

► Grade 3 Topic 13: Fraction Equivalence and Comparison

Big Conceptual Idea: [Number and Operations Fractions, 3-5](#) (pp. 3-5)

Prior to instruction, view the *Topic 13 Professional Development Video* located in *Pearson Realize* online. Read the *Teacher's Edition (TE): Cluster Overview/Math Background* (pp. 605A-605F), the *Topic Planner* (pp.669A-669C), all 8 lessons, and the *Topic Performance Assessment* (pp. 731-732A).

<p>Mathematical Background: Read Topic 12-13 Cluster Overview/Math Background (TE, pp. 605A-605F)</p>	<p>Topic Essential Question: What are different ways to compare fractions?</p> <p><i>Reference Answering the Topic Essential Question (TE, pp. 727-728) for key elements of answers to the Essential Question.</i></p>
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<p>Topic 13 Fraction Equivalence and Comparison</p>
<p>Number of lessons: 8</p>
<p>A/D/E: 3 days</p>
<p>NVACS Focus: N.F.A.</p>
<p>Total Days: ~11</p>

The lesson map for this topic is as follows:

13-1	13-2	13-3	13-4	13-6	13-5	13-7	13-8	Assessment
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3 A/D/E days used strategically throughout the topic.

[3rd Grade Curriculum](#)
[Pacing Framework:](#)
[Balanced Calendar](#)

Instructional note:

Topic 13's big idea is in developing the idea of fractions as numbers (fractional number sense) and equivalence. As you read on, you will find that the question, "What is the whole?" will be critical when working with fraction equivalence and comparison. Students must be able to identify, or make the assumption that they are working with the same size whole in order to identify equivalent fractions and make comparisons. Chapin and Johnson (2006) state the following in regards to fraction equivalence:

"Equivalence is one of the most important mathematical ideas for students to understand, particularly with regard to fractions. Equivalence is used in when comparing fractions, ordering fractions, and adding and subtracting fractions. Equivalent fractions are fractions that represent equal value; they are numerals that name the same fractional number. When represented using a number line, equivalent fractions represent the same distance" (p.114).

The importance of the whole is key to third grade fraction standards. Van de Walle, Karp, Lovin, Bay-Williams (2014) state, "Every fraction is equal to an infinite number of other fractions" (p. 220). This idea includes seeing whole numbers as fractions as outlined in the [Numbers and Operations—Fractions, 3-5](#) progression document (p. 4). Consider using a number line to model this understanding. Topic 13 does this throughout the topic using the double number line.

This topic explores strategies to compare nonequivalent fractions with the same whole. A key idea in comparing fractions is understanding that the larger the denominator the smaller the unit fraction, when comparing fractions with the same denominator the larger the numerator the larger the fraction. Emphasize and formalize these ideas as student discover them. Focus on providing many experiences for students to notice these patterns rather than explicitly teaching them.

Throughout this topic, fraction strips and number lines are heavily used to model the ideas of fraction equivalence and comparison. Van de Walle, et. al. states, "Sometimes it is useful to do the same activity with two different representations as they offer different opportunities to learn. For example, an area model helps students visualize parts of the whole, and a linear model shows that there is always another fraction to be found between any two numbers" (2014, p. 207). It may be beneficial to plan and select students with two different models to share and compare.

Consider spending more than 1 day on a lesson. The need to spend more than 1 day on a lesson should be a balance of being learner responsive and pacing considerations; therefore, making a lesson a 2-day lesson may be different from class to class. As a result, an additional *Solve & Share* for each lesson is offered. The *Another Look* videos could be used to fill-in for a *Visual Learning Animation*. Please note the intent is not that every lesson become a 2-day lesson, but rather, to provide a resource for when it is needed.

Focus Math Practice 3: Construct viable arguments and critique the reasoning of others

Focus on opportunities for students to develop Mathematical Practice 3 behaviors throughout the entire topic, as this is the focus of the *Math Practices and Problem Solving lesson* 13-8. To support students' development of MP. 3 consider using some of the language as a regular part of instruction. Giving students directions that include phrases such as, "construct an argument" and "justify your conjecture" may help students build understanding of what MP. 3 entails. Additional resources include the *Teacher's Edition* (pp. F23 - F23A) and the Nevada Academic Content Standards (NVACS) for Mathematical Practice.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
equivalent fractions	<i>fraction</i> * <i>numerator</i> * <i>denominator</i> <i>greater than</i>
	<i>less than</i> <i>conjecture</i>

Additional terminology that students may need support with: *compare*, *Consider using the definition described in Topic 12 Curriculum Guide’s *Instructional note* to avoid developing misconceptions.

