#### enVisionmath2.0

# ▶ Grade 2 Topic 2: Work with Equal Groups

### Big Conceptual Idea: Equivalence

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teachers Edition (TE): Cluster Overview/Math Background (pp. 77A-77E), the Topic Planner (pp. 77I-77J), the Topic Performance Assessments (pp. 117-118A), and all 5 lessons.

Mathematical Background:	Topic Essential Questions:
Read Cluster Overview (TE,	How can you show even and odd numbers? How do arrays relate to
pp. 77A-77F)	repeated addition?
	Reference Answering the Topic Essential Questions (TE, p. 115-116) for key elements of answers to the Essential Questions.

### The lesson map for this topic is as follows:

ļ	Z-1	Z-Z	2-3	Z-4	2-5	Assessment
	5	2.2	2.2	2.4	<u>о</u> г	Accorport

3 A/D/E days used strategically throughout the topic.

### Instructional note:

The big idea of Topic 2 is equivalence. This topic focuses on a) *securing* understanding of the categorization of numbers as even or odd, and b) developing understanding for finding the total objects in situations involving equal groups. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.OA.C which supports the Topic 1 cluster 2.OA.B.

## 2.OA.C Work with equal groups of objects to gain foundations for multiplication.

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Focus instruction on students' construction of a definition of even numbers as numbers that can be broken into two equal sets with no leftovers, reinforcing the big idea of equivalence. Likewise, investigate and discuss how odd numbers cannot be split into two equal sets. Do not *define* even and odd numbers by the patterns in the ones digits (e.g., 0, 2, 4, 6, 8 for even; 1, 3, 5, 7, 9 for odd). These patterns describe *attributes* of even and odd numbers but do not support the big idea of equivalence. Building on the work from Topic 1, students will apply their understanding of doubles to even numbers and their understanding of near doubles to odd numbers. It is important that students explore with concrete objects before moving to representations including drawings, arrays, bar diagrams and equations.

Work with equal groups requires students to apply the big idea of unitizing (Fosnot, Dolk, 2001). "Unitizing requires that children use number to count not only objects but also groups- and to count them both simultaneously" (Fosnot, 2007, p. 7). As students begin to work with arrays, they will connect their understanding of even and odd numbers. Even numbers can be represented using arrays with two equal groups (rows or columns), while odd numbers cannot be represented in arrays with two equal groups.

Students will also apply their ability to unitize by grouping objects in arrays by rows or columns, and develop the understanding that the total items in an array can be found through repeated addition of these units. Students will write an equation reflecting the sum of equal addends as equivalent to the total items in the array. Finally, students will apply these understandings to problem situations that involve equal groups. Look for opportunities to connect equal groups to students' real-world experiences. Work with arrays supports skip counting by 5s, 10s, and 100s to 1,000 in Topic 9, and the partitioning of rectangles into equal rows and columns of squares in Topic 15. Ultimately, work around the big idea of equivalence lays the foundation for algebraic reasoning and multiplication and division in grade 3.

### Math Practice 4: Model with mathematics

Focus on opportunities for students to develop MP.4 behaviors. This is the focus of the Math Practices and Problem Solving lesson 2-5. Reference the Teacher's Edition (TE, pp. F26-F26A) and the *Nevada Academic Content Standards for Mathematical Practice*.

**Note:** The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
even	equal		
odd	part		
equal groups*	whole		
* Add to vocabulary explicitly taught			
in lesson 2-3			
array			
rows			
columns			
bar diagram			

Additional terminology that students may need support with: addends, equation, model, pairs, sum

#### \*Collaborative Team Conversations (CTC)

Consider using one of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

#### Guiding questions:

"Are students developing conceptual understanding and moving their thinking counting all objects to repeated addition of arrays?" "Are students developing conceptual understanding of organizing models to represent math equations?"

