Grade 4 Topic 5: Use Strategies and Properties to Divide by 1-Digit Numbers

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Number of lessons: **10**

A/D/E: **4** days

NVACS Focus: NBT.B, OA.A

Total Days: ~14

[4th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 5 Professional Development Animation (located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 249A-249D), all 10 lessons, and the Topic Assessments (pp. 323-324A).

Mathematical Background: Read Cluster Overview (TE, pp. 43A-43F)	Topic Essential Questions: How can mental math be used to divide? How can quotients be estimated? What tools, models or strategies can be used to divide? (This essential question was modified to focus on division conceptually.) *Reference the Teacher Edition (p. 249) and Answering the Topic Essential Questions (TE, p. 319-320) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	5-9	5-10	Assessment

4 A/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on dividing multi-digit whole numbers by using place-value understanding and estimation to divide. Focus instruction on Nevada Academic Content Standard (NVACS) 4.NBT.B.6. Emphasis for standard 4.NBT.B.6 is to "find whole-number quotients and remainders up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division" (NVACS, 2010). The U.S. traditional algorithm is not required until Grade 6, give students multiple opportunities to learn and practice division conceptually.

Van de Walle, Karp, & Bay-Williams (2010), state there are different division problem types, "Problems in which the size of the sets is unknown are called fair-sharing or partition problems. If the number of sets is unknown but the size of the equal sets is known, the problems are called measurement or sometimes, repeated subtraction problems" (p. 155). Although we note the different problem types of division throughout the lessons, this is background knowledge for teachers. At this time, students do not need to know the names of the different types of division, partitive and measurement.

In most situations in the real world, division does not end with an equal amount whole number. According to Van de Walle, et al., (2010), "in real contexts, remainders sometimes have three additional effects on the answer; the remainder is discarded (leaving a smaller whole number answer), the remainder can "force" the answer to the next highest whole number and the answer is rounded to the nearest whole number for an approximate result" (p. 155). Van de Walle, et al., (2010) goes on to say, "students should not just think of remainders as "R 3" or "left over". Remainders should be put in context and dealt with accordingly" (p. 156).

Estimation is important as students use estimation strategies to check for reasonableness to their answers. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups that will make finding an estimate easier" (Van de Walle, et al., 2010, p. 247).

Focus Math Practice 4: Model with math

Focus opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 5-10. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
remainder partial quotients	*equation* *divisor* *dividend* *quotient* *compatible numbers*

Additional terminology that students may need support with: estimate, round, area, divisible, division, hidden question

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks or arrays) towards a more sophisticated understanding (partial quotient algorithm; area model)? Are students interpreting remainders based on the context of the problem?

Lesson	Evidence	Look for
5-5	*Higher Order Thinking* (student work samples) Item 13	Focus CTC around the big idea: • student strategies and models. • student interpretation of the remainder.
5-7	*Solve & Share* (student work samples)	Focus CTC around the big idea: • student connection between area model and partial quotients, as well as using properties of operations.

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 319-324	Use *Scoring Guide* TE pp. 319-324A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 5-1: Mental Math- Find Quotients		
4 NBT.B.6 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In third grade, students learned basic division facts and division strategies. In previous topics, students learned to use patterns within our Base Ten System to multiply numbers. Beginning of the Big Idea: In this lesson, students will learn to divide multiples of 10, 100 and 1,000 by one-digit numbers, using the inverse work they did in Topic 3.	Solve & Share: The *Solve & Share* is a partitive division problem. Students may be seen dealing out the 270 "hockey cards" one by one into the 9 boxes. Consider giving students the opportunity to use multiple tools or representations to complete the *Solve & Share*. Consider having students share and compare strategies who may have dealt using concrete tools, used a representation or solved abstractly by using a derived or known facts to solve the problem. Look Back: Consider having a discussion around the inverse operation of division, which is multiplication, when students write an equation 9 X n = 270. Have students understand they were finding 9 groups of *n* = 270. Visual Learning: The *Visual Learning* problem is a partitive division problem. Consider giving students the opportunity to use multiple tools or representations during the *Visual Learning Animation*. Convince Me: The *Convince Me!* presents three missing dividend problems. This may be the first time students have had experience with this type of division problem, this idea connects to place-value patterns and basic fact multiplication. Consider asking, "What basic facts can you use to solve the problem?" Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition (SE). Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. *-continues on next page-*

