Grade 4 Topic 2: Fluently Add and Subtract Multi-Digit Whole Numbers

Topic 2
Fluently Add and Subtract Multi-Digit Whole Numbers

Number of lessons: **6**

A/D/E: **3** days

NVACS Focus: NBT.B, OA.A

Total Days: ~9

[4th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: [Number and Operations in Base Ten] (pp. 12-15)

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 43I-43J), all 6 lessons, and the Topic Assessments (pp. 87-90A).

Mathematical Background: Read Cluster Overview (TE, pp. 43A-43F)	Topic Essential Questions: How can sums and differences of whole numbers be estimated? What are standard procedures for adding and subtracting whole numbers? *Reference the TE page 43 and Answering the Topic Essential Questions (TE, pp. 87-88) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

2-1	2-2	2-3	2-4	2-5	2-6	Assessment

3 A/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on addition and subtraction of multi-digit whole numbers. Focus instruction on Nevada Academic Content Standards (NVACS) 4.NBT.B.4 and 4.OA.A.3 (2010). Emphasis for standard 4.NBT.B uses place value understanding and properties of operations to perform multi-digit arithmetic. This topic begins a major cluster group. Topics 2-5 are clustered together focusing on 4.NBT.B. Topic 7 will come after Topic 2, which will break up this major cluster, as the focus of Topic 7 is 4.OA.B. This major cluster will continue after Topic 7 with Topic 3.

Again, the focus of Topic 2 is for students to fluently add and subtract multi-digit whole numbers by using the standard algorithm. As defined by the NVACS standards, fluency refers to "skill in carrying out procedures *flexibly*, *accurately*, *efficiently* and *appropriately*" (NVACS, 2010, p. 6). The National Council of Teachers of Mathematics (NCTM) states, "procedural fluency is more than just memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation" (2014, p. 1). When instruction focuses on memorization, students are less willing to think about numbers and their relationships and to apply and develop their number sense.

> *High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed tests stand in the way of number sense, giving students the impression that sense making is not important. (Boaler, 2015, para. 13)*

Rather, development of fluency occurs in three phases: 1) Constructing meaning and counting strategies (e.g., count on) 2) Reasoning strategies (e.g. doubles, near doubles) 3) Working toward quick recall. The third phase, quick recall is defined as allowing students to use a known fact to quickly derive an unknown fact without resorting to inefficient counting methods in about 3 seconds (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). For example, if students quickly say 5 + 6 = 11, they may have used a derived fact like 5 + 5 = 10 and 10 + 1 = 11 or 7 x 7 is 49, thus 8 x 7 is one more group of 7 so 49+7 or 56.

When assessing fluency, AVOID timed tests. Approximately one-third of students begin to experience math anxiety at the onset of timed testing (Boaler, 2014). In addition, timed tests do not tell us which strategies a student used or their level of flexibility. It is important that our instruction and assessments focus on numbers and their relationships. Kling and Bay-Williams state, "students can learn facts effectively without the use of timed testing" (2014, p. 490). Better options for assessment include student interviews, observations, journaling or quizzes based on strategies (Kling, Bay-Williams, 2014). For examples, reference "Assessing Basic Fact Fluency" and "My Fluency Progress" (Teaching Tool 30). Fluency Practice Activities are at the end of Topics 2-16 (TE, p. 83). There are online games to give students more opportunities for fluency practice in the Game Center and Center Games Lesson 2-1 and 2-5.

Along with fluency, a note of caution on developing meaning of addition and subtraction through keyword strategy instruction. Keyword strategy instruction is defined as assigning a mathematical operation to certain words. For example, terms like, each, as much, twice identify as keywords for multiplication. However, Karp, K., Bush, S., and Dougherty, B., state that, "reducing the meaning of an entire problem to a simple scan for key words has inherent challenges" (2014, p.21). Those challenges being:

- Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside the context.
- Unfortunately, many keywords are common English words that can be used in many different ways.

The focus of this topic is for students to develop the meaning of addition and subtraction. Focusing on simple keywords, rather than developing meaning through reasoning of the contexts and modeling the operations, will likely confuse students. This approach will also limit student understanding since many of the keywords for these operations overlap.

Focus Math Practice 2: Reasoning

Focus opportunities for students to develop *Mathematical Practice 2* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 2-6. Reference the Teacher's Edition (TE, pp. F22-F22A) and the NVACS (2010, SMP 2, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
variable algorithm	*commutative property of addition* *associative property of addition* *identity property of addition* *counting on* *compensation* *inverse operation*

Additional terminology that students may need support with: mental math, estimate, break apart, bar diagram, addends, sum, minuend, subtrahend, difference, regroup

*Consider using the additional terminology to label anchor charts used throughout this topic.

