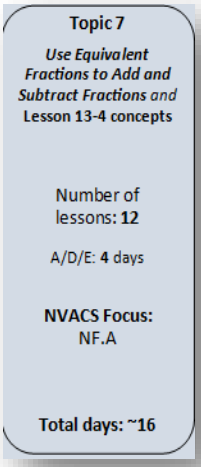
Grade 5 Topic 7: Use Equivalent Fractions to Add and Subtract Fractions; lesson 13-4 concepts



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 367A-367F), the Topic Planner (pp. 367I-367L), all 12 lessons, and the Topic Assessments (pp. 453-453A).

Mathematical Background: Read Topic 7 Cluster Overview/Math Background (TE, pp. 367A-367F)	Topic Essential Question: How can sums and differences of fractions and mixed numbers be estimated? What are standard procedures for adding and subtracting fractions and mixed numbers? *Reference Answering the Topic Essential Questions (TE, pp. 449-450) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

7-1	7-2	7-3	7-4	7-5	7-6	7-7	7-8	7-9	7-10	7-11	7-12	Assessment

4 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on Nevada Academic Content Standards (NVACS) cluster 5.NF.A, "Use equivalent fractions as a strategy to add and subtract fractions". Standards in this cluster include:

- 5.NF.A.1- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. (NVACS, 2010).

- 5NF.A.2- Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (NVACS, 2010).

Understanding equivalence is critical; in order to successfully work with fractions. It is a central idea for which students must have sound understanding and skill (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). In 4th grade, students added and subtracted fractions with like denominators to determine that these operations work when fractions have equal sized pieces. Fourth graders also expressed a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and used this technique to add the fractions. In 5th grade, students learn to add and subtract fractions (including mixed numbers) with unlike denominators by finding equivalent fractions. Connecting visual representations and concrete models is an important part of this process, which helps students understand equivalence and apply their knowledge to new situations (Van de Walle et al., 2014). These understandings take time to build but are important to students' future success. As equivalence understandings grow and emerge, students foster a sense of unity and connectedness in the study of mathematics (NCTM, 2003). It is important to start with concrete and semi-concrete experiences to build conceptual understanding before completing abstract work.

Students who understand the concept of fraction equivalence and the need for equal-sized pieces should also understand why algorithms for addition and subtraction of fractions work. Students who memorize a procedure before understanding equivalence are not likely to apply it successfully. Research suggests that students who have memorized and practiced procedures they do not understand have less motivation to understand their meaning or the reasoning behind them (Hiebert,1999). The focus of the NVACS is understanding how equivalence allows fractional pieces to be added and subtracted. In fact, "It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding fractions," (Common Core Standards Writing Team, 2013, p. 11). Initially, students will want to use the operations of addition and subtraction without paying attention to the size of the pieces. Students need time to think and reason about fractions and determine the need for equivalency.

> *However, about half of students will write* $\frac{3}{8}+\frac{2}{8}=\frac{5}{16}$, *even after drawing the model correctly. And they won't seem to be bothered that the two answers* ($\frac{5}{8}$ and $\frac{5}{16}$) *are different. In such a case, ask students to decide whether both answers can be right. Ask them to defend which is right and why the other answer is not right. You cannot just tell students which is right-the key is for them to overcome their misconceptions (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).*

Procedures alone do not help students to think conceptually about the operations and what they mean. Understanding equivalency will help students to apply understanding even when they encounter changes in fractions or problem types. Research summarizing successful teaching for fractions suggests four steps: (1) use contexts, (2) use a variety of models, (3) include estimation and informal methods, and (4) address misconceptions (Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, & Wray, 2010). Conceptual and procedural explanations from students look and sound different.

> *Concept: Two fractions are equivalent if they are representations for the same amount or quantity-if they are the same number.*
>
> *Procedure: To get an equivalent fraction, multiply (or divide) the top and bottom numbers by the same nonzero number (Van de Wall et al., 2014).*

Have students use tools, visual models and mathematical reasoning to find equivalent fractions until they are able to generalize or understand more abstract procedures. Poorly understood procedures will quickly be forgotten or confused, especially as students move to multiplication and division with fractions.

Justification for including 13-4 concepts: Lesson 13-4 focuses on interpreting expressions without calculating. How can you tell if an expression matches the context of the problem? How can you determine which expression is greater without calculating? If parentheses are added or removed how does that change the value of the expression? Students should realize that all expressions can have only one stable value. Consider using the Solve and Share, Visual Learning Bridge, or items from the Guided Practice pages from Lesson 13-4 to help students build understanding of these concepts.