		Assess and Differentiate/Intervention Activity: Be cautious when doing the *Intervention Activity* with students as it addresses the "zero trick". Instead, consider using place-value blocks to help students understand the patterns in place-value.
Lesson 5-2: Mental Math- Estimate Quotients		
4.NBT.B.6 MP.2 MP.3 MP.4	Access Prior Learning: In the previous topic, students used compatible numbers to estimate the product. Developing the Big Idea: In this lesson, students will think about multiplication and use compatible numbers to find the quotient with a 1-digit divisor.	Solve & Share: The *Solve & Share* is an estimation partitive division problem. Students may deal out the 248 tickets to 3 people one by one. Structure the discussion around students who may have used strategies of estimation. Look Back: Consider facilitating a discussion around the *Look Back!* as estimation is the focus and how it relates to division. Visual Learning: The *Visual Learning* is an estimation partitive division problem. Convince Me: Consider using the *Convince Me!* to facilitate a discussion regarding estimation strategies that may work best for the given situation. In the *Convince Me!* compatible numbers and rounding are compared. Estimation in division often involves compatible numbers as opposed to rounding. Ask students, "What is an easy number to divide by?" Consider reading the Prevent Misconception at the bottom of TE p. 260, as it explains there may be more than one way to use compatible numbers to find a quotient. Guided Practice: Consider having students complete items 1 & 2 in the *Guided Practice* and then facilitate a discussion around the item. Item 1 has students begin to think about the situation and remainders (leftovers). Consider also completing items 3, 4, 8 & 9 as the problems give students an opportunity to use compatible numbers and to think about the inverse operation, multiplication. Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate.
Lesson 5-3: Mental Math- Estimate Quotients for Greater Dividends		
4.NBT.B.6 MP.2 MP.3	Access Prior Learning: In the previous lesson, students estimated quotients by using various estimation strategies. Developing the Big Idea: In this lesson, students will continue estimating quotients.	Solve & Share: The *Solve & Share* is an estimation, measurement division problem. In measurement division problems, students chunk out the number in each group to figure out how many groups are needed or made. Look for students who may have used estimation, and did not find the exact amount of groups. Visual Learning: The *Visual Learning* is an estimation partitive division problem. Students use compatible numbers to estimate, and derive multiplication facts to solve the problem. Another Example: Consider using the *Another Example!* with the whole class as rounding in division is the focus. Compare what students learned in the *Visual Learning Animation* to the *Another Example!*. Read the note on bottom of the Teacher's Edition about the *Another Example!* before the lesson (pp. 267-268). Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students, especially if students are having difficulty with the division in the lesson. The *Intervention Activity* uses estimation and multiplication to divide quotients with greater dividends.
Lesson 5-4: Interpret Remainders		
4.NBT.B.6 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In third grade, students learned how to divide with basic facts. In previous lessons, students used basic facts to divide multiples of 10, 100 and 1,000.	Solve & Share: The *Solve & Share* is a measurement division problem. Students will chunk the 4 apples and group them to fill a basket. Child-watch for students as they interpret the 2 remaining apples. Students should conclude they cannot make another full basket. Visual Learning: The *Visual Learning* is a measurement division problem. Students interpret the remainder or leftover whole number". Consider discussing the context of the problem, how this helps to determine what to do with the remaining amount or leftover. Consider creating an anchor chart for the three different interpretations of remainders. *-continues on next page-*

