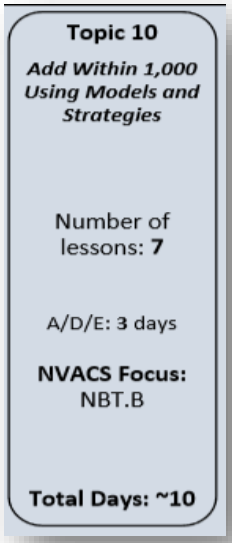
Grade 2 Topic 10: Add Within 1,000 Using Models and Strategies



Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 583A-583E), the Topic Planner (pp.583I-503K), the Topic Performance Assessments (pp. 633-634) all 7 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 583A-583E)	Topic Essential Question: What are strategies for adding numbers to 1,000? *Reference Answering the Topic Essential Question (TE, p. 631-632) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	10-5	10-6	10-7	Assessment

3 A/D/E days used strategically throughout the topic.

2nd Grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:

The big idea of Topic 10 focuses on using models and strategies to add within 100.

> *…there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, Karp, Lovin, Bay-Williams, 2014, p.176).*
>
> *It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).*

Topics 10-11 compose a major cluster focused on the big idea of the base-10 numeration system through addition and subtraction within 1,000. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.NBT.B.

> *2.NBT.B Use place value understanding and properties of operations to add and subtract.*
>
> *7. Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.*
>
> *8. Mentally add 10 or 100 to a give number 100-900, and mentally subtract 10 or 100 from a given number 100-900.*
>
> *9. Explain why addition and subtraction strategies work, using place value and the properties of operations.*

This work builds upon understandings developed in Topics 3-5 and Topic 9. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above and excerpted here, "it is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers" (National Council of Teachers of Mathematics, 2000, p.83). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding 100 as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that allow students to see 100 as 10 groups of ten or 100 singles (connecting cubes, beads in a jar, linked

paper clips, etc.) are essential. Groupable models allow children to move from a count-by-ones approach, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a hundreds flat, is worth 100 singles or 10 tens is ineffective. When considering language, help students connect standard language, "one hundred thirty-five", to base-ten language, "1 hundred 3 tens 5 ones; 1 group of a hundred 3 groups of ten 5 ones, etc". Also, it is recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

Topics 10-11 are mirrored topics, as the strategies used for addition in Topic 10 are later used for subtraction in Topic 11. Focus planning conversations to go beyond *what* strategies are used to *why* those strategies are important for students' development of the big idea. Reference the lesson level instructional notes below for content to support these conversations. In both topics, students will work with algorithms. The authors of enVisionmath2.0 placed the algorithms in sequence with other strategies, with the intent that students connect their understanding of place value strategies to construct meaning of the algorithms. They also intended for students to see algorithms as one of many strategies for addition and subtraction, not the pinnacle of addition and subtraction strategies.

As NVACS 2.NBT.B.7 states, "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; *relate the strategy to a written method*. Understand in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." The "Progressions for the Common Core State Standards in Mathematics" elaborate on what it means to "relate to a written method", by including the following examples for addition:

Addition: Recording newly composed units in separate rows

278	278	278	278
+147	+147	+147	+147
	300	300	300
		110	110
			15
			425

The computation shown proceeds from left to right, but could have gone from right to left. Working from left to right has two advantages: Many students prefer it because they read from left to right; working first with the largest units yields a closer approximation earlier.

Addition: Recording newly composed units in the same row

Add the ones, $8+7$, and record these 15 ones with 1 on the line in the tens column and 5 below in the ones place.

Add the tens, $7+4+1$, and record these 12 tens with 1 on the line in the hundreds column and 2 below in the tens place.

Add the hundreds, $2+1+1$, and record these 4 hundreds below in the hundreds column.

Digits representing newly composed units are placed below the addends, on the line. This placement has several advantages. Each two-digit partial sum (e.g., "15") is written with the digits close to each other, suggesting their origin. In "adding from the top down," usually sums of larger digits are computed first, and the easy-to-add "1" is added to that sum, freeing students from holding an altered digit in memory. The original numbers are not changed by adding numbers to the first addend; three multi-digit numbers (the addends and the total) can be seen clearly. It is easier to write teen numbers in their usual order (e.g., as 1 then 5) rather than "write the 5 and carry the 1" (write 5, then 1).

