Grade 5 Topic 3: Fluently Multiply Multi-Digit Whole Numbers; lesson 13-3 concepts

Topic 3
Fluently Multiply Multi-digit Whole Numbers and* Lesson 13-3 *concepts
Number of lessons: **7**
A/D/E: **3** days
NVACS Focus: NBT.B
Total days: ~10

[5th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-19)

Prior to instruction, view the Topic 3 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 109A-109C), all 7 lessons, and the Topic Assessments (pp. 161-162A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What are the standard procedures for estimating and finding products of multi-digit numbers? *Reference Answering the Topic Essential Questions (TE, pp. 159-160) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

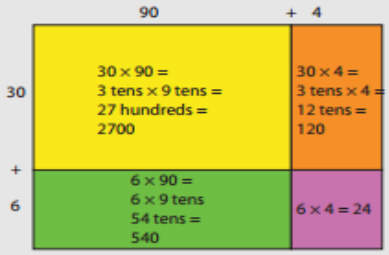
3-1	3-2	3-3	3-4	3-5	3-6	3-7	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:
This topic focuses on multiplying multi-digit whole numbers. Focus instruction on Nevada Academic Content Standard (NVAC) 5.NBT.B.5, "Fluently multiply multi-digit whole numbers using the standard algorithm" (2010). The Progression Documents add that students should also understand that "underlying this algorithm are the properties of operations and the base-ten system" (Common Core Standards Writing Team (CCSWT), 2015, p. 16).

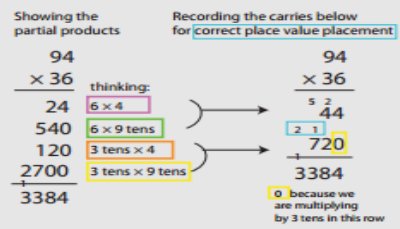
5.NBT.B.5 is the only standard in fifth grade that calls for fluency. As defined by the NVACS, fluency refers to "skill in carrying out procedures **flexibly, accurately, efficiently** and **appropriately**" (NVACS, 2010, p. 6). Development of fluency occurs in three phases: 1) Constructing meaning and counting strategies 2) Reasoning strategies and 3) Working toward quick recall (Van de Walle, Karp, Lovin & Bay-Williams, 2014). The foundations for fluency are built in the third and fourth grades as students apply multiplicative thinking and generalize about place value for multi-digit whole numbers (NVACS, 2010, 4.OA.A, 4.OA.B, 4.NBT.A). Additionally, standard 4.NBT.B.5 requires students to multiply multi-digit whole numbers by a one-digit whole number and to multiply two two-digit whole numbers using strategies based on place value and the properties of operations. Students must "**Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models**" (NVACS, 2010, p. 29). Maintain this focus on reasoning by discussing various strategies and algorithms and the connections between them.

Illustrating partial products with an area model



The products of base-ten units are shown as parts of a rectangular region. Such area models can support understanding and explaining of different ways to record multiplication. For students who struggle with the spatial demands of other methods, a useful helping step method is to make a quick sketch like this with the lengths labeled and just the partial products, then to add the partial products outside the rectangle.

Methods that compute partial products first



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from 30 × 4 = 120 is placed correctly in the hundreds place and the digit 2 from 30 × 90 = 2700 is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second row of the method on the right is there because the whole row of digits is produced by multiplying by 30 (not 3). Colors on the left correspond with the area model above.

The U.S. traditional algorithm for multiplication is based on the distributive property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand. **Instruction that includes the conceptual knowledge behind procedures is crucial**. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999).

Use of area models and the partial products algorithm rely on the distributive property and can be used to help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These standard algorithms allow students to see

how partial products are created while avoiding the errors that often occur when carrying and recording using the U.S. traditional algorithm. They also allow students to work horizontally or vertically and to multiply factors and add partial products in varying order. These "slight changes in recording will scaffold all students to fluency" (Bay-Williams, *Topic 3 PD video*, enVisionmath2.0).

Area models and the partial products algorithm can be very efficient. When using place value strategies and algorithms, students create separate values for each partial product. Often these strategies are just as efficient, and require minimal if any additional time as compared to other standard algorithms. The area model has many advantages over the U.S. traditional algorithm, especially as students begin using two-digit multipliers (Van de Walle, et al., 2014). Students should understand that multiple strategies can be used to solve these problems with some being more appropriate than others in certain instances.

Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014). Although the lessons of Topic 3 will quickly focus on use of the U.S. traditional algorithm, encourage students to use multiple procedures and connect what they are doing to place value understanding. Do not require students to work with only one algorithm. Instead, ask students to compare different algorithms using the same factors. They will see the connections and naturally move towards using more efficient strategies and algorithms. Students need time to build a conceptual understanding of multi-digit multiplication prior to using abstract procedures. Students experiences with the area model will also help build an understanding of division in later Topics. The area model can visually represent division contexts as "missing factor" problems and connect student understanding between these inverse operations.

Please note that the components of fluency do NOT include memorization. When instruction focuses on memorization, students are less willing to think about numbers and their relationships and to apply and develop their number sense (Boaler, 2009).

> *High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed testing stand in the way of number sense, giving students the impression that sense making is not important (Boaler, 2015, para.13).*

When assessing fluency, AVOID timed tests. Approximately one-third of students begin to experience math anxiety at the onset of timed testing. Brain research also concludes that stress blocks the working memory, preventing students from accessing math facts they know (Boaler, 2014). In addition, timed tests do not tell us which strategies a student used or their level of flexibility. It is important that our instruction and assessment focus on numbers and their relationships. Better options for assessment include student interviews, observations, journaling or quizzes based on strategies (Bay-Williams & Kling, 2014).

Justification for including 13-3 concepts: Concepts from Lesson 13-2 (grouping symbols) and 13-3 (writing numerical expressions) can be included in Topic 3 instruction. The focus of lesson 13-3 is to represent multi-step problems with numerical expressions. Students can analyze and interpret the Solve and Share as a numerical expression before solving. Multi-step problems that allow students to record the calculations needed to solve as a numerical expression may also be found in the *Visual Learning Bridge* and in the *Math Practices and Problem Solving* pages throughout Topic 3

Math Practice 3: Construct viable arguments and critique reasoning of others
Focus on opportunities for students to develop *Mathematical Practice 3* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 3-7. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
underestimate overestimate	*partial products* *variable* *numerical expression* *equation* *factor* *product*

Additional terminology that students may need support with: *exponent*

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to estimate products of multi-digit whole numbers and explain if their estimate is reasonable using place value understandings?"
"Are students using a partial products strategy to multiply multi-digit whole numbers?"

Lesson	Evidence	Look for
3-2	*Solve and Share* (student work samples)	Focus CTC around the big idea: • reasoning based on place value understanding to justify estimates • reasonable estimates for the context of a situation
3-2	*Quick Check* (digital platform) Items 1, 3 and 4	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
3-5	*Guided Practice* (student work samples) Item 2	Focus CTC around the big idea: • student strategies and models used to multiply • correct use of multiplication to model context
3-5	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 159-162	Use *Scoring Guide* TE pp. 159-162A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 3-1: Multiply Greater Numbers by Powers of 10		
5.NBT.A.2 MP.1 MP.3 MP.5 MP.6 MP.7	Access Prior Learning: Students practiced multiplying powers of 10 during Topic 1. Developing the Big Idea: Students extend place value understanding to multiply multi-digit numbers by powers of 10.	Solve and Share: Students will multiply a single digit by powers of 10 and analyze patterns of zeros in the products. They can model this problem using concrete tools such as base-ten blocks, representational tools (Teaching tools 4 and 5) and numerical expressions. The *Look Back!* can be used to help students extend knowledge of the patterns they describe to a similar context. Watch for students that use language such as "just add a zero" or "zero trick" to describe patterns seen in the products. Steering students towards precise language such as "a place value is added" will help to avoid misconceptions when students begin multiplying with decimal factors during the next topic. Visual Learning: Students will see how patterns of zeros in the products created by multiplying with powers of 10 extend to using two-digit factors. Powers of 10 are shown in exponent form. Students will need to recognize the equivalent powers of 10 written in standard form and exponent form. Both of these forms will be used to multiply two-digit factors on the *Guided and Independent Practice* pages. The *Convince Me!* can be used to facilitate a discussion that will help students connect the term annexing zeros to place value patterns. Assess and Differentiate: The *Homework and Practice* page is very similar to the *Guided practice*. Students showing proficiency with these item types will find more challenge on the *Math Practices and Problem Solving* and the "Common Core Performance Assessment" on *Homework and Practice* (SE, pp. 116 and 118). Consider using the "Common Core Assessment" problems (SE, p. 116) with students as part of the formative assessment process. Focus on using reasoning to explain solutions.