Math Practice 4: Model with mathematics
Focus on opportunities for students to develop *Mathematical Practice 4* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 7-12. Reference the Teacher's Edition (TE, pp. F24-F24A) and the NVACS (2010, p. 7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
mixed number	*equivalent fractions* *benchmark fraction* *common denominator* *numerator* *denominator*

Additional terminology that students may need support with: *annex zeros*

Collaborative Team Conversations (CTC)
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using benchmarks (ex. 0, ½, 1) to estimate sums and differences of fractions?"
"Are students replacing given fractions with equivalent fractions to find a common denominator when finding sums and differences of fractions with unlike denominators?"

Lesson	Evidence	Look for
7-1	*Math Practices and Problem and Solving* (student work samples) Item 14	Focus CTC around the big idea: • use of benchmarking to make a reasonable estimate. • appropriate estimate for the situation.
7-1	*Quick Check* (digital platform) Items 2, 3, 4 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
7-5	*Homework and Practice* (student work samples) Item 15	Focus CTC around the big idea: • student strategies and models used to find a common denominator. • understanding and use of equivalent fractions.
7-5	*Quick Check* (digital platform) Items 2, 3, 4, and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 449-454	Use *Scoring Guide* TE pp. 449-454A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 7-1: Estimate Sums and Differences of Fractions		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.8	Access Prior Learning: In 4th grade, students worked with benchmark fractions and learned to add and subtract fractions with like denominators (4.NF.A) (4.NF.B). Developing the Big Idea: Students extend knowledge of benchmark fractions to estimate sums of fractions with unlike denominators.	Solve and Share: Look for students able to use number sense to estimate the sums of the fractions and ask these students to explain their thinking. Important benchmarks such as 0, $\frac{1}{2}$, $\frac{1}{4}$, and 1 can be used to help with estimation. Have students use concrete tools such as fraction strips and representations such as number lines to support their thinking. How do these strategies help us to work with fractions with unlike denominators? Visual Learning: Benchmark fractions are modeled on a number line and then used to estimate a sum for two fractions with unlike denominators. The *Convince Me!* asks students to think about the relationship between the numerator and denominator and explain their reasoning. Consider revisiting the *Topic 7 Essential Question*. A number line can be used to help place benchmark fractions and estimate sums and differences of fractions with unlike denominators. Students may confuse rounding versus estimating with benchmark fractions. How are these different? Help students connect that rounding is one type of estimation. Teaching Tool 12 (blank number lines) can be used to assist students estimating on *Independent Practice* page questions. The *Intervention Activity, Reteach* page and *Homework and Practice* page offer practice using benchmark fractions and number lines. Pages 374 and 376 offer application problems. Remember that students who seem to be struggling without context are successful with engaging real-world problems. **CTC: Math Practices and Problem and Solving* (student work samples) Item 14 **CTC: Quick Check* (digital platform) Items 2, 3, 4 and 5
Lesson 7-2: Find Common Denominators		
5.NF.A.1 5.NF.A.2 MP.1 MP.3 MP.4 MP.7	Access Prior Learning: In 4th grade students generated equivalent fractions and compared fractions with unlike numerators and denominators (4.NF.A.1) (4.NF.A.2). Developing the Big Idea: Students generate equivalent fractions using visual models to build conceptual understanding and lay the foundation for adding and subtracting fractions with unlike denominators.	Solve and Share: Students are asked to cut a pan of cornbread so that two people get the size they requested. This context leads students to use an area model to visually represent the fractions. Look for students using two separate pans and remind them that there is only one pan of cornbread available. Some students may create $\frac{1}{2}$ and $\frac{1}{3}$ on different sides of the pan. Challenge these students to find a way to know how much cornbread is left in the pan. Ask students who find a common denominator to share their strategies and explain their thinking. Can their models be used to find out how much cornbread was eaten altogether? How was the common denominator created? Why are equal sized pieces needed to solve this problem? Visual Learning: Area models are used to create equivalent fractions. The *Convince Me!* gives two new fractions and asks students to use an area model to find a common denominator. Students will begin to observe that the common denominator created by the area model is the product of the two denominators. Will this always work? Is this the only way to find a common denominator? Multiplying denominators is explicitly modeled on *Another Example* (SE, p. 379). Consider holding off on introducing this strategy until you are confident that students have the necessary conceptual understanding to know how it worked. Many students may still need a tool or a visual model to confirm their thinking on *Independent Practice* questions 4-11 (SE, p. 379). Questions 16-17 can be used to challenge students to think beyond multiplying the denominators to understand that common denominators can be any common multiples (SE, p. 381). Assess and Differentiate: Finding common multiples as a strategy to find common denominators is shown on the *Homework and Practice* (SE, p. 381). For students who are struggling, consider spending more time with the area model building conceptual understanding before using a procedure.

