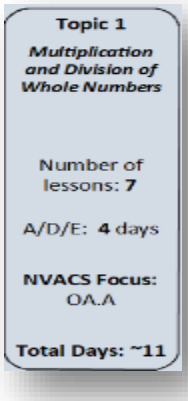
Grade 3 Topic 1: Understand Multiplication and Division of Whole Numbers



3rd Grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: K-5 Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 1 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 1A-1F), the Topic Planner (pp.1I-1K), all 7 lessons, and the Topic Assessments (pp. 55-56A).

Mathematical Background: Read Topic 1-2 Cluster Overview/Math Background (TE, pp. 1A-1F)	Topic Essential Question: What are the different meanings of multiplication and division? *Reference Answering the Topic Essential Question (TE, pp. 53-54) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

1-1	1-2	1-3	1-4	1-5	1-6	1-7	Assessment

4 A/D/E days used strategically throughout the topic

Instructional note:

This topic focuses on *beginning* to build the meaning of multiplication and division to meet the Nevada Academic Content Standards (NVACS) 3.OA.A cluster, "Represent and solve problems involving multiplication and division" (2010). This topic focuses on exploring multiplication as meaning equal groups and connects this understanding to 2nd grade when children "Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns; write an equation to express the total as a sum of equal addends" (NVACS, 2.OA.C.2, 2010).

Throughout this topic, children model various multiplication situations using tools including arrays and number lines. Additionally, students use strategies such as skip counting to explore the concept of equal groups and connect repeated addition to multiplication. Students are introduced to the Commutative Property of Multiplication which states that the order factors are multiplied in does not change the product. Students need to understand that multiplication equations model situations. For example, 4 teams of 6 players (4 x 6) is different from 6 teams of 4 players (6 x 4). Explore the equal groups within the array for each model of (4 x 6) and (6 x 4).

Conceptually, the focus is understanding multiplication as equal groups and relating this to division. In 2nd grade, students understand that they are able to add and subtract groups with varying group size (e.g. 36-14 or 45+76), now this knowledge is pushed to realizing that in multiplication the group size must be equal (2+2+2+2 is 4 groups of 2, or 4x2). Emphasize and help students connect "equal group size", "equal addends" and "equal subtrahends" throughout the topic.

Students develop understanding of division as a way of separating one group of objects into ***equal groups***. Division has two (2) different types of problems. Partitive division, also known as dealing or fair sharing, is when the number of groups is known but the size of each group is unknown. Measurement division, also known as chunking, is when the number in each group is known but the number of groups is unknown. Measurement division allows students to employ repeated subtraction. The two (2) types can only be determined when working with division in context or word problems and will be explored further in Topic 5.

This understanding of "equal groups" begins to develop the foundations for fluency and will build over the next few topics to support 3.OA.C.7, "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8x5=40, one knows 40/5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers" (2010). It is important to note that this expectation may not be secure until "the end of Grade 3." Additionally, Van de Walle, Karp, Lovin, and Bay-Williams (2014) state that, "…students' progress through stages that will eventually result in 'just knowing'" (p. 128). Those stages are:

- Phase 1: Constructing Meaning and Counting Strategies
- Phase 2: Reasoning Strategies
- Phase 3: Working toward quick recall

Detailed information about the three phases of development towards fact fluency can be found in the article Basic Math Facts: A Sequence of Learning. This topic focuses on phase 1: Constructing Meaning and Counting Strategies for multiplication and division. In this topic students develop understanding of multiplication as a way of joining equal groups by using repeated addition with arrays and skip counting with the open number line and arrays.

Tools are referenced several times in this topic in the *Look Back!*. Consider discussing these questions whole group to help establish a culture for learning mathematics with manipulatives seen as tools for working with, understanding and representing mathematics.

Focus Math Practice 5: Use appropriate tools strategically

Focus on opportunities for students to develop *Mathematical Practice 5* behaviors, as this is the focus of the Math Practices and Problem Solving lesson, 1-7. Reference the Teacher's Edition (TE, pp. F25-F25A) and the NVACS (2010, p. 6).