Lesson	Evidence	Look for
2-4	Solve & Share (student work samples)	Focus CTC around the big idea:
		<ul> <li>student strategies and models</li> </ul>
		use of repeated addition
		<ul> <li>understanding arrays as equal rows and columns</li> </ul>
2-4	Do You Understand: Show Me!	Focus CTC around data analysis and collection of student workspace
	(digital platform) *Optional in SE	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessment	Use Scoring Guide TE pp. 115-118
Assessments (summative)	SE pp. 115-118	

#### Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 2-1: E	Even and Odd Numbers	
2.OA.C.3 2.OA.B.2 MP.4 MP.5 MP.6 MP.7	Access Prior Learning: In first grade, students had the opportunity to work with the classification of even and odd numbers. Securing the Big Idea: In this lesson, students are securing understanding that numbers can be classified as even or odd by showing numbers as two equal parts.	<ul> <li>Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be represented with doubles facts.</li> <li><b>Topic Opener:</b> Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p.77), <i>Review What You Know</i> (TE, pp. 78-80) and the <i>Topic 2 Vocabulary Words Activity</i> with the words <i>even</i> and <i>odd</i>. Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. <b>Visual Learning:</b> Have students make cube towers to increase understanding and engagement. Although the <i>Visual Learning</i> discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that cannot. The patterns of ones digits should be understood as an attribute of even/odd numbers, not as their definition. (See instructional note at beginning of this document.) Encourage children to revise and add to their word webs for both concepts: even and odd.</li></ul>
		-continues on next page-

		Independent Practice/Math Practices and Problem Solving:
		Students do NOT need to do all of the problems in their Student Edition. Ask students to complete the <i>Quick Check</i> items (marked with a pink check mark) first and continue on to other items as appropriate. For item 12, consider using the <i>Problem Solving Recording Sheet</i> (Teaching Tool 1) to help students make sense of the problem. Allow students to work on this problem collaboratively in pairs with manipulatives before incorporating into whole group guided practice. Watch for students who use concrete objects, drawings, equations or tables to organize their thinking. Focus the discussion on the mathematical generalization that can be drawn from this work: that adding two whole odd numbers will always have an even sum. Explore why this works.
Lesson 2-2: 0	Continue Even and Odd Numbers	
2.OA.C.3	Access Prior Learning:	Have students revisit and add to the word webs for even and odd to assess understanding and
2.OA.B.2	In first grade, students had the opportunity to skip count by 2s and identify patterns in skip counting.	inform instructional decisions. Look for students who demonstrate understanding that even numbers can be broken into two equal parts, and that odd numbers cannot.
MP.2	identify patterns in skip counting.	Solve & Share:
MP.4 MP.6 MP.7 MP.8	In the prior lesson, second grade students broke apart cube towers to classify numbers as even or odd. Securing the Big Idea: In this lesson, students are securing understanding that numbers can be classified as even or odd by analyzing skip-counting patterns and writing even numbers as a sum of equal addends.	Before problem solving, ask students to make sense of the problem. Clarify vocabulary such as addends and sum as needed. During problem solving, child-watch for students who are able to use cubes to represent equations with two equal addends and demonstrate understanding that the addends in the equations represent the number of squares in each row. This idea will be reinforced in the <i>Visual Learning</i> . Strategically select two students to share their solutions building in sophistication or accuracy. In the discussion, focus your guiding questions on facilitating student connections between strategies and connections to the big mathematical idea of equivalence with questions such as, "What connections can you make to addition strategies we used in Topic 1?" [doubles facts]. "How can skip counting help us to answer the question?" [skip count by 2s] <b>Visual Learning:</b> Engage students in discussion throughout the <i>Visual Learning Animation</i> , intentionally connecting back to the <i>Solve &amp; Share</i> . Ask questions to help students connect the cube towers they built in the <i>Solve &amp; Share</i> to the representational drawings, arrays and equations seen in the animation. This will strengthen their conceptual understanding.
	Jse Arrays to Find Totals	Work with arrays and repeated addition develop students' understanding of againglance and law
2.OA.C.4 2.OA.B.2 MP.1 MP.3 MP.4 MP.7	Access Prior Learning: In the prior lesson, second grade students wrote equations to represent even numbers, connecting rows of objects to the addends in the equation. Beginning the Big Idea: In this lesson, students are <i>beginning</i> understanding that arrays show equal groups, and that equations using repeated addition can be used to find the total objects in an array.	Work with arrays and repeated addition develop students' understanding of equivalence and lay the foundation for multiplicative thinking in grade 3. Students work with equal groups in rows and columns, as well as equivalence in repeated addition equations to represent the total objects in an array (e.g., For a 4 x 3 array, 4 + 4 + 3 + 3 + 3 + 3 + 3 = 12.) After the <i>Solve &amp; Share</i> , introduce the vocabulary word "array" using the <i>Graphic Organizer 5: Frayer Model</i> (Teaching Tool 62). This organizer includes the definition, characteristics, examples and non-examples.  Solve & Share: In the teacher notes, omit step 2. <i>Build Understanding</i> (TE p. 93) as it provides too much up front and removes opportunities from students for problem solving. Allow students to work on the <i>Solve &amp; Share</i> without prior instruction. Child-watch for students who are able to unitize (work with equal rows or columns) and identify equal groups of 5 and equal groups of 3. If students count by 1s, support students in unitizing, by clarifying the terms: rows and columns and ask "How can you use equal groups to help you find how many circles in all more efficiently?" The use of two-sided counters (red/yellow) can help students to visualize rows or columns as equal groups on ten-frames.