Lesson 7-3: Add Fractions with Unlike Denominators		
5.NF.A.1 5.NF.A.2 MP.1 MP.3 MP.4 MP.5	Access Prior Learning: In 4th grade students added fractions with like denominators (4.NF.B). Students found common denominators in the previous lesson. Developing the Big Idea: Students connect visual models with procedures for finding common denominators and adding fractions with unlike denominators.	Solve and Share: Look for a range of strategies (concrete, representational, abstract) used to find a common denominator and add the fractions. Students should support their thinking with a visual model. Ask students to share their ideas and facilitate a discussion comparing strategies. What connections are seen between the visual models and more abstract strategies for finding a common denominator? Do the fractions need a common denominator to find a sum? Visual Learning: Fraction strips and finding common multiplies are modeled together as strategies for finding a common denominator. Replacing with equivalent fractions is shown as a strategy for adding fractions with unlike denominators. Consider having students use their own fraction strips rather than only looking at the pictures. Making fraction strips is a valuable experience for students. Consider using the *Convince Me!* to facilitate a discussion about equivalent fractions. Is 5/6 the only possible solution? Fraction strip manipulatives or Teaching Tool 13 (fraction bars) can be used to help students model their solutions for the *Guided and Independent Practice* page. Assess and Differentiate: *Homework and Practice* questions 1-4 ask students to find the least common multiple. This extra demand can impede learning for students still building understanding of how and why equivalent fractions are used to add. Consider moving students to question 5, which requires using fraction bars to model the context a problem.
Lesson 7-4: Subtract Fractions with Unlike Denominators		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In 4th grade students subtracted fractions with like denominators (4.NF.B). Students added fractions with unlike denominators in previous lessons. Developing the Big Idea: Students build conceptual understanding of how equivalent fractions are used to subtract fractions with unlike denominators.	Solve and Share: Look for a range of strategies (concrete, representational, abstract) used to find a common denominator and subtract the fractions. Students can support their thinking with a visual model. Ask students to share their ideas and facilitate a discussion comparing strategies. What connections exist between the visual models and more abstract strategies for finding a common denominator? Do the fractions need a common denominator to find a difference? Visual Learning: : Fraction strips and finding common multiples are used to model subtracting fractions with unlike denominators. Notice that while using fraction strips, the whole is used as a comparison to remind students about benchmarks. Students draw connections between the methods for adding and subtracting fractions with unlike denominators. Consider revisiting the *Topic 7 Essential Question* as part of the formative assessment process. What have students discovered about the procedures for adding and subtracting fractions? Question 18 on *Math Practices and Problem Solving* helps students model fraction addition using a bar diagram and a variable (SE, p. 392). How can modeling problems this way be helpful?
Lesson 7-5: Add and Subtract Fractions		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: Students added and subtracted fractions with unlike denominators in previous lessons. Securing the Big Idea: Students will build procedural fluency by applying strategies for adding and subtracting fractions with unlike denominators.	Solve and Share: Consider providing Teaching Tool 14 (circle fractions) as a visual model for this problem. Students need to make sense of the context of this problem before computing. Look for students who mistakenly subtract the amounts eaten. Unfortunately, this will result in a correct solution due to the numbers used. Refer students back to the context of the problem to make sure their strategy matches what is happening in the problem. Look for and use a range of student strategies to facilitate a discussion. What connection is seen in all strategies? Is a common denominator generated by all successful strategies? Visual Learning: A multi-step problem is modeled. Consider asking students what they would do next before it is shown in the video. Why are the denominators 15 and then 30 chosen while computing this problem? Question 29 on *Math Practices and Problem Solving* connects adding fractions to geometry (SE, p. 398). Assess and Differentiate: *Homework and Practice* requires students to add three fractions with unlike denominators (SE, p. 399). Can this be done more than one way? Students can explore how the Commutative Property of Addition applies to fractions. Take the time to discuss the various strategies that students used. **CTC: Homework and Practice* (student work samples) Item 15 **CTC: Quick Check* (digital platform) Items 2, 3, 4, and 5

