▸ Grade 4 Topic 15: Geometric Measurements: Understand Concepts of Angles and Angle Measurements

Topic 15
Geometric Measurement: Understand Concepts of Angles and Angle Measurement

Number of lessons: **6**

A/D/E: **4** days

NVACS Focus: MD.C, G.A

Total Days: ~10

[4th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 87-92)

Prior to instruction, view the Topic 15 Professional Development Video (located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 765A-765F), the Topic Planner (pp. 765I-765J), all 6 lessons, and the Topic Assessments (pp. 813-814A).

Mathematical Background:	Topic Essential Questions:
Read Cluster Overview- (TE, pp. 765A-765F)	What are some common geometric terms? How can you measure angles? *Reference TE, p. 765 and Answering the Topic Essential Questions (TE, pp. 811-812) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

15-1	15-2	15-3	15-4	15-5	15-6	Assessment

4 A/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on angles and angle measurements. Focus for standard 4.MD.C, "Geometric measurement: understand concepts of angle and angle measures" (Nevada Academic Content Standards (NVACS), 2010). 4.MD.C.5a-b, 4.MD.C.6 and 4.MD.C.7 focus on recognizing angles as geometric shapes that are formed wherever two rays share a common endpoint, understand concepts of angle measurement, measure angles in whole-number degrees using a protractor, and recognize angle measures as additive.

"Technically, a *measurement* is a number that indicates a comparison between the attributes of the object (or situation, or event) being measured and the same attribute of a given unit of measure. To measure means that the attribute being measured is "filled", "covered", or "matched". (Van de Walle, Karp and Bay-Williams, 2010, pg. 370). "The attribute of angle size might be called the "spread of the angle's rays." Angles are composed of two rays that are infinite in length with a common vertex. The only difference in their size is how widely or narrowly the two rays are spread apart" (Van de Walle, et al., 2010, pg. 386).

John Van de Walle, et al., (2014) states,

> The two tools commonly used for measuring angles are angle rulers and protractors. The protractor is one of the most poorly understood measuring instruments. Part of the difficulty arises because the units (degrees) are so small. It would be physically impossible for students to cut out and use a single degree to measure an angle accurately. In addition, the numbers on most protractors run clockwise and counterclockwise along the edge, making the scale hard to interpret without a strong conceptual foundation. Note that the units of degrees are based on an angle in which the vertex of the rays is located at the midpoint of a circle creating an arc. A "one degree" angle is one in which the arc is 1/360 of the circle (pp. 336-337).

Give students multiple experiences with angle measures before giving students a protractor, so they understand the reason for the half circle the protractor represents. Using a full circle protractor can help students with the angle measurements conceptually, as well as having them make one themselves.

Focus Math Practice 5: Use appropriate tools

Focus opportunities for students to develop *Mathematical Practice 5* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 15-6. Reference the Teacher's Edition (pp. F25-F25A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
point line segment ray acute angle obtuse angle straight angle reflex angle degree unit angle angle measure protractor	*line* *right angle* *vertex*

Additional terminology that students may need support with: measure, circle

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

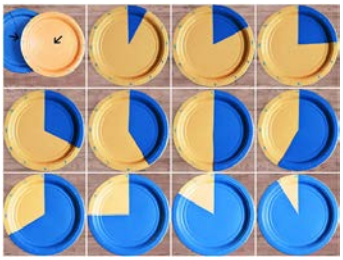
Guiding questions: "What tools or strategies are students using to determine the measurement of angles?"
"Are students able to find angle measurements based on the information provided?"

Lesson	Evidence	Look for
15-2	*Math Practice and Problem Solving* (student work samples) Item 13	Focus CTC around the big idea: • students use their understandings of angle measurements of a circle to find the unit angle.
15-4	*Quick Check* (digital platform) Items 1, 2 and 5	Focus CTC around the big idea: • students find the angle measurements based on the given information. • students use a protractor to find the angle measurement. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 811-814	Use *Scoring Guide* TE pp. 811-814A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Math Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 15-1: Lines, Rays, and Angles		
4.MD.C.5 4.G.A.1 MP.2 MP.4 MP.6 MP.7	Access Prior Learning: In the previous grades, students learned to draw shapes with a given number of angles or sides. Students analyzed shapes by their attributes such as sides and angles. Beginning of the Big Idea: In this lesson, students recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angles.	Note: Consider changing the orientation of the angles, so students do not develop a misconception that angles only open in one direction. Look Back: Consider having students complete the *Look Back!* as they work on the *Solve & Share*. Students will be drawing 3 angles; two less than 90 degrees and one greater than 90 degrees. Visual Learning: The *Visual Learning Animation* is heavy with mathematical language related to angles. Consider pausing the video after each new mathematical word and have students "act out" the words. The *Intervention Activity* has students using their bodies to represent the terms, so use the *Intervention Activity* during the animation. Connect what students did in the *Solve & Share/Look Back!* to the *Visual Learning Animation*. Convince Me: Have students do the *Convince Me!* in a math journal as students complete the figures to show the given angle. Independent Practice/Math Practices and Problem Solving: As students name line segments, rays and angles, consider making sure students are labeling the different line segments, rays and angles correctly. Consider facilitating a discussion around items 12-14, as students are looking for angles in a diagram and not just in isolation. *-continues on next page-*

