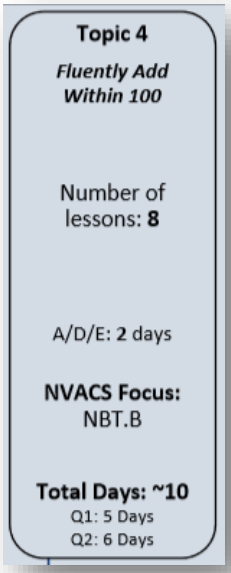
Grade 2 Topic 4: Fluently Add Within 100



2nd Grade Pacing Curriculum Framework: Balanced Calendar

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 4 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 119A-119E), the Topic Planner (pp.189A-189C), all 8 lessons, and the Topic Performance Assessments (pp. 251-252A).

Mathematical Background: Read Cluster Overview (TE, pp. 119A-119E)	Topic Essential Question: What are strategies for adding numbers to 100? *Reference Answering the Topic Essential Questions (TE, pp. 247-248) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	Assessment

2 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 4 focuses on fluently adding within 100 using efficient strategies.

> *…there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, et al, 2014, p.176).*
>
> *It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).*

Topics 3-6 compose a major cluster focused on the big idea of the base-10 numeration system. Focus instruction on Nevada Academic Content Standards (NVACS) cluster 2.NBT.B. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above, place-value instruction does not need to occur in isolation (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 176). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding one hundred as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that children can group into tens (connecting cubes, beans in cups, bundles of straws, etc.) are essential. Groupable models allow children to move from operating with ones only, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a tens rod, is worth ten is ineffective. When considering language, help students connect standard language, "thirty-five", to base-ten language, "3 tens and 5 ones; 3 groups of ten and 5 ones, etc". It is also recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

Topic 4, specifically, focuses on the following standards in NVACS cluster 2.NBT.B:

> *2.NBT.B Use place value understanding and properties of operations to add and subtract.*
>
> *5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.*
>
> *6. Add up to four two-digit numbers using strategies based on place value and properties of operations.*
>
> *9. Explain why addition and subtraction strategies work, using place value and the properties of operations.*

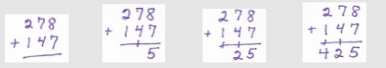
In Topic 4, students apply the strategies from Topic 3 for adding within 100 to addition algorithms including partial sums and the U.S. Traditional algorithm. (Focus instruction on NVACS cluster 2.NBT.B) The authors of enVisionmath2.0 placed the algorithms at the end of this sequence of strategies with the intent that students connect their understanding of place value strategies to construct meaning of the algorithms. They also intended for students to see the algorithm as one of many strategies for addition, not the pinnacle of addition strategies. Standard 2.NBT.B.5 expects students to "Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" (2010, pg. 19). Looking ahead to the focus of Topics 10-11, standard 2.NBT.B.7 states, "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; *relate the strategy to a written method*. Understand in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." The "Progressions for the Common Core State Standards in Mathematics" elaborate on what it means to "relate to a written method", by including the following examples:

Addition: Recording newly composed units in separate rows

278	278	278	278
+147	+147	+147	+147
	300	300	300
		110	110
			15
			425

The computation shown proceeds from left to right, but could have gone from right to left. Working from left to right has two advantages: Many students prefer it because they read from left to right; working first with the largest units yields a closer approximation earlier.

Addition: Recording newly composed units in the same row



Add the ones, $8 + 7$, and record these 15 ones with 1 on the line in the tens column and 5 below in the ones place.

Add the tens, $7 + 4 + 1$, and record these 12 tens with 1 on the line in the hundreds column and 2 below in the tens place.

Add the hundreds, $2 + 1 + 1$, and record these 4 hundreds below in the hundreds column.

Digits representing newly composed units are placed below the addends, on the line. This placement has several advantages. Each two-digit partial sum (e.g., "15") is written with the digits close to each other, suggesting their origin. In "adding from the top down," usually sums of larger digits are computed first, and the easy-to-add "1" is added to that sum, freeing students from holding an altered digit in memory. The original numbers are not changed by adding numbers to the first addend; three multi-digit numbers (the addends and the total) can be seen clearly. It is easier to write teen numbers in their usual order (e.g., as 1 then 5) rather than "write the 5 and carry the 1" (write 5, then 1).

