Grade 5 Topic 8: Apply Understanding of Multiplication to Multiply Fractions

Topic 8 ***Multiply Fractions*** Number of lessons: **8** A/D/E: **4** days **NVACS Focus: NF.B** **Total days: ~12**

5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 8 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 455A-455F), the Topic Planner (pp. 455I-455K), all 8 lessons, and the Topic Assessments (pp. 521-521A).

Mathematical Background:	Topic Essential Question:
Read Topic 8-9 Cluster Overview/Math Background (TE, pp. 455A-455F)	What does it mean to multiply whole numbers and fractions? How can multiplication with whole numbers and fractions be shown using models and symbols? *Reference Answering the Topic Essential Questions (TE, pp. 517-518) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	8-8	Assessment

4 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on 2010 Nevada Academic Content Standards (NVACS) cluster 5.NF.B, "Apply and extend previous understandings of multiplication and division to multiply and divide fractions". This cluster consists of four standards.

- 5.NF.B.4a- Interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b*. For example, use a visual fraction model to show *(2/3) x 4 = 8/3*, and create a story context for this equation. Do the same with *(2/3) x (4/5) = 8/15*. In general, *(a/b) x (c/d) = ac/bd* (NVACS, 2010).

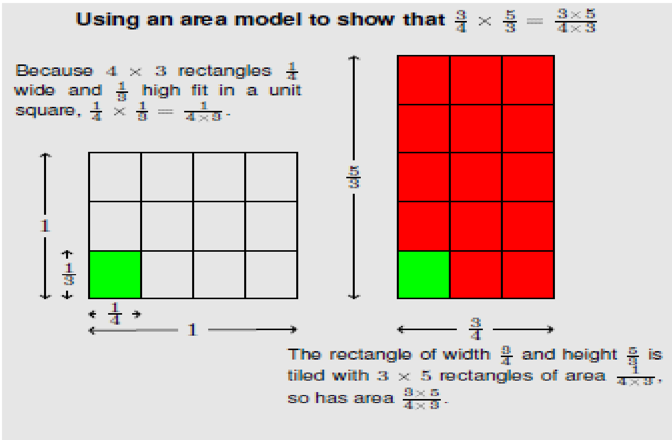
- 5.NF.B.4b- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fractions products as rectangular areas (NVACS, 2010).

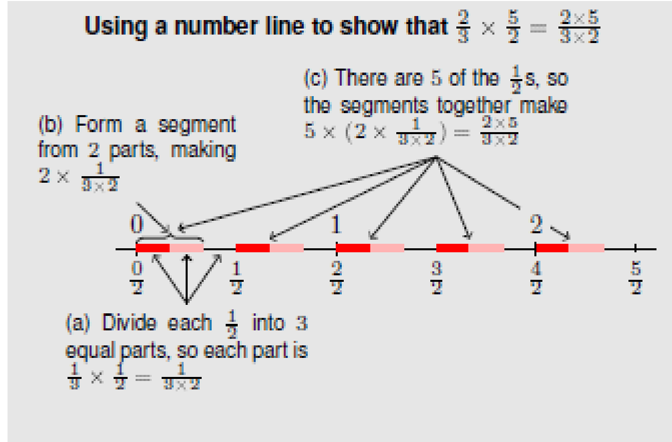
- 5.NF.B.5a- Interpret multiplication as scaling (resizing) by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication (NVACS, 2010).

- 5.NF.B.5b- Interpret multiplication as scaling (resizing) by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (n x a)/(n x b)* to the effect of multiplying *a/b* by 1 (NVACS, 2010).