Lesson 2-4:	Make Arrays to Find Totals	
2.OA.C.4 2.OA.B.2	Access Prior Learning: In the prior lesson, second grade	Solve & Share:
MP.2 MP.4 MP.5 MP.8 MP.8 students used find the total of Beginning the In this lesson, beginning under making arrays, addition can be	students used repeated addition to find the total objects in an array. Beginning the Big Idea: In this lesson, students are beginning understanding that	During problem-solving, child-watch for students who confuse columns and rows. During the share, ask two children to share their solutions, beginning with the student who built an array with 4 rows and 3 columns, followed by a student who built an array with 3 rows and 4 columns, as was asked. Engage children in a discussion regarding equivalence (both arrays have the same total). Then use this share as an opportunity to highlight MP. 6: Attend to Precision (TE, pp. F28-28A) by reviewing the meaning of columns and rows.
	making arrays, and using repeated addition can be used to solve addition problems.	Visual Learning: Use the <i>Do You Understand? Show Me!</i> to formatively assess students' understanding of arrays and equal groups.
		Independent Practice/Math Practices and Problem Solving: In preparation for item 9, look for opportunities for your students to interact with arrays in the real world. They can draw upon this experience to write a story problem using repeated addition. For example, have students walk in 2 equal lines when they come in from recess, put supplies away in an array formation, etc.
		*CTC: Solve & Share (student work samples) *CTC: Do You Understand: Show Me! (digital platform)
Lesson 2-5: I	Math Practices and Problem Solv	ing: Model with Math
2.OA.A.1 2.OA.C.4 MP.1 MP.3	Access Prior Learning: In first grade, students engaged in Math Practice 4, modeling with ten frames, number lines and open number lines.	MP.4: Model with Mathematics, encourages students to solve problems using the mathematics they know. Support students in determining whether their solutions make sense based on the context of the problem (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p.3). Also, focus on the MP.4 Model with Mathematics thinking habits included in the <i>Math Practices and Problem Solving Handbook</i> (TE, pp. 26-26A). Add these habits to the math focus wall for reference throughout the year.
MP.4 MP.5 MP.6 MP.7 MP.8	Developing the Big Idea: In this lesson, students are <i>developing</i> understanding that mathematicians use math they know to show and solve problems. They model problems with drawings, arrays, bar diagrams and equations.	Solve & Share: During problem solving, look for students who use their prior knowledge to solve the problem. Strategically select students who used different models to share. In the share, ask questions such as, "How does [student A's] model show the problem?" and "How does this model show that you used what you know to solve the problem?" After discussion, encourage students to write an explanation of their use of MP.4. Visual Learning: If students are having trouble understanding the bar diagram, reference <i>Error Intervention</i> : Item 1 note (TE, p.106).

#### References

- Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Fosnot, C.T. (2007). The t-shirt factory: Place value, addition, and subtraction. Heinemann: Portsmouth, N.H.
- Fosnot, C. T., & Dolk, M. (2001). Young mathematicians at work: constructing number sense, addition, and subtraction. Heinemann: Portsmouth, N.H.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2<sup>nd</sup> ed.). Boston, MA.: Pearson.