	Beginning of the Big Idea: In this lesson, students will use models or drawings to find quotients involving remainders.	Convince Me: Consider using the *Convince Me!* as a formative assessment to see if students are using derived facts to determine where the mistake was made. Another Example: The *Another Example* uses counters to make a model representation. Consider having students use the concrete tools or draw the representation to support their understanding of division. Guided Practice: Consider facilitating a whole class discussion around items 1 and 2. By doing these items, students begin to think about remainders and what to do with those remainders: keep them, drop 1 or add one. Item 2 also gives students more opportunity to work with measurement division problems. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students as it is a partitive division problem and supports students' conceptual knowledge of division. Consider supporting students who need more scaffolding regarding a partitive or dealing division problem with remainders.
Lesson 5-5: Division as Sharing		
4.NBT.B.6 4.OA.A.3 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In third grade, students learned the concepts of division as sharing and as repeated subtraction. In the previous lesson, students used models and representations to find quotients with remainders. Developing the Big Idea: In this lesson, students use representations to find quotients, with or without remainders.	Solve & Share: The *Solve & Share* is a partitive division problem as students deal patches into rows. Consider having students who used concrete tools (like counters) and representations (grouping) to solve the problem share their strategies or models. Structure the discussion around an efficient strategy, like mental math rather than long division algorithm. Consider using the *Solve & Share* as a Number Talk. Look Back: Consider facilitating a discussion around the *Look Back!* as students discuss dividing objects into equal groups, but have them think about what happens when objects are not divided equally. Visual Learning: The *Visual Learning* is a partitive division problem. Consider having students use place-value blocks or other representations to show dealing. Convince Me: Consider facilitating a discussion with the whole class around the *Convince Me!* or as a formative assessment, because it is a real-world problem with regrouping money. Another Example: The *Another Example!* shows regrouping to divide 55 into 4 groups. Consider having a whole class discussion around this problem while students use tools or representations to find the quotient. Independent Practice/Math Practices and Problem Solving: Consider having students estimate for items 5-8 before solving for the actual quotient. Consider facilitating a discussion around item 9, as students will use a bar diagram and write an equation to solve the problem. This gives students an opportunity to model with math using other representations. *CTC: *Higher Order Thinking item 13* (student work samples)
Lesson 5-6: Use Partial Quotients to Divide		
4.NBT.B.6 4.OA.A.3 MP.1 MP.2 MP.4 MP.7	Access Prior Learning: In third grade, students learned division as sharing and as repeated subtraction. Beginning of the Big Idea: In this lesson, students will use repeated subtraction and partial quotients algorithm to divide.	Solve & Share: The *Solve & Share* is a measurement division problem where students chunk 6 out of 72 to find how many times Sally's bird feeder can be filled. Consider having students who used place-value understanding or repeated subtraction to find the quotient share their strategies or models. Visual Learning: The *Visual Learning* is another measurement division problem, and encourages students to think about partial quotients. Consider having students think about ways they may chunk 63 ÷ 3 before sharing the animation. Consider creating an anchor chart with the partial quotients algorithm and area model. Convince Me: Consider using the *Convince Me!* as a formative assessment to child-watch for students who understand the multiplication and division relationship. *-continues on next page-*

		Another Example: Consider facilitating a discussion around the *Another Example!* as the focus is using the distributive property to divide. Students will draw an array to model the distributive property. Guided Practice: Consider having students complete items 1-3 in the *Guided Practice* as students use models and the distributive property to solve for the quotients.
Lesson 5-7: Use Partial Quotients to Divide- Greater Dividends		
4.NBT.B.6 4.OA.A.3 MP.2 MP.3 MP.7	Access Prior Learning: In the previous lesson, students used repeated subtraction and the partial quotients algorithm to divide. Developing the Big Idea: In this lesson, students will build on the work from the previous lesson as well as estimate to solve division problems.	(Possible 2-day Lesson) Note: Consider expanding this lesson over 2 days to elicit more time for students to work with the area model and partial quotients algorithm. Remember by taking two days for this lesson, it will be one of the A/D/E days on the WCSD Pacing Framework. Day 1: Solve & Share: The *Solve & Share* is an area problem, where students have to solve for one of the dimensions. Here is another opportunity to facilitate a discussion around area after students have solved the problem to prepare them for later topics. Look Back: Consider using the *Look Back!* to facilitate a discussion, or have students work together to solve for the answer to the problem. Visual Learning: Consider facilitating a discussion around the *Visual Learning Animation*, as students will interpret the quotient and remainder. Consider having students compare both the area model and partial quotients algorithm to make connections. Day 2: Another Example: Consider facilitating a discussion around the *Another Example!* as it asks students to use multiplication, estimation, place-value and an open array to solve for the quotient. Child-watch for ways students use various multiplication, estimation and place-value to solve the problem. Have students share the different ways. Independent Practice/Math Practices and Problem Solving: Consider having a discussion around items 14 and 15 as students use information from a table to solve problems; as well as use estimation. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with all students, as they are asked to use partial quotients and an open area model to solve a division problem. Have students compare the model and algorithm. Look for students who notice the place-value connection. Consider asking students if there are other ways to find the quotient using an open area model or partial quotient algorithm. *CTC: ***Intervention Activity*** (student work samples)
Lesson 5-8: Divide with 1-Digit Numbers		
4.NBT.B.6 4.OA.A.3 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In the previous lessons, students learned how to divide by using the partial quotients algorithm. Beginning of the Big Idea: In this lesson, students will learn to divide by using the partial quotient algorithm.	Note: Students will not need to know the U.S. traditional division algorithm for the Topic Performance Assessment, as this algorithm is not required until Grade 6. Consider spending time developing conceptual understanding by using the partial quotient algorithm or models. Solve & Share: The *Solve & Share* is a partitive division problem. Give students opportunity to use concrete tools or representational models to solve the problem. Consider having students share who may have used concrete tools, models, or the partial quotients algorithm in this order. Have students compare the tools, models or strategies. Look Back: Consider having students solve the *Look Back!* and facilitate a discussion regarding how to construct an argument. *-continues on next page-*