Lesson 3-2: Estimate Products		
5.NBT.B.5 MP.1 MP.2 MP.3	Access Prior Learning: Students have estimated in previous grades and topics. Make connections to the various types of estimation. Developing the Big Idea: Students will build number sense by estimating products and reasoning about how different estimation strategies produce products above or under the actual product.	Solve and Share: This lesson focuses on using estimation when multiplying in order to determine the relative size of the product. This practice will help students think about the reasonableness of their answers as they build procedural fluency. Students may need to be cued to the words in the problem that signal estimation. Consider asking students why shopping is a context that often requires estimation. As students share their estimates, draw out the idea that there are multiple ways to estimate. Different methods will give estimates that are above or below the actual answer. Visual Learning: The *Visual Learning* demonstrates rounding as one estimation strategy. *Another Example* uses compatible numbers to model an additional strategy (SE, p. 121). Ask students to think about whether an underestimate or an overestimate is created. Students may need clarification on what these terms mean and practice analyzing how rounding factors up or down changes the estimate. What happens when both factors are rounded down versus rounded up? How can we use this understanding to estimate more precisely? Consider using *Math Practice and Problem Solving* items 23 and 25 to formatively assess student reasoning. Assess and Differentiate: If needed, use the *Intervention Activity* in small groups or whole class if more practice with rounding and estimating. Use *Math Practice and Problem Solving* (SE, p. 122) or *Homework and Practice* (SE, p. 124) to challenge students to apply their understanding. *CTC*: Guided Practice* (student work samples) Item 2 *CTC*: Quick Check* (digital platform) Items 1, 3 and 4
Lesson 3-3: Multiply 3-Digit By 2 Digit-Numbers		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.7 MP.8	Access Prior Learning: In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5) Developing the Big Idea: Students learn to extend multiplication strategies to include larger multi-digit numbers. Emphasize the connections between place value and partial products using different multiplication algorithms/strategies.	Solve and Share: Students multiply a three-digit whole number by a two-digit whole number using a strategy or algorithm of their choosing. *Teacher's Edition* page 125 under *Analyze Student Work* shows an example of an area model and the partial products algorithm. These strategies are important for building a conceptual understanding of using the distributive property to solve multi-digit multiplication problems. Consider showing the same problem solved by an area model, the partial products algorithm and the U.S. traditional algorithm side by side. Ask students to look for connections between the strategies. Facilitate a discussion about what property is used in all three strategies and how the partial products are created in each. Visual Learning: The U.S. traditional algorithm for multiplication is modeled using multi-digit factors. Students ready for this strategy will find it to be an efficient way to multiply. Look for mistakes in "carrying" and placing the zero as a place holder. Students making these mistakes will greatly benefit from continued use of the area model or the partial products algorithm (see Topic 3 Instructional Note). These strategies can also become very efficient and will scaffold students towards fluency with standard algorithms. Consider providing a separate page to allow space for students to use visual models as they complete the *Guided and Independent Practic*e problems. Assess and Differentiate: The *Intervention Activity* and *Reteach* page focus only on the U.S. traditional algorithm. Students using other algorithms or strategies will benefit from completing the *Homework and Practice* items on a separate blank page that will allow room for visual models or alternative approaches. Although *Homework and Practice* items 1 and 2 reteach the standard algorithm, allow students to use multiple strategies (SE, p. 129). It is important to focus on understanding rather than using an algorithm.
Lesson 3-4: Multiply Whole Numbers with Zeros		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5). Developing the Big Idea: Students learn to extend multiplication strategies to include larger multi-digit numbers. Students will see how zeros within multi-digit whole numbers can create partial products when multiplying.	Solve and Share: Consider beginning with the *Look Back!* to focus on making sense of problems before solving. Students will multiply multi-digit numbers that contain a zero in one of the factors. *Analyze Student Work* shows an example of the Distributive Property used to decompose a problem into smaller simpler problems (TE, p. 131). Look for students using multiple strategies and facilitate a discussion noting how the partial products are created. What connections can be seen between the varying strategies? Look for the common misconception that the zeros in factors do not create partial products. *-continues on next page-*

