!* prompt if those ideas do not already come out during the whole class discussion. Visual Learning: Consider pausing and discussing after the *Visual Learning Animation* poses the question, "Why are all of these shapes quadrilaterals?" Independent Practice/Math Practices and Problem Solving: Consider assigning items 11 and 12 "Vocabulary" to provide students with additional opportunities to develop the mathematical language in this topic.
Enrichment		
3.G.A.1 MP.1 MP.3 MP.6	Access Prior Learning: In Topic 15, Grade 2, students learned about polygons and angles. Students have been working with prototypical and non-prototypical shapes and defining attributes since pre-K or Kindergarten. Developing the Big Idea: Students further *develop* geometric concepts by describing and classify different quadrilaterals using attributes.	Materials: Assorted Shape BLM 44,45 and 46 from: https://tinyurl.com/Topic-15-Shapes Opener: Students work in triads to cut out the shapes from the above masters. Students work as a group to sort the shapes into several categories (open sort of at least four categories). Once all the shapes have been placed into groups, have students rotate to another triad's sort. Have this triad analyze the classification by labeling each group of shapes. Have students create a list of all the attributes the shapes in the group have in common. Ask triads to write a statement on if they agree or disagree with the sort and why. Whole Group Discussion: Choose one triad's work to discuss whole group. Highlight attributes of shapes. When analyzing parallel lines, consider using two straight edges (meter sticks or rulers) to test lines that students think are parallel, yet end up intersecting (when the straight edges are placed on either supposedly parallel line to show the lines will eventually intersect even if they are not intersecting now).
Lesson 15-2: Classify Shapes		
3.G.A.1 MP.3 MP.5 MP.6 MP.7 MP.8	Access Prior Learning: In the previous lesson, students explored classifying shapes using their attributes. This builds from 2nd grade when students recognized and drew space when given specified attributes. Developing the Big Idea: Students further *develop* geometric concepts by classifying shapes by the number of sides, equal sides, parallel sides, size of angles, and concavity versus convexity.	Instructional note: In third grade, students do not formally measure angles. Students classify based on size of angles from "eye balling" right/square angles. Solve & Share: Consider assigning the *Look Back!* to extend students' reasoning of how they sorted the triangles. Visual Learning: During the *Visual Learning Animation*, consider pausing and discussing the questions: • "How are the groups different?" • "How are the groups alike?" After viewing the *Visual Learning Animation* consider discussing the prompt provided in *Prevent Misconceptions* (TE, p. 818). Assign the *Convince Me!* to check for understanding of the *Visual Learning* Animation and extend the ideas presented. Independent Practice/Math Practices and Problem Solving: Consider assigning item 16 "Algebra" to provide distributed practice of "Put Together/Addend Unknown" (see page 88 of NVACS, 2010 for more information of problem types) problem type and algebraic reasoning. Assess & Differentiate: Consider providing an opportunity for all students to interact with the *Math and Science Activity* (TE, p. 821A). This activity provides a meaningful experience with identifying and naming polygons based on their attributes. Child watch to identify students who need additional support and pull them into a small group to do the *Intervention Activity* (TE, p. 821A). Use triangles from master cutouts (see above lesson).