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The first written method for addition, Partial Sums, records intermediate steps and is helpful in building toward the second written method for addition, the U.S. Traditional algorithm. The Progression Documents go on to articulate that drawings, such as the one pictured below can be used by students in explaining the written methods above. Knowing that our trajectory is building toward the expectation that students will relate strategies to a written method when adding within 1,000, we can view the lessons in Topic 10 as building onto addition algorithms introduced in Topic 4. However, in regards to transitioning from the first written method (Partial Sums) to the second written method (U.S. Traditional Algorithm), the progression document also states, "Some students might make this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency requires a more compact method." Based on this, we should offer opportunities for our students to construct meaning of the algorithms, but we should not expect all students to transition to use of the *standard* algorithm in second grade. The progression documents recommend that students' solutions that involve count-on or add-on strategies continue to be discussed. It goes on to state that that the major focus "for addition within 1000 needs to be on methods such as those [pictured above] that are simple for students and lead toward fluency (e.g., recording new units in separate rows shown) or are sufficient for fluency (e.g., recording new units in one row)." (CCSWT, 2015, p.10).

Illustrating combining like units and composing new units

The drawing shows the base-ten units of 278 and 147. Like units are shown together, with boundaries drawn around ten tens and ten ones to indicate the newly composed hundred and the newly composed ten. The newly composed units could also be indicated by crossing out grouped units and drawing a single next-highest unit, e.g., crossing out the group of ten ones and drawing a single ten. Drawings like this can be used to illustrate and explain both of the written computations below.

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Math Practice 8: Look for and express regularity in repeated reasoning

Focus on opportunities for students to develop MP.8 behaviors. This is the focus of the Math Practices and Problem Solving lesson 10-7. Reference the Teacher's Edition (pp. F30-F30A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	break apart *compensation* *digit* *equals, =* *hundred* *mental math* *ones* *open number line* *partial sum* place-value chart *regroup* *sum* *tens* *thousand*

Additional terminology that students may need support with: *algorithm, models, patterns, standard algorithm, unit*

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding question: "Are students able to use different addition strategies and explain why they work?"

Lesson	Evidence	Look for
10-4	*Do You Understand: Show Me!* (student work samples)	Focus CTC around the big idea: • student strategies and models • use of multiple strategies to check work • explanation of strategy
10-1	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 631-634	Use *Scoring Guide* TE pp. 631-634

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 10-1: Add 10 And 100		
2.NBT.B.8 2.NBT.B.9 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In first grade, (1.NBT.C.5) given a 2-digit number, students found 10 more or 10 less without counting. Developing the Big Idea: In this lesson, students are *developing* understanding of adding 10 or 100 to 3-digit numbers using place value patterns and mental math.	Students can use basic facts to help them mentally solve problems when adding by 10 and 100. Using place value blocks will reinforce conceptual understanding that the tens digit goes up by 1 when adding ten, and that the hundreds digit goes up by 1 when adding 100. These patterns also build on learning opportunities from Topic 9. Topic Opener: Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE, p. 583), *Review What You Know* (TE p. 584), and *Vocabulary Review Activity* (TE, p. 584) only. Post the essential question and student strategies on your focus wall. Consider engaging students in skip-counting by 10s and 100s using place value blocks, place value charts or hundreds charts. Solve & Share: This *Solve and Share* builds on students' study of place value patterns when skip counting by 5s, 10s, and 100s in Topic 9. Child-watch for evidence of this understanding in students' mental math strategies and explanations. Visual Learning: Encourage students to generalize their understanding by identifying another equation that demonstrates the pattern in the animation. For example, when the animation shows that adding 10 makes the tens digit go up by 1, ask students to use a whiteboard and marker to write an equation for a different 3-digit number for which this also applies (e.g., 482 + 10 = 492). Students may also be asked to identify a basic fact, which helped them to solve their equation. Additional time may need to be spent on the final frame of Visual Learning, which identifies situations when adding 10 changes the tens and hundreds digits (e.g., 290 + 10 = 300). Independent Practice/Math Practices and Problem Solving: As previously indicated, students do NOT need to do all of the problems in their Student Edition (SE). However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate. *CTC: *Quick Check* (digital platform)
Lesson 10-2: Add On An Open Number Line		
2.NBT.B.7 2.NBT.B.9 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In Topics 3 and 5, second grade students used the open number line and break apart strategies to model addition and subtraction with 2-digit numbers. Developing the Big Idea: In this lesson, students are *developing* understanding that when adding 3-digit numbers, the numbers can be broken apart into hundreds, tens and ones and added with jumps on the open number line. This model allows students to keep track of their thinking.	Open number lines help students keep track of their thinking and allow students to add/subtract by groups of hundreds, tens or ones. The use of an open number line supports place value understanding as it involves decomposing and composing numbers. It also supports students' number sense and computational fluency. Solve and Share: Child-watch for students who make jumps of hundreds, tens and ones. Ask all students to solve the problem two different ways and evaluate their strategies for efficiency. If students use inefficient methods to add on the open number line, ask, "How can jumps of hundreds and tens help you solve the problem more efficiently? Does the number you start with affect your efficiency?" Also, child-watch for students who have trouble crossing into a new century, from 598 into the 600s and 700s. These students may need support with connecting their understanding of the repeated structure and patterns in our number system. Visual Learning: Give students time to solve 481 + 122 by drawing an open number line on a whiteboard, before interacting with the animation. Child-watch for evidence of students who increase their level of efficiency from the *Solve and Share*. Reference the "ways" in the *Visual Learning*, which progress from less to more efficient. Highlight these reflective students during the discussion. Although the animation presents two ways that both begin with jumps of 100, students may begin with smaller jumps to get to a landmark number when appropriate for the numbers. For example, in *Guided Practice*, item 2, students may begin with 670, make a jump of 30 to 700, and then jump the remaining 202 in a variety of ways. The use of landmark numbers supports students' work with compensation in lesson 10-3.