		Consider facilitating a discussion around the "Higher Order Thinking" item 19 with whole class, as students are problem solving and constructing an argument based on Nina's response. Assess and Differentiate/Intervention Activity: Consider using the *Center Games* at the bottom of page 775A as students identify and write descriptions about the types of line, rays and angles represented in the diagrams.
Lesson 15-2: Understand Angles and Unit Angles		
4.MD.C.5a MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In previous topics, students developed an understanding of the measurement process as a comparison of a unit to a whole. Beginning of the Big Idea: In this lesson, students learn that the unit used to measure angles is a degree and that a unit angle is an angle that turns through 1/360 of a circle.	Note: Consider breaking this lesson into 2 days. Remember by breaking the lesson into 2 days, this uses one of the A/D/E days allocated in the WCSD Pacing Framework. Day 1: Solve & Share: Consider changing the *Solve & Share* to the *Homework & Practice* item 7, "Janie served 4 same-size pizzas at the class party. Explain how to find how many slices of the pizza Janie served if the angle for each slice turns through a right angle". Consider having tools and representations available for students to use. After a discussion has been facilitated around the *Homework and Practice* item 7, consider having students complete the *Solve & Share*, students connect what they learned in the previous lesson and Topic 13 to be able to solve the problem. Consider having tools or representations available for students to use. As an extension for all students, ask, "What other angle is made by the two hands on the clock?" The purpose of this question is to get students to start thinking about the concept of *reflex angles*. See diagram at the end of this Curriculum Guide. Look Back: Consider facilitating a discussion around the *Look Back!* as students connect the *Solve & Share* to fractions. Students will be using this idea to think about a circle and degrees related to the fractions ¼ and ¾. Consider facilitating a discussion around the ***reflex angle*** by connecting ¾ to the degree amount on a circle. The reflex angle is 270 degrees. This idea will help support work in Day 2 when you begin the *Visual Learning Animation*. Day 2: Visual Learning: Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 778). In the *Visual Learning Animation*, students begin to think about angle measurements in degrees and the fractional turns through parts of the circle. Students think about the unit angle 1/360 degrees. Consider having students make a double paper plate circle. See the diagram below. Find the idea for this tool at https://www.mathlearningcenter.org/blog/interactive-paper-plate-fractions.  Convince Me: Consider facilitating a discussion around the *Convince Me!* as students compare their understanding of fractions and the same whole to circles and angle measurements. The size of the circle does not change the measure of the angle. Another Example: Consider facilitating a discussion around the *Another Example!* as students think about 45 degrees and the fraction of a circle the angle turns through. Independent Practice/Math Practices and Problem Solving: Consider having tools and other representations available for students to use as they work on the problems in the *Independent Practice/Math Practices and Problem Solving*. Tools may include clocks, and the circle made in the *Visual Learning*. *-continues on next page-*