Assessment Considerations:

On Topic Assessment item 6, students' arrays need to actually match the context described in the problem. For example, students must have 2 rows with 4 items in each row as their array to receive credit (2 x 4). Encourage students to continue to use tools while completing the assessment.

Both the Topic Assessment and the Topic Performance Assessment will provide opportunities to work at various DOK levels. Choose the assessment(s) that will provide the most information about student understanding. For Topic 1, consider scaffolding this resource by allowing students to work in groups throughout the topic and building in opportunities for discussion, peer feedback, and revision.

Finally, please note that lessons 1-1 and 1-2 indicate that these are possible 2-day lessons. Additional A/D/E days were built into the 2019/2020 WCSD 3rd Grade Pacing Framework to allow time to establish class routines and expectations for:

- Accessing and returning manipulatives
- Classroom discussion norms
- Mathematical Mindset
- Integrating ideas from the *Math Practices and Problem Solving Handbook* (TE p. F19-F35)

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
multiplication factors product equation unknown Commutative (order) Property of multiplication division	*equal groups* *number line* *array* *row* *column*

Additional terminology that students may need support with: repeated addition and repeated subtraction

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding question: "Are students using multiple strategies to multiply and divide whole numbers?"

Lesson	Evidence	Look for
1-3	*Solve & Share* (student work samples)	Focus CTC around big idea: • students can draw arrays to show joining groups. • students can use multiple tools, models or strategies (skip counting, repeated addition, arrays). • students understand multiplication means equal groups.
1-5	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). • students understand that multiple objects can be shared by separating into equal groups. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 53-56	Use *Scoring Guide* TE pp. 53-56A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 1-1: Multiplication as Repeated Addition		
3.OA.A.1 3.OA.A.3 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In Topic 2 of second grade, students used repeated addition to join equal groups in arrays. This lesson connects students' understanding of repeated addition as a way to think of multiplication. Developing the Big Idea: Students are *beginning* to build the understanding that real-world problems involving the joining of equal groups can be solved using multiplication or adding equal groups (repeated addition) (TE p. 8). This lesson has an emphasis on the language "groups of" to develop understanding of multiplication as being A groups of B items.	(Possible 2-day lesson) Day 1: Topic Opener: Consider having students complete the *Review What You Know* to assess student understanding and determine instructional responses to identified student needs. Using *the Item Analysis for Diagnosis and Intervention* (TE, p. 2) could be beneficial. Introduce vocabulary as students encounter words in the lessons rather than introducing all terms at the beginning of the lesson. Introduce the *Topic Essential Question*, "What are different meanings of multiplication and division?" (TE, p.1) and making this an anchor chart that ideas from *Developing the Big Idea* in each lesson could be added for assisting students in explicitly seeing the connections between lessons and the build of big mathematical idea for this topic. Solve & Share: Consider integrating ideas from the Problem Solving Guide (p. F29) and the Problem Solving Recording Sheet (p. F30). The master for the Problem Solving Recording Sheet is found in the *Teacher's Resource Masters, Volume 2:* Teaching Tool 1. Introduce routines for tool use and management. Students should have access to, and be encouraged to use tools throughout math instruction. Introduce routines for sharing student solution methods to the *Solve & Share* and whole class discussion structures for discussing the mathematical ideas developed from student reasoning. Whenever possible, it is beneficial to use the whole class discussion to highlight student solutions and reasoning and discover the mathematical understanding described in *Transition to the Visual Learning Bridge* (TE, p. 7). This provides students with a purpose for watching the *Visual Learning Animation* as they either confirm, clarify or correct their conjectures. Day 2: Visual Learning: Consider having students use counters to build the 3 groups of 8 to increase understanding and engagement. Item 2 in the *Guided Practice* confronts the common misconception that we do not need equal groups to multiply. Should students feel that this is still a multiplicative situation consider putting the problem into context and having students model the problem with counters. Pose the question, "Where are the equal groups that we're joining?" Independent Practice/Math Practices and Problem Solving: Students do NOT need to do all of the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first. Students can be asked to continue with other items based on individual needs. Consider including item 15 "Critique Reasoning" as it will provide evidence of learning that addends need to be the same in order for an addition equation to be written as a multiplication equation. Assess and Differentiate/Intervention Activity: If time permits, teach students how to play *Toss and Talk* (TE, p. 11A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.11A).