Lesson 10-3: Add Using Mental Math		
2.NBT.B.7 MP.1 MP.2 MP.6 MP.7	Access Prior Learning: In Topic 3, second grade students used the break apart strategy to add 2-digit numbers. In Topic 9, second grade students broke apart 3-digit numbers. In the prior lesson, second grade students broke apart 3-digit numbers to add with jumps on an open number line. Securing the Big Idea: In this lesson, students are *securing* understanding of break apart as a mental math strategy for adding 3-digit numbers. Developing the Big Idea: In this lesson, students are *developing* understanding of compensation as a mental math strategy for adding 3-digit numbers.	Solve & Share: Ask all students to solve the problem in two ways and to evaluate the efficiency of each strategy. Child-watch for students who use mental math strategies, as that is the focus of the day's lesson. During problem solving, look for opportunities to ask a student, "Can you solve this problem another way with mental math?" Visual Learning: Prior to interacting with the animation, have students solve the problem presented in the animation using a strategy of their choice. If students have difficulty understanding the strategies presented, refer to the *Prevent Misconceptions* note (TE, p.598) for support suggestions. Independent Practice/Math Practices and Problem Solving: The numbers in item 6 lend themselves nicely to the compensation strategy. Child-watch for students who change 250 + 298 to 250 + 300 = 550, then subtract 2, 550-2 = 548. Assess and Differentiate: The *Intervention Activity*, "Three-Digit Marathon" (TE, p.601A), provides students with support for the break apart strategy, as well as schema that they can draw upon in lesson 10-4 on Partial Sums.
Lesson 10-4: Add Using Partial Sums		
2.NBT.B.7 2.NBT.B.9 MP.3 MP.5 MP.7 MP.8	Access Prior Learning: In Topic 4, second grade students used the partial sums algorithm to solve addition problems with 2-digit numbers. In the prior lesson, second grade students broke apart 3-digit numbers using mental strategies to add hundreds and hundreds, tens and tens, ones and ones, and then added the partial sums. Developing the Big Idea: In this lesson, students are *developing* understanding of the partial sums algorithm when adding two (or more) 3-digit numbers.	Five A/D/E days have been paced into Topic 10. Consider using two of those days to differentiate and enrich students as they work with the Partial Sums algorithm. Ensure that all students model the partial sums algorithm with concrete place value blocks and place-value mats to ensure conceptual understanding as well as procedural understanding. Solve & Share: Child-watch for students who use the break apart strategy to add hundreds and hundreds, tens and tens, and ones and ones. If students use other strategies, honor those strategies, then ask, "How can you break apart by place value to solve the problem?" This will support their understanding of Partial Sums, the focus of the lesson, in the *Visual Learning*. Visual Learning: Prior to interacting with the animation, have students solve the problem, 518 + 327 using concrete place value blocks or drawings and a place value mat. Have students work in pairs during the animation. One student should model with place value blocks, and the other partner should record the step (Van de Walle, et al., 2014, p.219). Trade roles when solving the *Guided Practice* problems. Independent Practice/Math Practices and Problem Solving: Have students use the Partial Sums algorithm, and a second strategy of choice to check for accuracy. Connecting strategies will support students in making sense of the Partial Sums algorithm, as a way to *relate to a written method*. This is stated as an expectation in 2.NBT.B.7, and explained in more depth in the Instructional Note at the beginning of this document. *CTC: *Do You Understand: Show Me!* (student work samples)
Lesson 10-5: Use Models to Add		
2.NBT.B.7 2.NBT.B.9 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: In lessons 4-3 and 4-4, second grade students used the standard addition algorithm to add 2-digit numbers. Developing the Big Idea: In this lesson, students are *developing* understanding of the standard addition algorithm. Students progress from concrete to symbolic work with the algorithm. They also regroup ones and tens.	As indicated in the Instructional Note at the beginning of this document, the progression document states the following in regards to the standard algorithm: "Some students might make this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency requires a more compact method." Based on this, we should offer opportunities for our students to construct meaning of the algorithms, but we should not expect all students to transition to use of the *standard* algorithm in second grade. Solve & Share: Continue to encourage use of place-value blocks and place-value mats. Consider offering a blank workspace to allow students to strategically select an addition strategy. *-continues on next page-*