		Note: Students are not using protractors at this time. Assess and Differentiate/Intervention Activity: Consider having all students complete the *Intervention Activity* as students think about 6:00 on the clock (1/2) and think about the angle measure that is formed (180 degrees). Consider putting this before the *Another Example!*. *CTC: *Math Practices and Problem Solving item 13* (student work samples)
Lesson 15-3: Measure with Unit Angles		
4.MD.C.5b 4.MD.C.5a MP.1 MP.3 MP.4 MP.5 MP.8	Access Prior Learning: In the previous lesson, students learned that the unit used to measure angles is a degree, and a unit angle is an angle that turns through 1/360 of a circle. Developing the Big Idea: In this lesson, students begin applying the measurement process to measuring angles.	Solve & Share: Instead of giving the angle measures, consider having students figure out the smaller angle measure of a tan pattern block before solving the problem. The *Solve & Share* problem gives students the measurement of the angle degree, but consider having students figure out how many smaller angles on the tan pattern block fit in the 90-degree angle or edge of a paper. Then have students use the information found to solve the problem. Child-watch for students who use the rhombi and reasoning skills to show how three rhombi make 90 degrees. Students may determine one rhombus' angle is 30 degrees, because 90 divided by 3 is 30 degrees. Have students share their models, strategies or equations with the class. Look Back: Consider changing the *Look Back!* to, "How many right angles make a straight angle? How many 45-degree angles form a straight angle? Explain. Visual Learning: Consider providing students an opportunity to use pattern blocks as the discussion is facilitated around the *Visual Learning Animation*. Convince Me: Connect the *Convince Me!* to the *Visual Learning Animation* and the previous lesson as students think about one-degree angles in an angle measure. Guided Practice: Consider facilitating a discussion around the *Guided Practice* items 3 and 4 as students find angle measurements in squares.
Lesson 15-4: Measure and Draw Angles		
4.MD.C.6 MP.1 MP.2 MP.5 MP.6	Access Prior Learning: In the previous lesson, students used pattern blocks' angles to measure other angles. Developing the Big Idea: In this lesson, students are introduced to a tool designed for measuring angles, a protractor.	Solve & Share: Students read a protractor to find the measure of an angle. Have students think about if the angle is less than or greater than 90-degrees before figuring out the angle measurement. By estimating the angle, students may read the protractor correctly. Child-watch for students who may struggle when reading a protractor and do not connect the estimation to the angle measurement. Look Back: Consider facilitating a discussion around the *Look Back!* as students draw an angle measure of 110-degrees. Have students compare the angle from the *Solve & Share* to the angle in the *Look Back!*. Consider having students estimate how many 110-degree angles are in a circle. Visual Learning: Students learn how to use the protractor in the *Visual Learning Animation*. Consider giving students an opportunity to use the tool to make angles, including making a 45-degree angle and a 130-degree angle from the animation. Convince Me: Consider facilitating a discussion around the *Convince Me!* or using it as an opportunity for a formative assessment, as it helps clarify any misconceptions students may have reading a protractor. Independent Practice/Math Practices and Problem Solving: Consider creating the *Independent Practice* on a separate piece of paper for students to measure, as to change the orientation of the angles and the size of the angles to make it easier for students to measure. For example, item 11 change the 90-degree angle opening to face the opposite direction. This will help students read the protractor and not develop a misconception that angles only face one direction. *-continues on next page-*

		Assess and Differentiate/Intervention Activity: Consider facilitating a discussion around the *Intervention Activity*, as this will support students in the next lesson. *CTC: *Quick Check items 1, 2 and 5* (digital platform)
Lesson 15-5: Add and Subtract Angle Measures		
4.MD.C.7 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In third grade, students learned that area is additive and solved problems by adding areas. Developing the Big Idea: In this lesson, students learn angle measurements are additive. They add and subtract angle measurements to solve problems.	Solve & Share: Consider sharing the different ways students divided the angle into two angles. Look Back: Facilitate a discussion around the *Look Back!* and make connections to the *Solve & Share*. Have students write the equation they would use for the two angles used to make the angle of 70-degrees. Also, consider facilitating a discussion around the idea that no matter what students divided the angles into, they should still have the same total. Visual Learning: In the *Visual Learning Animation*, this is the first time students need to find the missing part of the total angle. Consider having students solve the problem after Part A, and then discuss student strategies before showing the other parts of the animation. Convince Me: Consider facilitating a discussion around the *Convince Me!* or use it as a formative assessment. Students find one unknown of three angles for the first time. Assess and Differentiate/Intervention Activity: Consider doing the *Intervention Activity* with all students, as students work by making their own angles.
Lesson 15-6: Math Practices and Problem Solving- Use Appropriate Tools		
4.MD.C.6 4.MD.C.7 MP.5 MP.1 MP.2 MP.4	Access Prior Learning: In previous topics and lessons, students use appropriate tools strategically. Developing the Big Idea: In this lesson, students will focus on thinking habits good problem solvers use when they use appropriate tools strategically to solve problems involving angle measures and distances.	Solve & Share: Consider having various tools available for students to use to solve the problem. Give them opportunity to use various tools and to problem solve which tools would be best in helping them solve the problem. Visual Learning: In the *Visual Learning*, students learn to use a protractor and a ruler in different ways. Consider providing students an opportunity to measure the red trapezoid pattern block and then discuss students' findings before moving on in the animation. Convince Me: Consider facilitating a discussion around the *Convince Me!*, as students work on the *Visual Learning* problem. Guided Practice: In the *Guided Practice*, students are not asked to find angles, but how to use appropriate tools when adding mixed fractions. Assess and Differentiate/Intervention Activity: Consider having all students complete the *Intervention Activity* as students use appropriate tools to find angle measurements.

An angle

name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

Common Core Standards Writing Team. Measurement and data progression.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). Measurement and data, measurement.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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