***Collaborative Team Conversations (CTC)**

Consider using **one** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: “Are students finding equivalent fractions and comparing fractions based on the whole using multiple strategies?”
(Fraction strips, number lines, models, benchmark fractions)

Lesson	Evidence	Look for
13-2	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under “Teacher Resources”. • ability to use a number line to identify equivalent fractions
13-8	Math Practices and Problem Solving (student work samples) Items 7 and 9	Focus CTC around the big idea: • student understanding of the whole (item 7) constructing a math argument based on comparison of fractions (item 9)

Learning Cycle Assessments (summative)	Topic Assessments SE pp. 727-732	Use <i>Scoring Guide</i> TE pp. 727-732A
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Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 13-1: Equivalent Fractions- Use Models		
3.NF.A.3a 3.NF.A.3b MP.2 MP.4 MP.5 MP.7	Access Prior Learning: In Topic 12, Grade 3, students learned that fractions name a part of a whole. Beginning of the Big Idea: Students are <i>beginning</i> to develop an understanding that the same fraction amount can be represented by an infinite set of different but equivalent fractions.	Topic Opener: Introduce the <i>Topic Essential Question</i> , “What are different ways to compare fractions?” (TE p. 669). Consider making this an anchor chart in your classroom that allows for the addition of new ideas so that students can see the development and connections throughout the topic. Consider having students complete the <i>Review What You Know</i> prior to beginning instruction on Topic 13 so that you can respond to students’ instructional needs using the <i>Item Analysis for Diagnosis and Intervention</i> prior to beginning the topic (TE, pp. 670-672). Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson. Solve & Share: After introducing the <i>Solve & Share</i> consider asking students what tool might be helpful (e.g., fraction strips) before asking the provided question in <i>Build Understanding</i> (TE, p. 673). Watch for students that do not partition the rectangle(s) into equal parts. Convince Me: Consider discussing the <i>Convince Me!</i> to make explicit a strategy for identifying equivalent fractions for $\frac{1}{2}$. The <i>Convince Me!</i> also asks students to look for patterns and relationships between the numerator and denominator of fractions equivalent to $\frac{1}{2}$.
<i>-continues on next page-</i>		

		<p>Assess and Differentiate: If time permits, teach students how to play <i>Display the Digit</i> (TE, p. 677A). All students should have the opportunity to play this game as it provides engaging and meaningful practice of a key concept.</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 677A).</p> <p>Possible Day 2 Solve & Share: (Read the Instructional note at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using items 9 and/or 10 from <i>Math Practices and Problem Solving</i>.</p>
Lesson 13-2: Equivalent Fractions- Use the Number Line		
<p>3.NF.A.3a 3.NF.A.3b</p> <p>MP.3 MP.4 MP.5</p>	<p>Access Prior Learning: In Topic 12, Grade 3, students learned to represent fractions on number lines. In the previous lesson students learned about equivalent fractions.</p> <p>Developing the Big Idea: Students further <i>develop</i> their understanding of equivalent fractions by finding that there are limitless number of fraction names for each point on a number line.</p> <p>These points can be used to name equivalent fractions.</p>	<p>Solve & Share: A common misconception when working with number lines is for students to count the hash marks instead of the sections to name the fractions. Consider discussing "Drew's Work" (TE p. 679) to confront this misconception.</p> <p>Since both the fraction strips (area) and number line (linear) models have been introduced, it may be beneficial to have a class discussion on how both models represent the same mathematics through their similarities and differences.</p> <p>Look Back: Consider wrapping up the class discussion of students' solutions and reasoning by discussing the <i>Look Back!</i> prompt.</p> <p>Independent Practice/Math Practices and Problem Solving: Consider discussing item 8 as it offers an opportunity to discuss and develop understanding of equivalent names for 1.</p> <p>Consider discussing item 12 <i>MP.3 Construct Arguments</i> to support students' development of MP.3, and to develop schema for the next lesson.</p> <p>Assess and Differentiate: If time permits, you may consider replacing the <i>Math and Science Activity</i> with the game <i>Display the Digit</i> (TE, p. 677A) or the <i>Fluency Practice Activity</i> (TE, p. 721).</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 783A).</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional Note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using the <i>Convince Me!</i></p> <p>*CTC: <i>Quick Check</i> (digital platform)</p>
Lesson 13-3: Use Models to Compare Fractions- Same Denominator		
<p>3.NF.A.3d</p> <p>MP.2 MP.3 MP.5 MP.6 MP.8</p>	<p>Access Prior Learning: In previous lessons, students used fraction strips and number lines to find equivalent fractions.</p> <p>Developing the Big Idea: Students are further <i>developing</i> fractional sense by comparing fractions with the same denominator.</p> <p>Students use quantitative reasoning to determine that when comparing fractions with the same denominator, the fraction with the greater numerator is the greater fraction.</p>	<p>Solve & Share: Consider reviewing the meaning of comparison symbols (<, >, and =) after introducing the <i>Solve & Share</i>, to reinforce the significance of the whole when comparing fractions. Consider asking students if we have the same size whole (e.g., yes, the whole is 1 mile for both joggers) being compared. Then ask, "Can we compare if they were different size wholes (e.g., no because the wholes have to be the same size in order to compare)?"</p> <p>Visual Learning: Considering pausing and discussing after the question is posed, "Which is greater 4/6 or 2/6?"</p> <p>Independent Practice/Math Practices and Problem Solving: Consider discussing item 18 <i>MP.8 Generalize</i> to begin to develop schema for the next lesson.</p> <p>Assess and Differentiate: If time permits, you consider teaching students how to play <i>Tic Tac Toe</i> (TE, p. 689A). All students should have the opportunity to play this game it provides engaging and meaningful practice of a key concept.</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 17 from the <i>Independent Practice/Math Practices and Problem Solving</i>.</p>