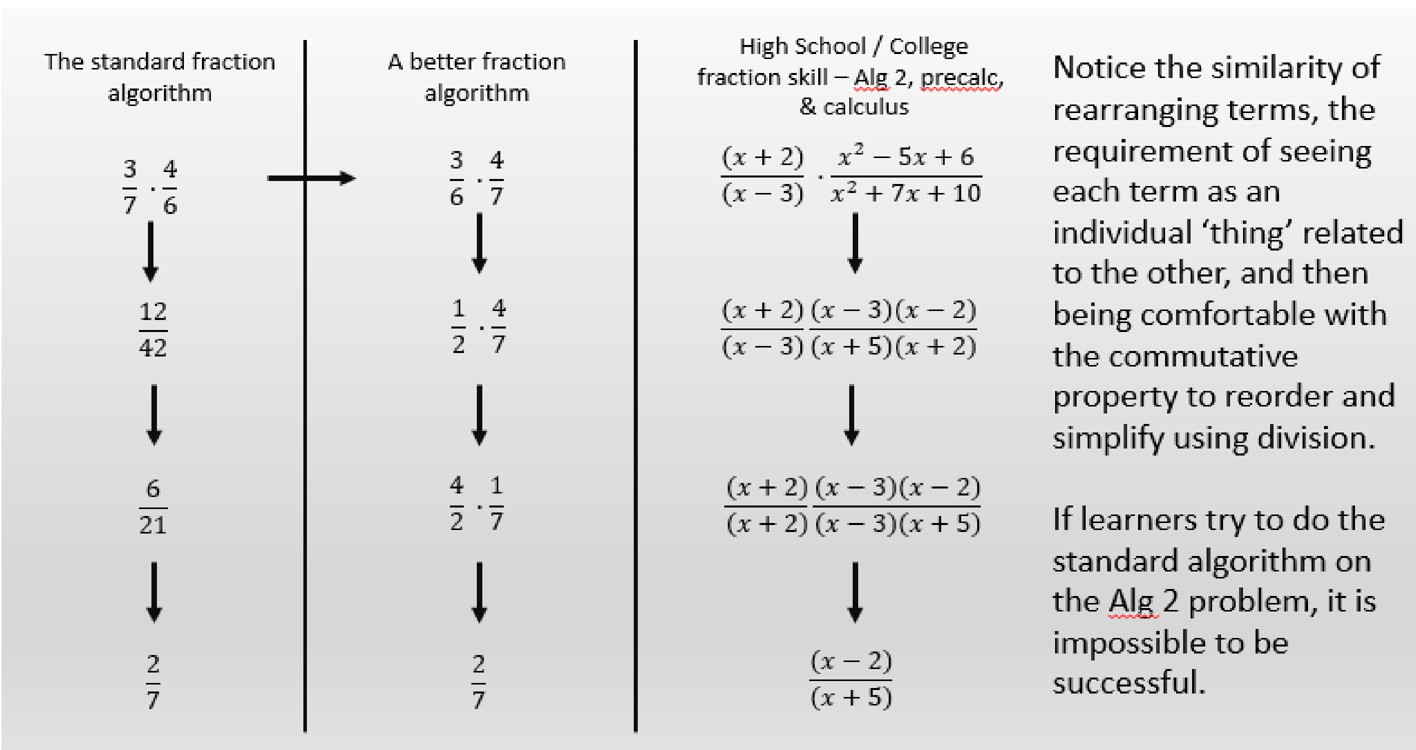
Students extend understanding of multiplication with whole numbers to include multiplication with fractions. A common misconception that requires immediate attention is that multiplication always creates greater numbers (Karp, Bush & Dougherty, 2014). Working with whole numbers, we would describe a multiplication problem such as 3 x 4 as "3 groups or sets of 4" (set models), "3 rows of 4" (area model or array), or "3 four times" (number line, linear model, skip count). When fractions are included in these models, products can be smaller than one *or* both factors. Thinking of multiplication as scaling connects working with whole numbers to multiplication with fractions. When we multiply 3 x 4, the 3 becomes 4 times bigger. However, when we multiply 3 by $\frac{1}{2}$, the 3 becomes half of its original size. Building on student understanding of whole number multiplication, we can say that 3 x $\frac{1}{2}$, is "3 groups of $\frac{1}{2}$,", "3 rows of $\frac{1}{2}$,", or even "3 half of a time" (jump halfway to 3 on a number line). Helping students to interpret multiplication as scaling will address the common misconception that multiplication always makes greater numbers and increase student's ability to determine whether solutions are reasonable.