Enrichment		
3.G.A.1 MP.1 MP.3 MP.6	Access Prior Learning: In the previous lesson, students explored classifying quadrilaterals by using their attributes. Developing the Big Idea: Students further *develop* geometric concepts by classifying shapes into formalized categories in this case parallelograms and quadrilaterals.	Materials: Assorted Shape BLM 44,45 and 46 from: https://tinyurl.com/Topic-15-Shapes (see enrichment after 15-1). Parallelogram vs. Quadrilateral sorting mat: https://tinyurl.com/Topic-15-sort-mat Opener: Students work in partners or triads to sort the shapes using the Parallelogram vs. Quadrilateral sorting mat. After about 6 minutes of exploration, pull the group together and have students share the attributes of a parallelogram. Post or list these on an anchor chart for students to reference. Post the word quadrilateral and have students discuss what a quadrilateral is in their triads (yet don't post attributes yet).Have students analyze their work so far and then continue working. Whole Group Discussion: Have students do a gallery walk to look at the other triads work. Bring students together to discuss their findings. What did students find? What is interesting about this sort? Can they think of a shape that could go into just the parallelogram spot (that would not fit both or either too)? Highlight attributes of shapes. Use two straight edges (meter sticks or rulers) to test lines that students think are parallel. Full lesson available at: https://tinyurl.com/Topic-15-enrichment-lesson)
Lesson 15-3: Analyze and Compare Quadrilaterals		
3.G.A.1 3.MD.C.5b MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In previous lessons, students described and classified quadrilaterals. Developing the Big Idea: Students further *develop* geometric concepts by analyzing and comparing quadrilaterals and group them by their attributes.	Instructional note: A common struggle for students is recognizing that while all squares are rhombuses not all rhombuses are squares. Consider using an ADE day to develop a class anchor chart that represents the hierarchical inclusion of these shapes. This idea refers to the hierarchical inclusion of geometric shapes, for more details on this read the *Instructional Note* at the beginning of this topic. Solve & Share: After introducing the *Solve & Share* consider asking the questions provided in *Build Understanding* activate prior learning. Consider assigning the *Look Back!* prompt to extend student reasoning from the *Solve & Share*. Visual Learning: Consider pausing and discussing after the *Visual Learning Animation* poses the question, "How do you know all the shapes are quadrilaterals?" Consider assigning the *Convince Me!* to check for understanding of the *Visual Learning Animation* and extend the ideas presented. Independent Practice/Math Practices and Problem Solving: Consider having students work in partners and triads and to reason about and discuss the problems. Encourage students to discuss, debate, define possible solutions using appropriate and precise mathematical terminology. Assess & Differentiate: Child watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 827A). *CTC: *Quick Check* (digital platform)
Lesson 15-4: Math Practices and Problem Solving- Precision		
MP.6 MP.1 MP.3 MP.5 MP.7 3.G.A.1	Access Prior Learning: In previous lessons, students described, classified, analyzed, and compared quadrilaterals. Securing the Big Idea: Students *secure* the geometric concepts of describing, classifying, analyzing, and comparing quadrilaterals by using accurate language and using appropriate tools.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 6. Refer to the *Math Practices and Problem Solving Handbook* for suggestions on how to develop, connect and assess this Math Practice (TE, pp. F26-F26A, F29). Also reference the handbook in the Student Edition (SE, p. F26). Solve & Share: Consider reintroducing MP. 6 Thinking Habits (SE p. F26) before introducing the *Solve & Share*. Also consider using time students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.6, that are listed in the *Math Practices and Problem Solving Handbook* (TE, p. F26A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. Have grid paper, rulers, and index cards (or other tools for making right angles) available. After introducing the *Solve & Share*, consider discussing the questions provided in the section *Build Understanding* to help students develop a possible strategy for solving (TE, p. 829). After discussing students' solution methods and reasoning, consider discussing the *Look Back!* prompt to support students' development of MP.6. *-continues on next page-*

		Assess & Differentiate: Consider providing an opportunity for all students to interact with the Math and Science Activity (TE, p. 833A). This activity provides a meaningful experience with identifying and naming polygons based on their attributes. Child watch to identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p. 833A). *CTC: *Convince Me!* (student work samples)

References

Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). K-6, Geometry.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Wiest, L. (2015). *Lecture 3: Polygons.* Reno, NV: University of Nevada, Reno

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). New York, NY: Pearson.

Additional enrichment activities:

- Graham Fletcher's "Geo-Dotting" (DOK 2 potential when incorporated into whole class discussion)
- Shape-Match (DOK 1, limited to prototypical standard orientations)
- Quadrilateral Riddle Creator (DOK 1)
- Quadrilateral & Polygon Sort (DOK 2)

Polygons On The Geoboard

1) Make a 3-sided polygon with 1 square corner and no 2 sides the same length.

2) Make a 4-sided polygon with no parallel sides.

3) Make a 4-sided polygon with all sides different lengths.

4) Make a 4-sided polygon with no square corners but with two pairs of sides parallel.

5) Make a 5-sided polygon that has exactly one pair of parallel sides.

6) Make a 6-sided polygon with three pairs of parallel sides.

7) Make a 6-sided polygon with one pair of sides perpendicular.

8) Make a polygon that is not a square and looks the same no matter on which side you rest the geoboard.

9) Make a polygon with as many sides as is possible on the geoboard.