		Visual Learning: Have students solve the problem in the animation using concrete place value blocks or drawings and a place value mat. Have students work in pairs during the animation. One student should model with place value blocks, and the other partner should record the step (Van de Walle, et al., 2014, p.219). Trade roles when solving the *Guided Practice* problems. Independent Practice/Math Practices and Problem Solving: Encourage students to try either the Partial Sums or standard algorithm, and use a second strategy of choice to check for accuracy. Connecting strategies to the algorithm will support students in sense making. Offering students a blank workspace is also helpful. Assess and Differentiate: The *Intervention Activity*, "Regroup to Add" (TE, p. 613A) may be modified to support students with the Partial Sums algorithm, rather than the standard algorithm.
Lesson 10-6: Explain Addition Strategies		
2.NBT.B.9 2.NBT.B.7 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In Topics 3 and 4, second grade students used several addition strategies and algorithms. Securing the Big Idea: In this lesson, students are *securing* understanding of addition strategies with 3-digit numbers. Students will select a strategy and explain why it works using place value and properties of operations. Students may not be secure in *every* strategy, but should demonstrate security with a variety of strategies.	Solve & Share: Ask students to solve the problem with two different strategies, and then encourage them to evaluate which was more efficient. Select and sequence the share to include a variety of student strategies that increase in efficiency. As students explain their strategies, encourage them to use precise academic vocabulary, referring to the math focus wall as needed. Independent Practice/Math Practices and Problem Solving: Item 8 offers students an opportunity to engage in MP.3 behaviors. Some students may benefit from making a visual representation of Tommy's strategy before writing an explanation.
Lesson 10-7: Math Practices and Problem Solving: Repeated Reasoning		
2.NBT.B.7 2.NBT.B.9 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In first grade, students engaged in the Standards for Mathematical Practice including MP. 8 Look For and Express the Regularity in Repeated Reasoning. Developing the Big Idea: In this lesson, students are *developing* understanding of Math Practice 8: Look For and Express the Regularity in Repeated Reasoning by thinking about things that repeat in a problem, and using one problem to help them solve others.	Consider using the *Math Practice 8 Animation* on Pearson Realize Online for an example of MP.8 behaviors. Refer to the *Math Practices and Problem Solving Handbook* for ideas on developing, connecting and assessing MP.8 (TE, pp. F30-F30A). MP. 8 Behaviors: • Notices and describes when certain calculations or steps in a procedure are repeated • Generalizes from examples or repeated observations • Recognizes and understands appropriate short cuts • Evaluates the reasonableness of intermediate results Visual Learning: Prior to interacting with the animation, have students solve the problem, 235 + 489 with a strategy of their choice. During the animation, have students model with place value blocks to determine if regrouping is needed to make a ten or a hundred. Independent Practice/Math Practices and Problem Solving: Place a sticky note over the workspace for items 7-8 to allow students to choose their own strategy for determining if a problem requires regrouping.