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The first written method, Partial Sums, records intermediate steps and is helpful in building toward the second written method, the U.S. Traditional algorithm. The progression documents articulate that drawings such as the one pictured below, can be used by students in explaining the written methods above. Knowing that the trajectory is building toward the expectation that students will relate strategies to a written method, we can view the lessons in this topic as offering an entry point into algorithms. However, in regards to transitioning from the first written method to the second written method, the progression document also states, "Some students might make this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency requires a more compact method."

Based on this, we can offer opportunities for our students to construct meaning of the algorithms, but we should not expect all students to transition to use of the standard algorithm in second grade. Just as in Topic 3, although the lessons focus on a particular strategy, encourage students to use the strategy but do not require them to do so. A requirement such as this removes the reasoning from strategy development (Van de Walle, et al., 2014). Instead, honor student strategies by emphasizing their ability to use strategies based on place value understanding, properties of operations, and the relationship between addition and subtraction. Continue to encourage the use of manipulatives throughout math instruction.

Illustrating combining like units and composing new units

The drawing shows the base-ten units of 278 and 147. Like units are shown together, with boundaries drawn around ten tens and ten ones to indicate the newly composed hundred and the newly composed ten. The newly composed units could also be indicated by crossing out grouped units and drawing a single next-highest unit, e.g., crossing out the group of ten ones and drawing a single ten. Drawings like this can be used to illustrate and explain both of the written computations below.

Math Practice 4: Model with mathematics

Focus on opportunities for students to develop MP.4 behaviors. This is the focus of the Math Practices and Problem Solving lesson 4-8. Reference the Teacher's Edition (pp. F26-F26A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
partial sum regroup compatible numbers	*tens* *ones* *open number line)*

Additional terminology that students may need support with: *algorithm, model*

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students developing conceptual understanding of part/part/whole and how it relates to an equation?"
"Were students able to effectively communicate their strategies?"

Lesson	Evidence	Look for
4-7	*Show Me!* (student work samples)	Focus CTC around the big idea: • mental math strategies • ability to communicate thinking through writing
4-6	*Quick Check* (digital platform) Items 2, 3, and 4	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 247-252	Use *Scoring Guide* TE pp. 247-252

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 4-1: Add With Partial Sums		
2.NBT.B.5 2.NBT.B.9 MP.1 MP.4 MP.5 MP.6 MP.7	Access Prior Learning: In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones. In Topic 3, second grade students used addition strategies to add 2-digit numbers within 100. Beginning the Big Idea: In this lesson, students are *beginning* understanding of the partial sums algorithm when adding within 100.	As indicated in the instructional note at the beginning of this document, encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Although the text offers tens-and-ones charts as an intended support for students, it might send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper to solve problems on. A blank workspace reflects the value we place on students' selection of appropriate strategies. Topic Opener: Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE, p. 189) and *Review What You Know* (TE, pp. 190-192) and the *Topic 4 Vocabulary Words Activity* with the words *compatible numbers*. Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. Solve & Share: As indicated under *2. Build Understanding* (TE, p. 193), show students how to draw place value blocks efficiently, using sticks for tens and circles/dots for ones. To extend early finishers, ask them to solve the problem using a second model that demonstrates place value understanding and compare the efficiency of each (e.g., jumps of tens and ones on the open number line). Visual Learning: To facilitate student connections between concrete and abstract models, have students use place value blocks to solve 57 + 28 before showing the animation. During the animation, consider having students model the steps of partial sums using manipulatives or drawings. Focus the discussion around making sense of the algorithm, avoiding a rote, procedural approach. *-continues on next page-*