Students initial experiences with fraction multiplication should rely heavily on using visual models. These models allow students to build a conceptual understanding of how and why multiplication with fractions results in resizes through partitioning. Students gained experience with set models, number lines and area models while working with whole numbers. Students in grades 3 worked with linear and area models to explore fraction understandings. In grade 4, students worked with all three models with denominators of 2, 4, 5, 6, 8, 10, 12 and 100. These same models are used to illustrate fraction multiplication. For more challenging problems, the area model is particularly useful. Students are able to create a rectangle with fractional side lengths or visually demonstrate using the Distributive Property to multiply mixed numbers (Common Core Standards Writing Team (CCSWT), 2013).





Connecting visuals with operations will be important to help students understand why the U.S. traditional algorithm for multiplying fractions works. Memorized procedures are likely to be forgotten or confused with other operations. Students must be able to compute with fractions flexibly and accurately. Success with fractions in particular is closely related to success in Algebra (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). An example of flexible thinking to solve a fraction multiplication problem is shown below. In 5th grade this student is using the Commutative Property to switch the numerators on the fractions and create a much easier problem. In Algebra 2, use of the U.S. traditional algorithm would result in a harder problem and incorrect solution. Student's conceptual understanding of this content is crucial to their future success in higher levels of mathematics.



(Beckam & Waddell, 2017)

Math Practice 1: Make sense of problems and persevere in solving them
Focus on opportunities for students to develop *Mathematical Practice 1* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 8-8. Reference the Teacher's Edition (TE, pp. F21-F21A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	Associative Property *Iteration* *Unit Fraction*

Additional terminology that students may need support with: annex zeros

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Can students use a model to demonstrate and explain how a denominator partitions both factors when multiplying with a fraction?"
"Are students applying knowledge of multiplication to estimate products when multiplying with fractions?"

Lesson	Evidence	Look for
8-2	*Math Practices and Problem Solving* (student work samples) Item 11	Focus CTC around the big idea: • reasonable estimate for the context of the situation. • understanding of the connection between multiplying with a fraction and finding parts of a set.
8-2	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
8-4	*Homework and Practice* (student work samples) Item 16	Focus CTC around the big idea: • ability to estimate a product based on the sizes of factors.
8-4	*Convince Me!* (student work samples)	Focus CTC around the big idea: • appropriate use of an area model to multiply two fractions. • ability to demonstrate and explain how denominators partition the whole.

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 517-522	Use *Scoring Guide* TE pp. 517-522C

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 8-1: Use Models to Multiply a Whole Number by a Fraction		
5.NF.B.4a **5.NF.B.6** MP.2 MP.3 **MP.4** MP.6 MP.7	Access Prior Learning: In 4th grade students multiplied a whole number by a fraction using visual models and equations (4.NF.B.4). Developing the Big Idea: Students will build conceptual understanding of multiplying with a fraction. A visual model is connected to the repeated addition interpretation of multiplication.	Solve and Share: In 4th grade students iterated or repeatedly added with unit fractions to find that a fraction such as $\frac{3}{4}$, could be decomposed to ($\frac{1}{4}$, + $\frac{1}{4}$ + $\frac{1}{4}$). The number line model used in this problem can represent the context as 5 jumps of $\frac{1}{2}$. This model helps students connect multiplying with fractions to repeated addition and iterating with unit fractions. Visual Learning: The *Visual Learning Bridge* uses bar diagrams to model repeated addition as multiplication with a whole number and a unit fraction The example above ($\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$) can be reinterpreted as 3 x $\frac{1}{4}$. This understanding allows students to use the Associative Property to reinterpret whole number by fraction multiplication. If $\frac{3}{4}$ is written as (3 x $\frac{1}{4}$), then 4 x $\frac{3}{4}$ can be reinterpreted as 4 x (3 x $\frac{1}{4}$). Now the whole numbers can be multiplied, (4 x 3) x $\frac{1}{4}$, to create 12 x $\frac{1}{4}$. Since the fraction is a unit fraction, students will see that it partitions the whole into 4 equal parts. This creates the fraction 12/4 or 12 ÷ 4. Assess and Differentiate: Use fraction strips to model repeated addition as fraction multiplication in *Another Look,* the *Homework and Practice* page, the *Intervention Activity* and the *Reteach* page. Students can try using this model with concrete tools; as well as, representations such as Teaching Tool 13 (fraction strips).