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding question: "How are students applying place value understanding and the properties of operations to perform multi-digit addition and subtraction?"

Lesson	Evidence	Look for
2-3	*Solve & Share* (student work samples)	Focus CTC around the big idea: • students estimation, strategies and models. • use of regrouping and place value understanding.
2-5	*Quick Check* (digital platform or student work samples) Item 5	Focus CTC around the big idea: • students solve multi-step problems with both addition (regrouping) and subtraction (across zeros). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 87-90	Use *Scoring Guide* TE pp. 87-90A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 2-1: Mental Math: Find Sums and Differences		
4.NBT.B.4 MP.2 MP.3 MP.6 MP.7	Access Prior Learning: In previous grades, students learned different properties for addition. Beginning in second and third grade, students used mental math strategies such as, breaking apart, compensation and counting on to add and subtract whole numbers mentally.	Note: Consider spreading this lesson over two days. If choosing to do the lesson over two days, know this is one of the A/D/E days in the WCSD suggested pacing framework. This lesson gives various estimation strategies that students may apply mentally. Solve & Share: Consider removing the "think bubble" when students are working on this problem independently. The "think bubble" gives students clues of what to do when solving this problem. Use an anchor chart to display students' strategies. Watch for students who use place value understanding, and other mental math strategies like compensation to add to the anchor chart. *-continues on next page-*

	Developing the Big Idea: In fourth grade, students will continue using addition properties and mental math strategies to add and subtract greater whole numbers. Look Ahead: Estimation is a recurring concept throughout 4th grade. In this topic, consider having students estimate before finding the actual sums and differences. This helps students to determine if their answers are reasonable.	Visual Learning: Connect strategies used in the *Solve & Share* to the properties used in the *Visual Learning Animation*. Consider pausing after each addition property to discuss how the properties are used when adding mentally. Make an anchor chart with the addition properties so students can reference. Convince Me: In the *Convince Me!* students are asked to use mental math to add 150 + 2,300 + 250. Students may use addition properties (given as a sample response) to add mentally. Honor other mental compensation strategies used to solve the problem. Consider also doing the following Number String with your students (Fosnot and Uittenbogaard, 2007): 102 - 5 102 - 97 1003 – 5 1003 – 997 152 – 49 152 – 3 10,002 – 5 10,002 – 9999 Do one problem at a time and record students' strategies. Use the Number String as a formative assessment to see the strategies students use to subtract whole numbers mentally. The focus of this string is to have students use landmark or anchor numbers to support strategies; such as, break apart constant difference into two groups then add ("Think Addition") or engage in compensation strategies. Another Example: In the *Another Example!*, students will develop mental math strategies; break apart, counting on and compensation to add and subtract whole numbers. Consider facilitating a class discussion around the mental math strategies by having students attempt the problems on their own by posing the problems individually without the Student Edition (SE) open. Encourage students to solve in different ways. After the discussion, add the mental math or estimation strategies like breaking apart and adding-on to the anchor chart. Independent Practice/Math Practices and Problem Solving: Students do NOT have to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with students who may need place-value blocks or other tools to "count on" and/or to compensate.
Lesson 2-2: Mental Math: Estimate Sums and Differences		
4.NBT.B.4 4.OA.A.3 MP.1 MP.3 MP.5 MP.6	Access Prior Learning: In third grade, students used place value understanding to round whole numbers to the nearest 10 or 100. In Lesson 1-4, fourth grade students rounded whole numbers to place value positions. Developing the Big Idea: In this lesson, students will continue to use their place value understanding of rounding whole numbers to estimate sums or differences of whole numbers.	Solve & Share: Consider removing the bar diagram to see what tools, models or strategies students might use to solve the problem. Some students may find the actual sum for the problem. Choose students who may have estimated and another student who may have found the actual solution to share. Facilitate a discussion regarding the language of estimation (about, approximate, etc.). As well as, the purpose of estimation and that estimation helps mathematicians make sense of the problem. Guide students in understanding when estimation is appropriate and when to find the actual solution. Visual Learning: Note: Van de Walle, Karp and Bay-Williams (2010) state, "when several numbers are to be added, it is usually a good idea to round them to the same place value" (p. 246). In the *Visual Learning Animation*, students determine which estimate is correct. Ensure that students understand that either estimate is correct however, there may be an estimate that is closer or more reasonable to the actual answer. Convince Me: Use the *Convince Me!* to facilitate a discussion in regards to which estimate is more precise for the given situation. Be cautious in comments made in regards to who has a more precise estimate. Students need to understand that their estimation is not incorrect, but there may be a more precise estimate that is closer to the actual answer. *-continues on next page-*