Lesson 13-4: Use Models to Compare Fraction- Same Numerator		
<p>3.NF.A.3d</p> <p>MP.2 MP.3 MP.4 MP.6</p>	<p>Access Prior Learning: In the previous lesson, students used fraction strips and pictorials to compare fractions with the same denominator.</p> <p>Developing the Big Idea: Students are further <i>developing</i> fractional sense by comparing fractions with the same numerator.</p> <p>Students use quantitative reasoning to determine that when comparing fractions with the same numerator, the fraction with the greater denominator is less than the other fraction.</p>	<p>Convince Me: Consider discussing the <i>Convince Me!</i> to support student's development of MP.3 Construct Arguments, as well as, to extend the ideas covered in the <i>Visual Learning Animation</i>.</p> <p>Independent Practice/Math Practices and Problem Solving: Item 18 <i>Higher Order Thinking</i> from the <i>Quick Check</i> addresses a common misconception that results from students applying reasoning with whole numbers to rational numbers.</p> <p>Assess and Differentiate: If time permits, you may consider replacing the <i>Math and Science Activity</i> with the game <i>Display the Digit</i> (TE, p. 677A), <i>Tic Tac Toe</i> (TE, p. 689A), or the <i>Fluency Practice Activity</i> (TE, p. 721).</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 695A).</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 15 from <i>Math Practices and Problem Solving</i>.</p>
Lesson 13-6: Compare Fractions- Use the Number Line		
<p>3.NF.A.3d</p> <p>MP.2 MP.3 MP.6 MP.7</p>	<p>Access Prior Learning: In lessons 13-1 through 13-4, students used quantitative reasoning and models to compare fractions with either the same numerator or the same denominator. Students have also represented fractions on a number line.</p> <p>Developing the Big Idea: Students further <i>develop</i> their fractional sense by comparing fractions using a number line.</p>	<p>Instructional note: Lesson 13-6 is recommended to be taught before 13-5, as it offers stronger visual representations for comparing fractions and establishing benchmark fractions, which will be explored further in 13-5. Watch for students that are confusing the comparison symbols. Students may have understanding of which fraction is greater than or less than, but still have confusion about which symbol accurately communicates their relationship. Interview these students to determine if they understand the mathematics.</p> <p>Look Back: After students have shared their solution methods and reasoning, consider discussing the <i>Look Back!</i></p> <p>Assess and Differentiate: If time permits, you may consider replacing the <i>Math and Science Activity</i> with the game <i>Display the Digit</i> (TE, p. 677A), <i>Tic Tac Toe</i> (TE, p. 689A), <i>Think Together</i> (TE, p. 701A), or the <i>Fluency Practice Activity</i> (TE, p. 721).</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 707A).</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using the <i>Convince Me!</i></p>
Lesson 13-5: Compare Fractions- Use Benchmarks		
<p>3.NF.A.3d</p> <p>MP.1 MP.2 MP.3</p>	<p>Access Prior Learning: In lessons 13-1 through 13-4, students used quantitative reasoning and models to compare fractions with either the same numerator or the same denominator. Students have also represented fractions on a number line.</p> <p>Developing the Big Idea: Students are further <i>developing</i> fractional sense by comparing fractions using the benchmark fractions 0, $\frac{1}{2}$, and 1 to reason the larger fraction.</p>	<p>Solve & Share: Consider discussing the <i>Look Back!</i> to support students' quantitative reasoning about fractions.</p> <p>Visual Learning: Consider assigning the <i>Convince Me!</i> to reinforce ideas shared in the <i>Visual Learning Animation</i>.</p> <p>Assess and Differentiate: If time permits, consider teaching students how to play <i>Think Together</i> (TE, p. 701A). All students should have an opportunity to play this game it provides engaging and meaningful practice of a key concept.</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 701A).</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional Note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using item 18 <i>MP. 3 Critique Reasoning</i> from the <i>Independent Practice/Math Practices and Problem Solving</i>.</p>