Lesson 8-2: Use Models to Multiply a Fraction by a Whole Number		
5.NF.B.4a 5.NF.B.6 MP.2 MP.3 MP.4	Access Prior Learning: Students multiplied a whole number by a fraction in the previous lesson. Developing the Big Idea: Students build conceptual understanding of multiplication with fractions. They will multiply a fraction by a whole number using models and equations.	Solve and Share: Students view that 5 x $\frac{1}{2}$ could be interpreted as adding $\frac{1}{2}$ five times in the previous lesson. Multiplying a fraction by a whole number is similar because of the Commutative Property, yet the meaning changes. $\frac{2}{3}$x 6 means to take $\frac{2}{3}$ of a set of 6. Look for students representing the context of this problem correctly using visual models and equations. Facilitate a discussion connecting the different strategies. What happens to the numbers in this problem? The *Look Back*! asks why multiplying a fraction and a whole number results in a product smaller than the whole number. Visual Learning: Bar diagrams are used to visually demonstrate multiplying a fraction by a whole number. Draw out the idea that the denominator partitions the whole amount while the numerator determines how many parts should be taken. The *Convince Me*! gives students a chance to practice using a set model. These models will also be used on the *Guided and Independent Practice* pages. Fraction strips or other concrete tools can also help students complete these problems with understanding. Assess and Differentiate: The *Reteach* page uses an area model instead of set models. Consider bringing this model into the class discussion earlier in the lesson to allow comparison and analysis by all students. Does this accurately represent all contexts? How is it similar to a set model? Some students will prefer the area model as they become more efficient. *CTC: *Math Practices and Problem Solving* (student work samples) Item 11 *CTC: *Quick Check* (digital platform)
Lesson 8-3: Multiply Fractions and Whole Numbers		
5.NF.B.4a MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students multiplied fractions and whole numbers in previous lessons. Developing the Big Idea: Students will build procedural skill through practice. They will be shown two abstract strategies for multiplying a fraction and a whole number.	Solve and Share: Consider starting this lesson with the *Look Back!* to give students the opportunity to practice reasoning and sense making with this relatively new content. Look for students modeling and solving this problem using a range of strategies. Facilitate a class discussion using student work to connect visual models and abstract strategies. Why do we need both multiplication and division when multiplying fractions and whole numbers? Visual Learning: The *Visual Learning Animation* shows two methods of multiplying a whole number by a fraction. The first example builds on understanding that fractions can be interpreted as a whole number times a unit fraction and incorporates use of the Associate Property. The second method demonstrated is the U.S. traditional algorithm for multiplying fractions. Students should justify why it works with visual models. Placing a 1 under a whole number to create a fraction can create misconceptions. Students may need clarification with the idea that a/b = a ÷b. This means that 7/1 is equivalent to 7 ÷ 1 which is still a whole 7. Consider having students work on a problem such as problem 9 on the *Independent Practice* (SE, p. 471). Use strategic questioning to draw out student understanding around the idea that it is possible to use the denominator to partition the whole number before multiplying with the numerator. Ask, "When is this an appropriate strategy for this problem type?"
Lesson 8-4: Use Models to Multiply Two Fractions		
5.NF.B.4a MP.2 MP.4 MP.6	Access Prior Learning: Students used models to multiply two decimals in Topic 4. Developing the Big Idea: Students build conceptual understanding using two visual models to multiply fractions.	Solve and Share: Students can literally model the context of this problem if they fold a piece of paper in half, color $\frac{1}{4}$ of one side and then determine how much of the paper is colored. Other students will represent the problem using fractions strips or visual representations. Help students connect the meaning of multiplication to the numbers in this problem. Why is the solution smaller than either of the factors? This question is also explored in the *Look Back!.* Visual Learning: Students may need clarification on why the overlap created on the area model is the product. This is an excellent opportunity to think more deeply about the meaning of multiplication. If multiplying 3 x 4 means to take 3 sets of 4, then $\frac{1}{2}$ x $\frac{3}{4}$ means to take $\frac{1}{2}$ of a set $\frac{3}{4}$. We are taking a "part of a part" which is shown by the overlap on the area model. Students are given opportunity to practice using an area model with the *Convince Me!* problem. Encourage students to experiment with various models while completing the *Guided and Independent* practice problems to build conceptual understanding. *CTC: *Homework and Practice* (student work samples) Item 16 *CTC: *Convince Me!* (student work samples)