		Guided Practice: Consider doing items 1 and 2 of the *Guided Practice*. Have students explain their thinking in regards to why they estimated to a specific place value position. For item 2, ask students, "Which place value position would we round to, in order to have a more precise estimate?" Independent Practice/Math Practices and Problem Solving: Consider doing item 12 in the *Independent Practice* before having students complete *Quick Check* items (marked with a pink check mark). Have students discuss how they would estimate the sum. See item 12 "Coherence" for an explanation of this problem (TE, p. 55-56). Assess and Differentiate/Intervention Activity: In the *Intervention Activity*, place-value blocks are used, consider also having students use a number line to estimate.
Lesson 2-3: Add Whole Numbers		
4.NBT.B.4 4.OA.A.3 MP.3 MP.7 MP.8	Access Prior Learning: In second and third grade, students added whole numbers with sums to 1,000. Securing the Big Idea: In this lesson, students will find the sum of multi-digit whole numbers by using the standard algorithm. Students will also estimate the sum by using various strategies.	Note: Encourage students to estimate sums of multi-digit whole numbers before finding the actual answer. This will help students develop reasonableness of numbers. Also, encourage students to use mathematical language like, addends and sum when discussing addition of whole numbers. Solve & Share: Consider having students use an estimation strategy before finding the actual sum. Look for students who use their understanding of place value and regrouping to find the sum. This would include partial sums. See example for partial sums for *Solve and Share* 4,219 + 3, 472 + 4, 436 4,000 + 200 + 10 +9 3,000 + 400 + 70 +2 + 4,000 + 400 + 30 +6 Look Back: In the *Look Back!,* students determine the property used to group numbers to add. In Lesson 2-1, the addition properties were discussed. Visual Learning: In the *Visual Learning,* students find the sum by using the U.S. Traditional standard algorithm. Consider having students use a standard algorithm and partial sums to compare strategies and support place-value understanding when using algorithms. Facilitate discussion around place value when using standard algorithms to add whole numbers. See questions in the *Prevent Misconceptions,* such as, Why do you need to align place values before adding? Explain how you regroup after adding the thousands. Why do you not regroup when adding the ten thousands? (TE, p. 60). Consider making an anchor chart with multiple addition strategies, like partial sums algorithm and the U.S. traditional algorithm. Note: During the *Visual Learning Animation*, the mathematical word ***variable*** appears. Consider using variables throughout the rest of this topic and beyond. Visual models, such as bar diagrams, are great representations to build conceptual understanding around this idea. See Teacher's Edition pages F31-F34 for various addition and subtraction situations with bar diagrams. Independent Practice/Math Practices and Problem Solving: Facilitate a discussion around students' estimation and reasonableness compared to their actual sums. Assess and Differentiate/Intervention Activity: Place-value charts are great tools to guide understanding and should include values related to the different place value positions. Consider also having students use the partial sums algorithm to compare to the U.S. traditional algorithm. *CTC: *Solve & Share* (student work samples)
Lesson 2-4: Subtract Whole Numbers		
4.NBT.B.4 4.OA.A.3 MP.1 MP.2 MP.3	Access Prior Learning: In third grade, students fluently subtracted whole numbers to 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationships between addition and subtraction (3.NBT.A.2).	Note: Karp, Bush and Dougherty (2014), state to be cautious with the phrase, "You cannot take a bigger number from a smaller number" as this is a rule that expires (p. 21). Meaning, this does not work when adding and subtracting negative numbers. A rule that stays true is "subtracting a larger number from a smaller one results in a negative number, an integer that is not in the set of whole numbers" (Karp, et al., 2014, p. 21). Keep focus on place value. See MP.3 Construct Arguments for example questions (TE, p.66). *-continues on next page-*