		Independent Practice/Math Practices and Problem Solving: Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper to solve problems on. A blank workspace reflects the value we place on students' selection of appropriate strategies.
Lesson 4-2: Continue To Add With Partial Sums		
2.NBT.B.5 2.NBT.B.9 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones. In the prior lesson, second grade students used partial sums to add 2-digit numbers within 100. Beginning the Big Idea: In this lesson, students are *beginning* understanding of the partial sums algorithm when adding within 100.	As indicated in the instructional note at the beginning of this document, encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper to solve problems on. A blank workspace reflects the value we place on students' selection of appropriate strategies. Visual Learning: To facilitate student connections between concrete and abstract models, have students use place value blocks to solve 38 + 59 before showing the animation. During the animation, have students model the steps of partial sums using their place value blocks. Focus the discussion around making sense of the algorithm, avoiding a rote, procedural approach. Assess and Differentiate: Modify the *Intervention Activity Sum of Sums!* (TE, p.203A). Students should build the addends using manipulatives, and later combine them while the teacher records the corresponding equations. This allows students to focus on constructing the concept of partial sums, without following a rote procedure for recording the steps. Replace the *On-Level* and *Advanced Activity Centers* (TE, p.203A) with the lesson 4-6 *On-Level* and *Advanced Activity Centers*, "Play a Game" (TE, p. 227A).
Lesson 4-3: Models to Add 2-Digit Numbers		
2.NBT.B.5 2.NBT.B.9 MP.2 MP.3 MP.4 MP.5 MP.6	Access Prior Learning: In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones. Earlier in this topic, second grade students used partial sums to add 2-digit numbers within 100. Beginning the Big Idea: In this lesson, students are *beginning* understanding of the standard addition algorithm, using place value blocks to model the math, when adding within 100.	As indicated in the instructional note at the beginning of this document, students are expected to relate addition strategies to a written method by the end of second grade (2.NBT.B.7). These written methods include expanded algorithms (e.g., Partial Sums) and standard algorithms (e.g., U.S. Traditional Algorithm). However, the Progression Documents clarify that although some second grade students will transition from an expanded algorithm such as partial sums, to the standard algorithm, some won't make this transition until third grade. Therefore, do not expect all students to understand and use a standard algorithm at this point in the year. Focus instruction on student strategies based on place value, properties of operations and the relationship between addition and subtraction. As Zachary Champagne, one of the enVision authors states, "there is power in the blank page". Throughout Topic 4 you will find tens-and-ones charts and 2-digit addition guides. These structures, intended to support students, may be perceived as limiting students' approach to algorithm work only. Instead, offer students a blank space to work in, allowing students to self-select strategies with understanding. Blank paper, post-its, math journals, and whiteboards are a few examples of such workspaces. Solve & Share: Students are introduced to the vocabulary word, *regroup* in *Visual Learning*. During the share, highlight and discuss the use of regrouping in students' solutions as an entry point for students to connect to the concept of regrouping as presented in the *Visual Learning Animation*. Regrouping is not limited to "carrying the 1" in the U.S. Traditional and other standard algorithms. This is better illustrated in students' work with manipulatives in *Analyze Student Work*: Conor's Work (TE, p.205). Visual Learning: Have students model using concrete manipulatives to begin to develop conceptual understanding of the steps in the standard algorithm. Independent Practice/Math Practices and Problem Solving: Offer students a blank workspace conducive to a variety of strategies.

Lesson 4-4: Add 2-Digit Numbers		
2.NBT.B.5 2.NBT.B.9 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones. Earlier in this topic, second grade students used partial sums and place value blocks to add 2-digit numbers within 100. Beginning the Big Idea: In this lesson, students are *beginning* understanding of the standard addition algorithm, using place value blocks to model the math, when adding within 100.	A note of CAUTION: The Coherence section for lesson 4-4 (TE, p.211A) states, "Now that students understand the concepts behind the standard addition algorithm, students can use symbols alone to perform addition at the abstract level. Drawings of place-value blocks are used during instruction to reinforce conceptual understanding." Do NOT push students to the abstract level only. Continue to give students opportunities to build conceptual understanding through the use of manipulatives and drawings. Independent Practice/Math Practices and Problem Solving: Continue to offer students a blank workspace and encourage them to use place value strategies. If you choose to have students try the standard algorithm, allow them to use it as a second strategy and draw connections between both approaches. Students should perceive the standard algorithm as another strategy they can choose when solving addition problems. Assess and Differentiate: The *Intervention Activity:* Missing Parts (TE, p. 215A) focuses only on the abstract algorithm. Instead, have students play "Play a Game" (Lesson 4-6, TE, p. 227A).
Lesson 4-5: Add More Than Two 2-Digit Numbers		
2.NBT.B.6 2.NBT.B.9 MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning: In first grade, (1.OA.A.2) students solved word problems that involved addition of three whole numbers with a sum within 20, using objects, drawings, and equations with a symbol for the unknown. In Topics 1 and 3, second grade students used strategies for addition. Earlier in this topic, students were introduced to algorithms. Developing the Big Idea: In this lesson, students are *developing* understanding that strategies and algorithms can be used to add more than two 2-digit numbers. They are also developing understanding that numbers can be added in any order using the Commutative (order) and Associative (grouping) Properties of Addition.	Standard 2.NBT.B.6 expects students to "Add up to four two-digit numbers using strategies based on place value and properties of operations." Look for evidence of place value understanding as students work with more than two addends. For example, students may break apart tens and ones, make jumps of tens and ones, use the Make 10 strategy, etc. Look for students' understanding and use of the properties of operations. Associative Property of Addition: $(a + b) + c = a + (b + c)$ Commutative Property of Addition: $a + b = b + a$ Solve & Share: As students problem solve, ask them to explain how they used place value to add three numbers. After students complete the directions given, encourage students to solve the problem a second time, adding the numbers in a different order to facilitate understanding of the Associative Property of Addition. Ask, "Did adding the numbers in a different order change the sum? Why or why not?" This work will offer students an entry point into the content of the *Visual Learning* animation. Visual Learning: Have students model the addition problem using manipulatives or drawings to support conceptual understanding. In Guided Practice, circling the digits added first is an opportunity for students to show their reasoning and use of prior learned strategies such as Make 10 and other compatible numbers.
Lesson 4-6: Practice Adding		
2.NBT.B.5 2.NBT.6 2.NBT.9 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In first grade, (1.OA.A.2) students solved word problems that involved addition of three whole numbers with a sum within 20, using objects, drawings, and equations with a symbol for the unknown. In Topics 1 and 3, second grade students used strategies for	Review the vocabulary words, *compatible numbers*. Use students' experiences from the prior lesson to draw examples of compatible numbers that are easy to add or subtract with mental math. For example, in the lesson 4-5 Solve & Share, students used the compatible numbers: 6 and 4 to make a 10 when adding 24 + 16 + 14 + 15 = ? Add compatible numbers to the math focus wall and include student-generated examples. *-continues on next page-*