Lesson 1-2: Multiplication on the Number Line		
3.OA.A.1 3.OA.A.3 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In Grade 2 students learned to add using the number line by making jumps to the right. In Topic 9, Grade 2, they learned to skip count by 5s, 10s, & 100s to join equal groups. In lesson 1-1, Grade 3, they learned to think of multiplication as repeated addition. Developing the Big Idea: Students *begin* to build the understanding of multiplication as repeated addition and skip counting by making equal jumps on the number line that are the size of the multiplier (the 2nd factor in a multiplication equation). Each jump represents the repeated addition or skip counting.	(Possible 2-day lesson) Day 1: Solve & Share: Ask students how they could use a number line to show the repeated addition from yesterday's *Solve & Share*. For students that demonstrate they are not ready to use the number line, consider allowing them to solve the *Solve & Share* any way they choose and then attempt to solve using the number line. Continue to build routines for accessing and returning math tools as well as whole class discussion structures and norms. Day 2: Visual Learning: Before going on to the *Try It*, consider discussing the equation that could represent this problem and what students would need to skip count by to get the product. During the *Try It*, discussing how skip counts are modeled in the number line can help to make connections. Key questions to help students understand how the equation relates to the model are: • "What number do you need to skip count by?" • "Why is each arrow 3 units long?" • "How does the equation show 5 x 3?" Independent Practice/Math Practices and Problem Solving: The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate. Assess and Differentiate/Intervention Activity: Teach students how to *Toss and Talk* (TE, p. 11A). All students should have the opportunity to play this game. Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.17A).
Lesson 1-3: Arrays and Multiplication		
3.OA.A.3 3.OA.A.1 MP.1 MP.2 MP.3 MP.5 MP.7	Access Prior Learning: In Grade 2, students learned that arrays could be used to find the total number of objects in equal sized groups. Lesson 1-1, Grade 3, connected students' understanding of repeated addition to find the total by introducing that the total number can be found by multiplying the number of rows by the number in each row. Developing the Big Idea: Students work with the *developing* understanding that one way to think of multiplication is by using arrays in situations where we are joining equal groups. This understanding helps students connect ideas about arrays and repeated addition and/or skip counting to a way of thinking about multiplication.	Solve & Share: Students should be encouraged to use appropriate tools (MP.5) to model multiplication and division situations. Visual Learning: *United States convention for representing multiplication as an array is that the first factor is the number of rows and the second factor is the number of columns (objects in each row).* Teacher's Edition p. 20 poses the question, "How would the array look if the multiplication equation were 5 x 4 = 20?" Consider discussing as a class and modeling this difference with counters. The *Visual Learning Animation* would explain this as; there are 4 objects in a row and that row is repeated 5 times, thus we have 4 "times" 5. It is important that students understand that a multiplication equation represents a context. Lesson 1-4 will explore the Commutative Property of Multiplication with arrays which helps students develop the understanding that factors multiplies in a different order will result in the same product. Guided Practice: During the *Guided Practice*, consider asking students to write the skip count for items 3 & 4. Currently, students are used to showing the skip counts on the number line. They have made a list of the skip counts in 2nd grade; yet have not done so in 3rd grade. Including this will prepare students to be able to show on *Independent Practice* item 5 that they understand skip counting as multiplication. Assess and Differentiate/Intervention Activity: If time permits, teach students how to play "Teamwork" (TE, p. 23A). All students should have the opportunity to play this game. Consider having students record and turn in an explanation for 1 of the equations they draw as a formative assessment. Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.23A). *CTC: *Solve & Share* (student work samples)