	Developing the Big Idea: In this lesson, students will subtract greater whole numbers using the standard algorithm and place value.	Solve & Share: Look for students who use place-value understanding; like partial differences algorithm to subtract whole numbers. Consider having these students share their strategy before having students who used the U.S. Traditional algorithm share. Facilitate a discussion around the similarities and differences between the strategies. Make a subtraction strategies anchor chart with strategies like the partial difference algorithm, U.S. traditional algorithm and other standard algorithms. Visual Learning: Connect student strategies from the *Solve & Share* to the *Visual Learning*. In the *Visual Learning Animation,* students discuss the inverse operation. Convince Me: Consider having students do the *Convince Me!* independently and then facilitate a discussion around the errors made. You may use this formatively to help guide the rest of the lesson. Another Example: Consider doing this activity with whole class, because it discusses estimation when subtracting. Independent Practice/Math Practice and Problem Solving For item 16, students have to estimate to decide if their answer is reasonable. Assess and Differentiate/Intervention Activity: Consider doing the *Intervention Activity* with the whole class, so students see the minuend, 725, written as a different whole number using a place-value chart.
Lesson 2-5: Subtract Across Zeros		
4.NBT.B.4 MP.2 MP.3 MP.5 MP.8	Access Prior Learning: In the previous lesson, students subtracted whole numbers using the standard algorithm and place value. Securing the Big Idea: In this lesson, students will continue subtracting greater multi-digit whole numbers using the standard algorithm and place value.	Solve & Share: Students will be subtracting whole numbers across zeroes. Look for students who use place-value understanding; like partial differences algorithm to subtract whole numbers. Consider having these students share their strategy before having students who used the standard algorithm share. Facilitate a discussion around the similarities and differences between the strategies (Share and Compare). Visual Learning: Connect student strategies from *Solve & Share* to the *Visual Learning*. In the *Visual Learning*, students will learn two different regrouping strategies to subtract whole numbers across zeroes. Facilitate a discussion around specific questions such as, "Explain the regrouping in 6,000 – 4,678" (TE, p. 72). Have students use estimation to check if their answer is reasonable. Independent Practice/Math Practice and Problem Solving: Consider having students estimate as they work through the *Independent Practice* items as mentioned in the Think Bubble on pg. 73. Item 26, discusses the mathematical terminology, variable from lesson 2-3, consider using this item to reinforce the use of variables. Also in the *Math Practices and Problem Solving* pages, item 28 "Higher Order Thinking" is a multi-step problem that uses information from a table to solve the problem. Consider using this formatively, as part of a whole-class discussion or during a Gallery Walk (Discussed in Topic 1 and ELL Toolkit). Assess and Differentiate/Intervention Activity: The *Intervention Activity* gives students an opportunity to use concrete manipulatives like place-value blocks or Teaching Tool 4 and 5 to solve subtraction problems across zeroes. This is a great place for all students to continue their understanding conceptually. *CTC: *Quick Check* (digital platform)
Lesson 2-6: Math Practices and Problem Solving- Reasoning		
4.NBT.B.4 4.OA.A.3 MP.2 MP.1 MP.4 MP.8	Access Prior Learning: Students have used Math Practice 2: Reasoning throughout the instructional materials and in previous grades. Securing the Big Idea: In this lesson, students will take previously learned concepts and skills related to addition and subtraction of whole numbers to	Solve & Share: Students will reason about whole number addition and subtraction to find sums and differences. Give students the opportunity to use a variety of tools, strategies or models to solve this problem. Consider having students "share and compare" student work of those who used concrete tools, drew a representation or solved abstractly by using an algorithm in this order. Look Back: Use the *Look Back!* to encourage students to use bar diagrams to write numerical expressions and equations. Bar diagrams are the focus model in this lesson and will help students understand using variables to write numerical expressions and equations. *-continues on next page-*

	reason abstractly and make sense of quantities and their relationships in problem situations.	Visual Learning: The *Visual Learning Animation* continues the use of bar diagrams and writing equations through reasoning about the problem. Discuss good mathematical thinking habits when reasoning, as well as using correct mathematical language when discussing. Convince Me: Have students do the *Convince Me!* as students are asked to create their own problem related to the bar diagram. Have students critique the reasoning of others to see if their problem works for the bar diagram given. Assess and Differentiate/Intervention Activity: If students are struggling with understanding bar diagrams, consider having students work on the *Intervention Activity*. A bar diagram can help students move from concrete tools to representational models. Guided Practice: Consider using items 1 and 2 whole class, as this lends itself to different problem situations. See Teacher's Edition pages F31-F34, page 88 in the 2010 Nevada Academic Content Standards or Table 2: Common Addition and Subtraction Situations below. If students are struggling with this problem, consider what they know and build upon their knowledge from there. Independent Practice/Math Practice and Problem Solving: To expose students to different problem situations, consider using items 3-5 or items 6-9. See Table 2 below for problem situations.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

References

Boaler, J. (2016). Fluency without fear. Retrieved from https://www.youcubed.org/fluency-without-fear.

Fosnot, C. & Uittenbogaard, W. (2007). *Minilessons for extending addition and subtraction: A yearlong resource*. Orlando, FL: Harcourt School Publishers.

Karp, K., Bush, S., & Dougherty, B. (2014). 13 Rules that Expire. *Teaching children mathematics, 21*(1), 18-25.

Kling, G., and Bay-Williams, J. (2014). Assessing Basic Fact Fluency. *Teaching children mathematics,* 20(8), 488-497.

Van De Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Pearson.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). Boston, MA: Pearson.

This page is intentionally left blank.