Lesson 8-5: Multiply Two Fractions		
5.NF.B.4a MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students multiplied two fractions in the previous lesson. Securing the Big Idea: Students build procedural skill through practicing use of strategies to multiply two fractions.	Solve and Share: Look for students using a range of strategies including models and equations to solve this problem. Facilitate a class discussion to help students connect the models to the equations. How do the models represent the equations and vice versa? How do both represent the context of the problem? Visual Learning: This revisits the idea of estimation to determine reasonableness of solutions. The U.S. traditional algorithm for multiplying two fractions is shown. How does this algorithm connect to other models and strategies? Giving answers in simplest form is not required by the 5th grade standards. This allows students to focus on building conceptual understanding and procedural fluency. Although, it might be appealing to present solutions in simplest form, there is no mathematical imperative to do so. Problems 31-34 on *Math Practices and Problem Solving* offer opportunity for students to demonstrate understanding of multiplication with fractions (SE, p. 484). Assess and Differentiate: *Homework and Practice* page problems 1-6 require use of the U.S. traditional algorithm. Consider moving students using other strategies to problems 7-24. Do not rush students to use the U.S. traditional algorithm, focus on building conceptual understanding. Use a small sample of problems to check for understanding before moving students to more challenging problems. Watch for students using subtraction. Consider scaffolding problem context by replacing fractions with whole numbers.
Lesson 8-6: Area of a Rectangle		
5.NF.B.4b MP.1 MP.2 MP.3 MP.5 MP.6	Access Prior Learning: Students used area models to multiply whole numbers, decimals and fractions in previous topics. Developing the Big Idea: Students build conceptual understanding and procedural skill by connecting use of an area model to the standard algorithm for multiplying two fractions.	Solve and Share: A hundredths grid is provided in the Student Edition. Students may prefer to draw their own area model to represent this problem. However, important understandings about multiplication can be built through use of the hundredths grid. For example, some students may realize that they will need to create a 4 x 4 array (representing 1 whole yard) within the grid to solve this problem. This connects to multiplying the denominators when using the U.S. traditional algorithm. Visual Learning: Using area models with fractional side lengths is modeled. Draw out the idea that the denominators help to determine the size of a single unit (a tile) and the numerators how many units (or tiles) can be used to fill in the total area. These visual models demonstrate how the U.S. traditional algorithm works to multiply fractions. Students are given models to practice with on the *Guided and Independent Practice* page. Help students connect the models to the procedures used in the U.S. traditional algorithm. Assess and Differentiate: Review of finding area using whole numbers is given on the *Reteach* page.
Lesson 8-7: Multiply Mixed Numbers		
5.NF.B.6 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In previous topics students used area models to multiply multi-digit whole numbers and decimals. In previous lessons students used area models to multiply fractions. Developing the Big Idea: Students build conceptual understanding and connect previous learning to multiplying mixed numbers through use of an area model.	Solve and Share: Students may initially have difficulty deciding how to multiply mixed numbers. Watch for students that only multiply the whole numbers and only multiply the fractions. Help these students to think about what making a quantity 2 and 3 times bigger means. Asking students if there is more than one way to represent a mixed number may lead some to rename the mixed number as a fraction greater than one, and multiply. Some students may use repeated addition to make the quantity greater. Facilitate a discussion to share student ideas and celebrate how they were able to apply what they know about multiplying fractions to this new content. Visual Learning: An area model is used to multiply two mixed numbers. Students have seen this model used with whole numbers and decimals. It allows use of the Distributive Property to multiply mixed numbers and create partial products. It is an effective model for building conceptual understanding and procedural skill for these problem types. Renaming the mixed numbers as fractions greater than one is also demonstrated. Do not rush to proceduralize this understanding, focus on building understanding of what this means and why this works. *Independent Practice* page problems 2-9 require use of this strategy (SE, p.495). Assess and Differentiate: *Homework and Practice* page problems 1-4 require students to rename mixed numbers. Problems 5-12 remind students to estimate and allow use of multiple strategies (SE, p. 497).