Lesson 1-4: The Commutative Property		
3.OA.B.5 3.OA.A.3 MP.3 MP.7	Access Prior Learning: In Grade 2, students were secure in their understanding of the commutative (order) property of addition. This property states that order of addend does not change the sum. In lesson 1-3, Grade 3, students learned that arrays can represent multiplication and that the order of the factors gives the number of rows and the number in each row. Beginning of the Big Idea: In this lesson students work with their *secure* understanding of the Commutative (order) Property of Addition to *beginning* to build the understanding of the Commutative (order) Property of Multiplication. The property states that we can multiply the factors in any order and the product stays the same.	Instructional note: Throughout this lesson bring attention to how switching factors creates a different multiplication situation. The Commutative Property gives us a strategy for finding products. Therefore, if 2 x 7 is an easier fact to recall, I can use 2 x 7 to get the product for 7 x 2. You may consider having students return to modeling the differences in the expressions while still seeing that the product remains the same. Solve & Share: To continue to support students' development of behaviors in selecting appropriate tools (MP.5), consider waiting to distribute the 25 counters until after posing the question, "What tools or strategies can you use to solve this problem?" (TE, p. 25). Look Back: In the event that a conjecture supporting the Commutative (order) Property is not developed during sharing of student solution methods and reasoning from the *Solve & Share,* consider discussing the *Look Back!* and relating it back to models for the *Solve & Share*. Visual Learning: Consider pausing the *Visual Learning Animation* before it shows the repeated addition in the array if students are still struggling with connecting repeated addition to a multiplication equation. After discussing, "Did the order of the factors change?" consider posing the question, "Do we have the same multiplication situation for both arrays?" See the above instructional note for ways to expand this discussion. Independent Practice/Math Practices and Problem Solving: To formatively assess if students are able to distinguish a multiplicative situation from an additive situation consider posing item 13, without students having access to the student edition since it leads students to knowing it is an additive situation. Assess and Differentiate/Intervention Activity: If time permits, you may consider replacing the *On* and *Advanced Activity Center* with either the games *Toss and Talk* (TE, p. 11A), *Teamwork* (TE, p. 23A) or the *Fluency Practice Activity* (TE, p. 49). Child-watch to identify students who need additional support and pull them in a small group to complete the *Intervention Activity* (TE, p.29A).
Lesson 1-5: Division as Sharing		
3.OA.A.2 3.OA.A.3 MP.1 MP.3 MP.4 MP.5 MP.6	Access Prior Learning: In Grade 2, students explored the idea of fair shares in geometry by dividing a shape into equal parts. Throughout this topic, students have been working to understand multiplication as the joining of equal groups. Beginning of the Big Idea: This lesson *begins* to build the understanding that multiple objects can be shared by separating into equal groups and is one way to think of division.	Instructional note: This lesson works with partitive division (fair sharing or dealing) scenarios (known groups, unknown number in each group). Solve & Share: To continue to support students' development of behaviors in selecting appropriate tools (MP.5), consider waiting to distribute the 20 counters until after posing the question, "How can we represent this problem?" (TE, p. 31). Encourage students to model with a tool or a pictorial representation. Discuss where the mathematics is represented in their model (e.g. Where are the apples? Where are how many each person gets?). Visual Learning: Connecting student solution methods to the bar diagram and how their models are shown, can help students better understand the bar diagram as a model for division. Convince Me: Consider discussing the *Convince Me!* question to initiate a discussion about remainders. While third grade students do not work with remainders, it is important for them to know that we can still divide when the divisor doesn't exactly go into the dividend (leftovers). Independent Practice/Math Practices and Problem Solving: Consider discussing item 11 as it demonstrates the relationship between the size of the divisor (e.g., 15 ÷ 3 = 5, 3 is the divisor) and the quotient (e.g. 15 ÷ 3 = 5, 5 is the quotient). Consider discussing item 12 to provide students with the opportunity to reason with a problem that does not provide enough information to solve. *-continues on next page-*