Lesson 7-6: Estimate Sums and Differences of Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.6 MP.8	Access Prior Learning: In 4th grade students added and subtracted mixed numbers with like denominators (4.NF.B) Students found sums and differences of fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use benchmarks to build number sense by estimating sums and differences of mixed numbers with unlike denominators.	Solve and Share: Students need to extend understandings of benchmark fractions to include mixed numbers. Students will need to see that although a mixed number is written using a whole number and a fraction, it represents one quantity. This means that $1\frac{3}{4}$ is close to 2 wholes. Number lines and visual models will help students to round mixed numbers to the nearest whole. Connecting mixed numbers to working with decimals can also enhance students' ability to visualize mixed numbers. Visual Learning: A number line is used to round mixed numbers to the nearest whole number. Estimation is a key strategy for determining whether answers are reasonable. How can using benchmark fractions help us to estimate when working with mixed numbers? Consider using the *Convince Me!* to facilitate a discussion on considering context when estimating. Questions 25 and 26 on the *Math Practices and Problem Solving* page ask which estimate is best for a given problem (SE, p. 404). Consider using these items as an opportunity for students to justify their strategies and the solution choice as "best".
Lesson 7-7: Use Models to Add Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In 4th grade students added mixed numbers with like denominators (4.NF.B) Students found sums and differences of fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use visual models to build conceptual understanding of adding mixed numbers with unlike denominators.	Solve and Share: Fraction strips and visual models will help students see that although mixed numbers are greater than 1, they are still composed of fractional pieces as denoted by the denominator. These tools also help students see that mixed numbers can be decomposed similarly to whole numbers. For example, in the same way that 5 can be rewritten as (4 + 1), a mixed number such as $1\frac{3}{4}$ can be rewritten as $(1 + \frac{3}{4})$, $(\frac{4}{4} + \frac{3}{4})$ or $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{3}{4})$. This flexible thinking helps students build procedural fluency working with fractions and mixed numbers. Visual Learning: Fraction strips are used to model adding mixed numbers with unlike denominators. In the *Visual Learning Bridge*, the wholes are added separately from the fractions. Being able to decompose a mixed number as described above is a crucial stepping stone to using this strategy. Students are encouraged to use fraction strips or another visual model while completing the *Independent Practice* problems (SE, p.409). Problems 22 and 23 use larger numbers and can be used to formatively assess students to see which are moving to more abstract procedures (SE, p. 410). Assess and Differentiate: Fractions strip representations are used in the *Intervention Activity*, the *Reteach* page, and on the *Homework and Practice* page. These models let students visualize how mixed numbers join to form a new whole. Manipulatives are appropriate for all students during this learning process.
Lesson 7-8: Add Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.7	Access Prior Learning: Students added mixed numbers with unlike denominators in the previous lesson. Securing the Big Idea: Students continue using models to build conceptual understanding of adding mixed numbers with unlike denominators and connect ideas to procedures for adding fractions with unlike denominators.	Solve and Share: Look for a wide range of student strategies (manipulatives, visual models, abstract procedures) used to solve this problem. Orchestrate a discussion to assist students in connecting these strategies. As a class, formulate a generalization of a mathematical rule for computing with fractions and mixed numbers with unlike denominators. Students may observe that no matter what strategy is used; a common denominator must be found before the equal sized pieces can be joined. Visual Learning: Adding mixed numbers with unlike denominators is shown and modeled with fractions strips. Why are the fractions added before the whole numbers? Is this the only way this type of problem can be solved? The *Convince Me!* can be used to help students think and talk about what makes an estimate reasonable. The *Independent Practice* problems remind students to estimate before computing. Can they explain why their estimates are reasonable?