		Visual Learning: In the *Visual Learning Animation*, students will be exposed to the U.S. traditional division algorithm. Rather than showing the animation, have students use the partial quotients algorithm to solve the problem. Students may need more time to develop the conceptual understanding of division before moving into a procedural understanding. Students will have more opportunity with the U.S. traditional division algorithm in future grades. Another Example: Consider giving students more opportunities to work with the partial quotients algorithm using an open array instead of the U.S. traditional division algorithm. Independent Practice/Math Practice and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students opportunities to estimate and use place-value strategies to solve problems in the *Independent Practice and Math Practice and Problem Solving*.
Lesson 5-9: Continue to Divide with 1-Digit Numbers		
4.NBT.B.6 MP.2 MP.3 MP.7	Access Prior Learning: In the previous lesson, students continued developing the conceptual understanding of division using the partial quotient algorithm. Developing the Big Idea: In this lesson, students will continue using the partial quotient algorithm to solve division problems.	Note: Students will not need to know the U.S. traditional division algorithm for the Topic Performance Assessment, and this algorithm is not required until Grade 6. Consider spending time developing conceptual understanding by using the partial quotient algorithm or models. Solve & Share: The *Solve & Share* is a partitive division problem. Consider having students share and compare multiple strategies used to find the quotient. Have students share the concrete tools, representation (area model) and the partial quotient algorithm (abstract) in that order. Look Back: Consider having students answer the question in the *Look Back!*, and then facilitate a discussion in regards to patterns in multiplication and division. Visual Learning: The *Visual Learning* is a measurement division problem. Estimation is an important concept for students, so consider having students continue estimating. Give students an opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation,* as the animation continues exposing students to the U.S. traditional division algorithm. Convince Me: Consider using the *Convince Me!* to facilitate a discussion around estimation strategies. Have students think about place value as they solve the problem. Independent Practice/Math Practice and Problem Solving: Students do not need to do all the problems in their Student Edition. Continue on to other items as appropriate. Consider giving students opportunity to use any strategy or model they want to solve the division problems. Have students complete items 20 and 22 as they reinforce estimation. Assess and Differentiate/Intervention Activity: Consider having all students do the *Intervention Activity*, as students continue work with compatible numbers to estimate each quotient.
Lesson 5-10: Math Practices and Problem Solving- Model with Math		
4.NBT.B.6 4.OA.A.3 MP.4 MP.1 MP.2 MP.6 MP.7 MP.8	Access Prior Learning: In Topic 3, students learned thinking habits that good problem solvers use to model with math. Developing the Big Idea: In the lesson, students will continue to use thinking habits when modeling with math for division.	Solve & Share: Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. Look Back: Consider facilitating a discussion around hidden questions. To find some answers, the hidden questions need to be exposed in order to solve the problem. Hidden questions are something not explicitly asked, but which must be done before the explicit question can be answered. In Topic 6, students will have more opportunities with hidden questions. Independent Practice/Math Practice and Problem Solving: Consider having students complete item 3 and 4, as 4 asks to check for reasonableness. *-continues on next page-*

		Assess and Differentiate/Intervention Activity: Consider doing the *Intervention Activity* with all students as it reinforces the bar diagram model. Students also find the hidden question to solve the problem. They will be doing more of this in Topic 6.

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	$A > 1$ Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	$A < 1$ Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

References

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