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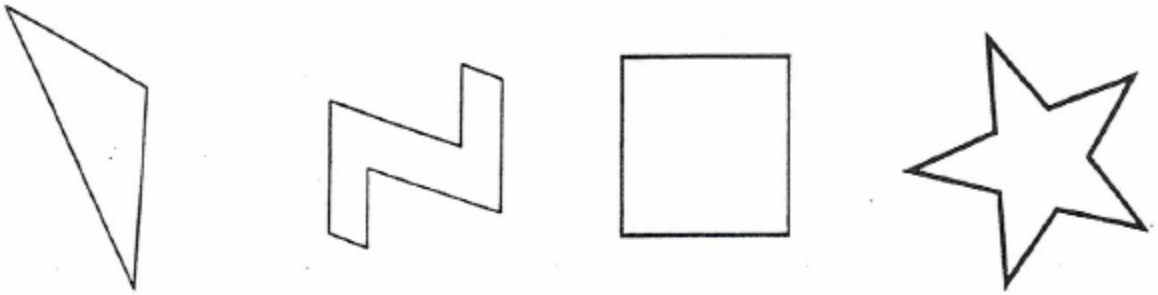
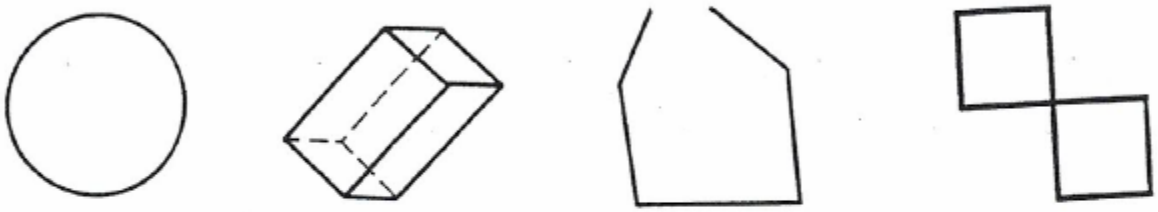
* To incorporate technology and encourage discussion on 2-dimensional shapes consider using, https://apps.mathlearningcenter.org/geoboard/.

These are polygons.

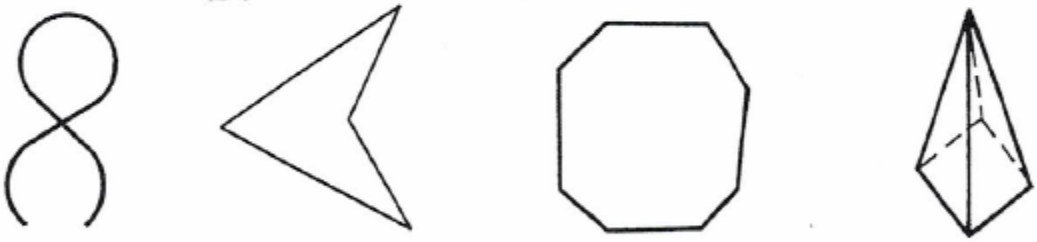


These are not polygons.



Which of these are polygons?

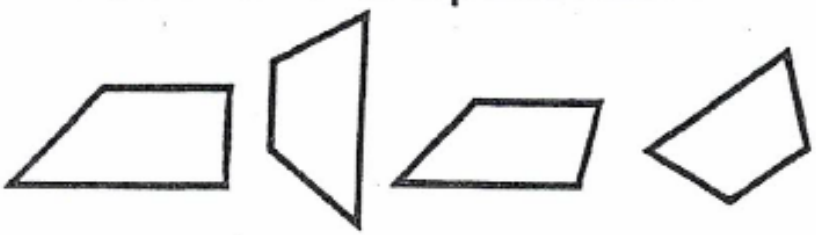
A. B. C. D.



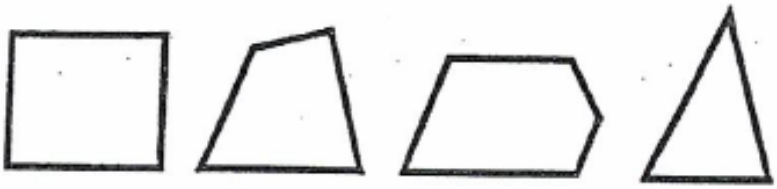
Draw some polygons.

Define "polygon."

These are trapezoids:



These are not trapezoids:



Which of these are trapezoids?

a.  b.  c.  d. 

Draw something that is a trapezoid.

Draw something that is not a trapezoid.

What is a trapezoid?

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