MP.7	addition. Earlier in this topic, students were introduced to algorithms. Developing the Big Idea: In this lesson, students are *developing* understanding that strategies and algorithms can be used to add more than two 2-digit numbers. They are also developing understanding that numbers can be added in any order using the Commutative (order) and Associative (grouping) Properties of Addition.	Continue to look for students' understanding and use of the properties of operations. Associative Property of Addition: (a + b) + c = a + (b + c) Commutative Property of Addition: a + b = b + a Solve & Share: During the share, highlight student solutions that make use of compatible numbers, regrouping, or adding the numbers in a different order to create a "bridge" for students into the content of the *Visual Learning*. Visual Learning: During the guided practice, if students use regrouping with a standard algorithm, ensure that they are able to explain their thinking accurately using place value understanding. This means they can convey that 10 ones have been regrouped into 1 ten. In doing so, we are distinguishing between students who *use* standard algorithms versus students who *understand* standard algorithms. If students cannot demonstrate understanding, redirect them to use other place value strategies such as break apart or partial sums. *CTC: *Quick Check* (digital platform)
Lesson 4-7: Solve One-Step and Two-Step Problems		
2.OA.A.1 MP.1 MP.4 MP.5 MP.8	Access Prior Learning: In first grade, (1.OA.A.1) students used addition and subtraction within 20 to solve word problems using objects, drawings, and equations with a symbol for the unknown number. In lesson 3-8, second grade students solved one-step and two-step word problems using bar diagrams and equations. Developing the Big Idea: In this lesson, students are *developing* understanding of one-step and two-step word problems using bar diagrams, equations and tens-and-ones addition charts.	Visual Learning: Consider using the *Problem Solving Recording Sheet* (Teaching Tool 1) to make sense of the problem (MP.1) presented in the *Visual Learning* animation. Avoid key word strategies as they send a message to students that sense-making is not important, they are often misleading, and cannot be used to solve multi-step problems (Van de Walle, et al., 2014, p. 148). For example, if students have been taught that "join" means to add, students may approach this problem incorrectly (36 + 53 = ? instead of 36 + ? = 53). Independent Practice/Math Practices and Problem Solving: Students may notice that the items include two different bar diagrams. Refer to the Item 3 note on TE, pp. 231-232 for clarification between the bar diagram and the comparison bar diagram. Reference Teaching Tools 15 and 23 for blackline masters of both bar diagrams. Assess and Differentiate: In the *Intervention Activity* "Let's Solve and Check!", have students build the addends on ten frames to support understanding. *CTC*: Show Me!* (student work samples)
Lesson 4-8: Math Practices and Problem Solving: Model with Math		
2.OA.A.1 2.NBT.B.5 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In first grade, students engaged in the Standards for Mathematical Practice including MP. 4 Model with Math. In Topic 2, second grade students focused on MP.4 Model with Math. Developing the Big Idea: In this lesson, students are *developing* understanding that they can make models to help them solve problems.	Students focused on MP4. Behaviors in Topics 2 and 4. Consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.

References

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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