Lesson 8-8: Multiplication as Scaling		
5.NF.B.5a 5.NF.B.5b MP.2 MP.7	Access Prior Learning: In previous lessons students have multiplied with whole numbers, decimals, fractions and mixed numbers. Securing the Big Idea: Students strengthen conceptual understanding of multiplication and increase procedural precision through reasoning about the size of a product based on the sizes of the factors.	Solve and Share: Students will likely want to compute the problems to find the correct solution. However, the goal of this lesson is to strengthen student's number sense and understanding of multiplication as scaling. Building on what they know about multiplication, students can use reasoning to determine the size of each product based on the size of the factors. Can students justify their solutions? Orchestrate a discussion to draw out the idea that in a multiplication problem, the first factor can be interpreted as a quantity while the second factor is a scaler that changes the size of the quantity to create a product. Visual Learning: Multiplication by fractions less than 1 and greater than 1 are discussed. These problems reveal an important understanding for students about the relationship between the size of the factors and the product. Can students use a model to prove these ideas? If students are having difficulty using reasoning to complete the *Independent Practice* problems, model estimating the size of a product using whole number factors and then easier fractions such as $\frac{1}{2}$. How do these same ideas carry over to working with mixed numbers? Assess and Differentiate: The *Intervention Activity* gives students a chance to discuss and justify their thinking about the sizes of products. Use of number lines is modeled on the *Reteach* page and *Homework and Practice* page.
Lesson 8-9: Math Practices and Problem Solving- Make Sense and Persevere		
5.NF.B.6 5.NF.B.5a 5.NF.B.5b MP.1 MP.3 MP.4 MP.6	Access Prior Learning: Students have practiced the thinking habits of MP.1 in previous lessons and grades. Students have made sense and persevered solving word problems in previous lessons and grades. Securing the Big Idea: Students apply knowledge of multiplication to solve a real world multi-step problem.	Solve and Share: Consider starting with the *Look Back!* to remind students to estimate before calculating. Look for students making sense of this problem and modeling the context. Can they explain how their model represents the context of the problem? Is there more than one way to solve this problem? How can estimation help to determine the reasonableness of their solutions? Facilitate a class discussion focused on how students are using the thinking habits of MP.1 to solve this problem. Celebrate students' growth and success as problem solvers. Visual Learning: A multi-step problem is modeled and solved. How are the thinking habits modeled in the *Visual Learning Bridge* similar to those students celebrated during the *Solve and Share*? Multi-step real world problems are given on the *Independent Practice* and *the Math Practices and Problem Solving* page. Can students explain why their strategy choices are appropriate for these problems? How did they create a plan before solving?

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