References

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

3-Digit Addition Game

Materials:

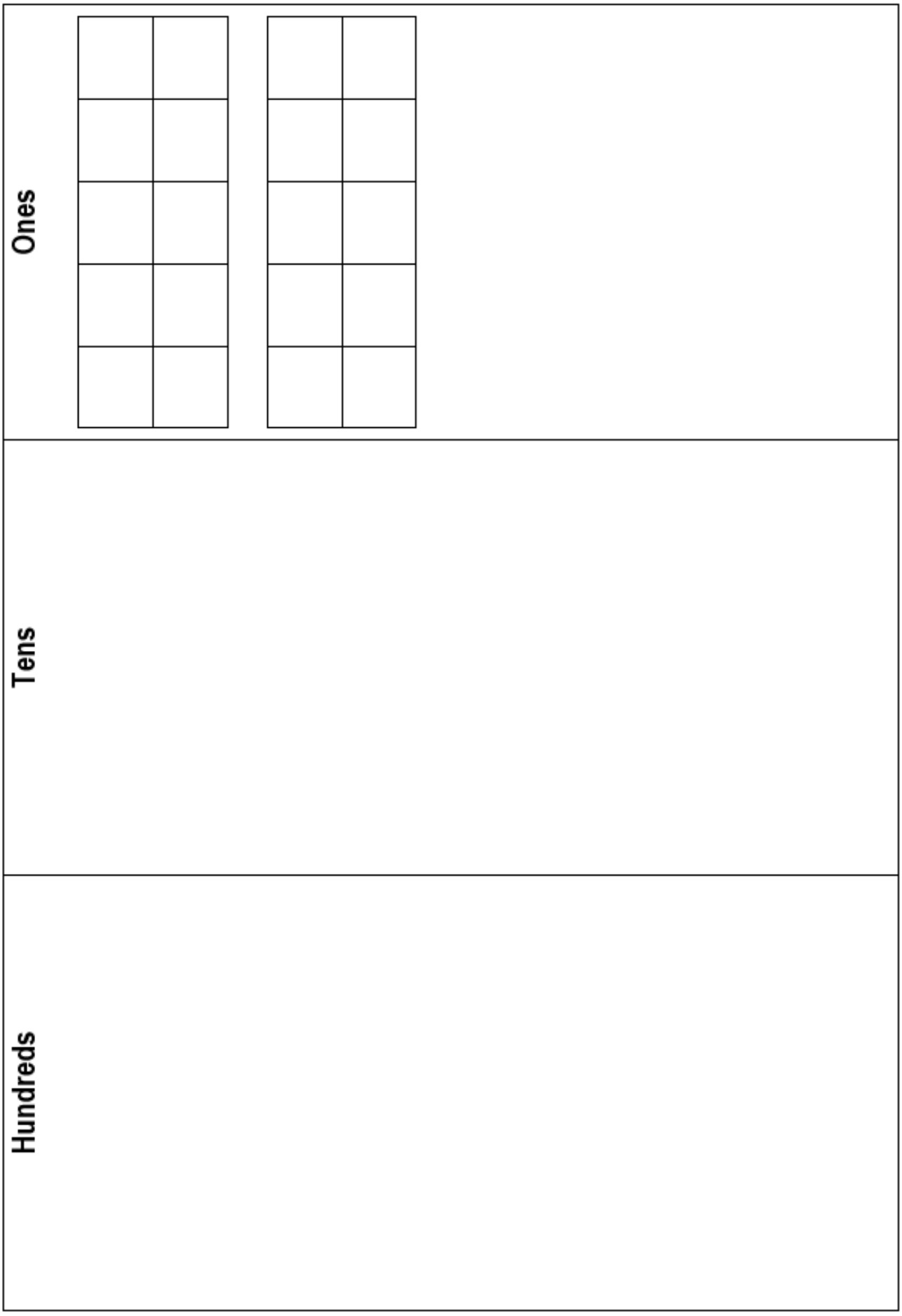
- Place value mat (Blackline master included on the next page, one per player)
- Playing cards: Digits 0-9 cards only
- Tools to support strategies (Place value blocks, whiteboards, etc.)

Players: 2+

Object of the game: To collect the most cards

Directions:

1. Place the card deck face down on the table.
2. Each player draws 6 cards and builds two 3-digit addends.
3. Players use a strategy of choice to find the sum. Players explain their strategy and check each other's work for accuracy.
4. The player with the largest sum takes the cards. In the event of a tie, players draw one more card to add to their sum.
5. Play ends when there are not enough cards for both players to make two 3-digit addends.
6. The player with the most cards wins.


Ones
Tens
Hundreds

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