Lesson 7-9: Use Models to Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.5 MP.8	Access Prior Learning: In 4th grade students subtracted mixed numbers with like denominators (4.NF.B). In 4th grade students decomposed fractions into a sum of fractions with like denominators (4.NF.B.3). Students subtracted fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use visual models to build conceptual understanding of subtracting mixed numbers with unlike denominators.	Solve and Share: Fraction strips and visual models will help students see that although mixed numbers are greater than 1, they are still composed of fractional pieces as denoted by the denominator. These tools also help students see that mixed numbers can be decomposed similarly to whole numbers. The ability to work with these numbers flexibly is key to understanding when and how it is necessary to rename the whole before subtracting mixed numbers. Renaming the whole is modeled in the *Visual Learning Bridge*. Visual Learning: Students see that subtracting wholes and fractions separately will not always work as it does with addition. If the fractional piece of the subtrahend is larger than that of the minuend, a negative number would be created. An example of renaming the whole is modeled. This creates a larger fraction before subtracting on the minuend. The example used shows that 2 6/12 is renamed as (1 + 12/12 + 6/12) and then as 1 18/12. Understanding these different forms as equivalent is key to students building procedural fluency. Fraction strips and visual models help students see the equivalence and explore why these steps are sometimes necessary to subtract mixed numbers with unlike denominators. Focus on building understanding with manipulative tools. The *Convince Me!* gives students another chance to practice renaming the whole. Consider facilitating a discussion comparing the similarities and differences in the processes used for adding and subtracting mixed numbers. Assess and Differentiate: The *Intervention Activity*, *Reteach* page, and *Homework and Practice* page encourage students to use tools to model and compute subtraction problems with mixed numbers. Tools help students build conceptual understanding and lay the groundwork for procedural fluency and application. Use the additional practice subtracting a fraction from a whole number formatively to identify students needing support with decomposing.
Lesson 7-10: Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: Students subtracted mixed numbers with unlike denominators in the previous lesson. Securing the Big Idea: Students use visual models and written methods to build conceptual understanding and procedural skill for subtracting mixed numbers.	Solve and Share: The part-part-whole diagram is used to mathematically model the context of the problem. Can students identify what the numbers in this model represent? Look for a wide range of student strategies (manipulatives, visual models, abstract procedures) used to solve this problem. A discussion will help students connect these strategies and generalize a mathematical rule for computing with fractions and mixed numbers with unlike denominators. Visual Learning: Subtracting mixed numbers with unlike denominators is shown and modeled with fraction strips. How did $4\frac{3}{12}$ become $3\frac{15}{12}$? Is this the only way this type of problem can be solved? The *Convince Me!* can be used to help students think and talk about what makes an estimate reasonable? The *Independent Practice* problems also remind students to estimate before computing. Can they explain why their estimates are reasonable?
Lesson 7-11: Add and Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students added and subtracted mixed numbers with unlike denominators in previous lessons. Securing the Big Idea: Students apply strategies and procedures for adding and subtracting mixed numbers with unlike denominators to build procedural skill.	Solve and Share: Look for students who make sense of this multi-step problem through estimation and modeling. Facilitate a discussion using their ideas to help students generalize that multiple operations may be necessary to solve a single problem. Have students share their strategies and thinking. Can students explain why their strategies are appropriate for this problem? Visual Learning: Would it be better to overestimate or underestimate the amount of fabric needed? Use this problem to discuss why considering the context when creating estimates is important. Consider revisiting the *Topic 7 Essential Question*. Consider using the problem shown in the *Visual Learning* as a jumping off point to discuss both questions. Students practice computing expressions with mixed operations on the *Guided and Independent Practice* pages. Remind students to use estimation to check the reasonableness of their solutions. Assess and Differentiate: Estimation is used as a tool check for reasonableness and build conceptual understanding in the *Intervention Activity*. This activity could be used whole class or in small groups to explore different estimation techniques and improve precision.

Lesson 7-12: Math Practices and Problem Solving- Model with Math		
5.NF.A.2 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students have used models in previous lessons and topics. Students have added and subtracted mixed numbers with unlike denominators in previous lessons. Securing the Big Idea: Students build conceptual understanding and procedural skill through application.	Solve and Share: Encourage students to use tools and sense making skills to model this problem before computing. How do their models represent the context of the problem? Consider changing the problem to ask "Which is longer, the Scotch Bonnet sea shell or the combined lengths of two Alphabet Cone Seashells? How much longer? How do you know? Focus class discussion on the thinking habits of MP.4 and connect back to student thinking and strategies. How did they model the thinking habits of MP.4? Visual Learning: A multi-step problem is solved. Consider, how do bar diagrams and equations help us to model and solve problems? Students are asked to use equations and bar diagrams to model similar problems in the *Convince Me!* and the *Guided and Independent Practice* page. Nudge students to model and explain their thinking using the multi-step problem on the *Math Practices and Problem Solving* page (SE, P. 440). Assess and Differentiate: Students practice modeling context with a part-part-whole bar diagram in the *Intervention Activity*. Observe students who struggle with transferring between a model and mathematical notations.

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