Lesson 13-7: Whole Numbers and Fractions		
<p>3.NF.A.3c 3.NF.A.3a</p> <p>MP.2 MP.3 MP.7</p>	<p>Access Prior Learning: In lesson 12-5, students represented fractions on a number line, including fractions greater than 1. In lessons 13-1 and 13-2, students learned that when using fractions to name quantities, a quantity can have more than one equivalent fraction.</p> <p>Developing the Big Idea: In this lesson, students are further <i>developing</i> their fractional sense by finding that whole numbers can be represented by many different fraction names.</p>	<p>Instructional note: Consider making this a 2-day lesson as the <i>Developing the Big Idea</i> understanding in this lesson is critical for future grade level work with fractions and has much depth; especially when connecting these understandings to division concepts.</p> <p>Solve & Share: Consider covering the given model so students are unable to see it and thus more inclined to represent the mathematics using tools, models or other representations that makes the most sense to them.</p> <p>Visual Learning: Consider pausing and informally assessing students' connections to Topic 12 and lesson 13-2 when the question is posed, "What are some equivalent fraction names for 1, 2, and 3? How do you know?"</p> <p>After viewing the <i>Visual Learning Animation</i> consider asking students what is the difference between $\frac{1}{2}$ and $\frac{2}{1}$ and create a model to prove the difference.</p> <p>Assess and Differentiate: If time permits, you may consider replacing the <i>Problem Solving Reading Mat</i> with the game <i>Display the Digit</i> (TE, p. 677A), <i>Tic Tac Toe</i> (TE, p. 689A), <i>Think Together</i> (TE, p. 701A), or the <i>Fluency Practice Activity</i> (TE, p. 721).</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 713A).</p> <p>Possible Day 2 Solve & Share: (Read the <i>Instructional Note</i> at the beginning of this topic for guidance on making a lesson more than 1 day.) Consider using items 18 <i>Higher Order Thinking</i> from the <i>Homework & Practice</i>.</p>
Lesson 13-8: Math Practices and Problem Solving- Construct Arguments		
<p>3.NF.A.3b 3.NF.A.3d</p> <p>MP.3 MP.1 MP.4 MP.5 MP.6</p>	<p>Access Prior Learning: In previous lessons, students have developed an understanding that when using fractions to name quantities, a quantity can have more than one equivalent fraction.</p> <p>Developing the Big Idea: Students are further <i>developing</i> their understanding of equivalent fractions and comparing fractions by constructing arguments to solve problems in real-world contexts.</p>	<p>This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 3. Refer to the <i>Math Practices and Problem Solving Handbook</i> (TE, pp. F23-F23A, F29) for suggestions on how to develop, connect and assess this Math Practice.</p> <p>Solve & Share: Consider reintroducing MP. 3 Thinking Habits (SE, p. F23) before introducing the <i>Solve & Share</i>. Also consider using time students are working on the <i>Solve & Share</i> as an opportunity to child-watch for behaviors associated with MP.3 that are listed in the <i>Math Practices and Problem Solving Handbook</i> (TE, p. F23A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice.</p> <p>Look Back: After discussing students' solution methods and reasoning, consider discussing the <i>Look Back!</i> if these ideas did not already come out from the classroom discussion.</p> <p>Visual Learning: Consider pausing to discuss, "Why are 2 number lines a good drawing to justify the conjecture?" and, "Is this a representation you might try to use? Why or why not?"</p> <p>Assess and Differentiate: If time permits, consider teaching students how to play the game <i>Teamwork</i> (TE, p. 719A).</p> <p>Child-watch to identify students who need additional support and consider the <i>Intervention Activity</i> provided (TE, p. 719A).</p> <p>*CTC: <i>Math Practices and Problem Solving</i> (student work samples)</p>

References

- Chapin, S. H., & Johnson, A. (2006). *Math matters: Understanding the math you teach, Grades K-8*. Sausalito, CA: Math Solutions Publications.
- Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). 3-5, Number and Operations—Fractions*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.
- Empson, S. B., & Levi, L. (2011). Extending children's mathematics: Fractions and decimals. *Mathematics Education*, 27(4), 403-434.
- Small, M. (2014). *Uncomplicating fractions to meet common core standards in math, K-7*. New York, NY: Teachers College Press, Nelson Education.
- Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). New York, NY: Pearson.