		Assess and Differentiate/Intervention Activity: If time permits, teach students how to play "Toss and Talk" (TE, p. 35A). All students should have the opportunity to play this game it provides engaging and meaningful practice of a key concept. Child-watch to identify students who need additional support with these ideas and pull them into a small group to do the *Intervention Activity* (TE, p.35A). *CTC: *Quick Check* (digital platform)
Lesson 1-6: Division as Repeated Subtraction		
3.OA.A.2 3.OA.A.3 MP.2 MP.4 MP.5 MP.8	Access Prior Learning: In lesson 1-1, Grade 3, students saw that repeated addition is a way to think of multiplication and a way to join equal groups. In lesson 1-5, Grade 3, students learned to think of division as sharing. Developing the Big Idea: In this lesson, students work with their *developing* understanding that division involves separating one group of objects into equal groups. Students also *beginning* to build understanding that one way to think about division is as repeated subtraction of the divisor from the dividend. Additionally, this lesson *develops* the understanding that we have 2 different types of division situations (fair sharing and chunking).	Instructional note: Using repeated subtraction to divide can support students in recognizing the inverse relationship that exists between multiplication and division. Having students work with repeated subtraction as a way of thinking about division also emphasizes the connections between repeated addition as multiplication and repeated subtraction as division. In this lesson, students are working with measurement division (chunking) scenarios, which allows repeated subtraction to be used as a strategy. Solve & Share: To continue supporting students' in selecting appropriate tools (MP.5), consider waiting to distribute the 12 counters until after posing the question, "What tools can you use to solve this problem?" (TE, p. 37). For students that incorrectly use 12 – 2 = 10 to solve this problem, consider asking them to model the problem with counters instead. Can they explain how many friends will get tacos using the counters? Can they connect the concrete modelling to repeated subtraction? During the whole class discussion of student solution methods and reasoning, consider posing the question, "We used repeated addition to help us find the solution to a multiplication equation, how could we use repeated subtraction to find the solution to this division situation?" and "where are our equal groups in repeated subtraction?" "How many groups do we have? Visual Learning: The *Visual Learning Animation* uses the bar diagram to model the division situation presented in this problem. This lesson is focusing entirely on measurement (chunking) division. In this type of division, we know the number of objects in the equal groups and need to find the total number of equal groups. Consider discussing the differences in the bar diagrams to develop the understanding that we have 2 different types of division situations (fair sharing and chunking). Convince Me: Consider assigning the *Convince Me!* and discussing to help students understand the role of the dividend, divisor, and quotient. Assess and Differentiate/Intervention Activity: If time permits, you may consider replacing the *On* and *Advanced Activity Center* with the game *Toss and Talk* (TE, p. 35A) or the *Fluency Practice Activity* (TE, p. 49). Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p. 41A).
Lesson 1-7: Math Practices and Problem Solving- Use Appropriate Tools		
3.OA.A.3 3.OA.A.1 3.OA.A.2 MP.5 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: Previous lessons in this topic have developed students' understanding of multiplication and division situations, and how to use appropriate tools strategically as they have modeled with counters. Securing the Big Idea: This lesson *secures* the idea that we can use appropriate tools strategically to model multiplication and division situations.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 5. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F25-F25A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the Student Edition (SE, p. F25). Solve & Share: Consider reintroducing MP. 5 Thinking Habits (SE, p. F25) before introducing the *Solve & Share*. Use the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.5 that are listed in the *Math Practices and Problem Solving Handbook* (p. F25A). After discussing student solution methods and reasoning, ask students to self-score for the behaviors associated with this math practice. Assess and Differentiate: If time permits, you may consider replacing the *On* and *Advanced Activity Center* with the game *Toss and Talk* (TE, p. 35A) or the *Fluency Practice Activity* (TE, p. 49). Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.47A).

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