		Visual Learning: Emphasize estimating before calculating to help students determine the reasonableness of their solutions. The U.S. traditional algorithm is modeled. Once again, look for mistakes in "carrying" and placing the zero as a place holder. Students making these mistakes will greatly benefit from continued use of the area model or the partial products algorithm (see Topic 3 Instructional Note). Students should use a strategy they understand to build understanding before moving to the more abstract standard algorithms. Assess and Differentiate: The *Intervention Activity* asks students to find the partial products created from a multi-digit multiplication problem. Have students use concrete, representational and/or abstract strategies. This helps students to understand the creation of partial products. The *Reteach* and *Homework and Practice* page focus on use of the U.S. traditional algorithm. Consider giving students a separate page to model their strategies. Use *Homework and Practice* item 25 to assist students in thinking about how partial products from the area model or partial products algorithm combine to form the larger partial products seen in other standard algorithms; including the U.S. traditional algorithm.
Lesson 3-5: Multiply Multi-Digit Numbers		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4	Access Prior Learning In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5) Developing the Big Idea Students build procedural skill with multi-digit multiplication while deepening conceptual understanding through modeling contexts with multiplication.	Solve and Share: Students must create a context that matches a numerical equation with multiplication. Look for students who use a real world context versus restating the equation in words (e.g. multiply 36 by 208 and find the answer). Note the use of a variable in this problem. Facilitate a whole class discussion around student contexts and how these contexts represent the given equation. Draw out generalizations about the types of contexts that require multiplication. The *Look Back!* highlights how the Commutative Property allows us to multiply factors in any order, however doing so may change the context. Visual Learning: A bar diagram is used to model a multiplication problem. Consider building on the *Solve and Share* discussion to determine how this model represents the given context. The *Convince Me!* gives students a chance to practice mathematical reasoning with justification (MP.2). The *Guided and Independent Practice* page require use of the U.S. traditional algorithm for items 3 - 10. Students building understanding using other strategies can begin with items 11-22. Students demonstrating proficiency can move to *Math Practices and Problem Solving* for practice modeling contexts with multiplication (SE, p. 140). Assess and Differentiate: The *Intervention Activity* provides additional practice modeling situations with multiplication. The *Reteach* and *Homework and Practice* page scaffold using the distributive property to decompose multi-digit factors and create partial products. *CTC: *Guided Practice* (student work samples) Item 2 *CTC: *Quick Check* (digital platform)
Lesson 3-6: Solve Word Problems using Multiplication		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning In 4th grade, students used multiplication to solve word problems (4.OA.A.3). Securing the Big Idea Students will apply knowledge of multiplication to model and solve word problems.	Solve and Share: Challenge students to model a real world context as a math problem (mathematical model). Initially, students may use many strategies to make sense of this context. Challenge students to check their thinking by modeling and solving this problem more than one way (MP.4). Facilitate a discussion using student models and strategies. Focus on why multiplication can be used to model this situation. Quickly have students connect the factors and product of their mathematical model (equation) back to the context of the problem to assist with solidifying the understanding of what a mathematical model is. The *Look Back!* will remind students that they should always use estimation to determine whether their answer is reasonable. Visual Learning: Students will continue to extend their understanding that multiplication problems are modeled and solved in different ways. Draw out the idea that students could use the properties, multiple strategies and models, and estimation to check the accurateness of their answers. The *Guided and Independent Practice* problems ask students to apply understanding of multiplication to model and solve real world problems. Assess and Differentiate: The *Intervention Activity* teaches a keyword strategy which has been shown to be ineffective, consider omitting. The *Reteach* and *Homework and Practice* page items ask students to model word problems using a bar diagram.

Lesson 3-7: Math Practices and Problem Solving- Critique Reasoning		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.6	Access Prior Learning: Students should have prior experience communicating their own reasoning and critiquing the reasoning of others. Make connections to prior class discussions. Securing the Big Idea: Students will apply understanding of multiplication to solve word problems and critique the mathematical reasoning of others.	Solve and Share: Challenge students to determine and justify their answer to this problem using estimation and reasoning before calculating. Facilitate a discussion about how students can determine if someone else's solution makes sense (MP.3). How can students make sure they are clearly communicating their own mathematical ideas? How can you decide which situations require estimation versus an exact answer? Visual Learning: Ideas on how to determine the reasonableness of other's thinking are given. Consider comparing/connecting these to the *Solve and Share* class discussion. Incorporate the thinking habits tied to MP.3 (SE, p. 149). The *Convince Me!* provides an opportunity to practice reasoning, explaining and critiquing the mathematical reasoning of others. The *Guided and Independent Practice* provide opportunities for students to critique reasoning with a focus on MP 3 (SE, pp. 151). Consider choosing a limited number of items for practice and encourage students to give complete and thoughtful answers. The *Math Practice and Problem Solving* provides an opportunity for students to work with multiple math practices (SE, p. 152). Assess and Differentiate: The *Intervention Activity* and *Reteach* page ask students to analyze and critique mathematical reasoning using the concepts of underestimate/overestimate. Although it is important to analyze and critique these written mathematical arguments, consider spending more time creating and critiquing arguments within your class discussions.

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