5th Grade

WCSD Curriculum Guides

Elementary Mathematics

Version 3: 2019/2020

Curriculum is one component of a larger mathematics instructional program in Washoe County School District (WCSD) for Kindergarten through 5th grade students. The purpose of curriculum guides is to bridge the district's K-5 Philosophy of Mathematics Education with the Nevada Academic Content Standards (NVACS) through a connection of the Curriculum Pacing Frameworks, instructional materials (*Bridges in Mathematics* or ***enVision**math**2.0***), research based instructional practices and clarification of the standards when necessary. The following describes a course of study for the specified grade for one year. ***ALL*** students must receive quality instruction in ***ALL*** grade level standards in one instructional year.

This guide is designed to be **used with the instructional materials** during planning. *This guide is not meant to supplant any portion of the instructional materials*. Teachers will continue to read through Units/Topics during instructional planning.

Guide language:

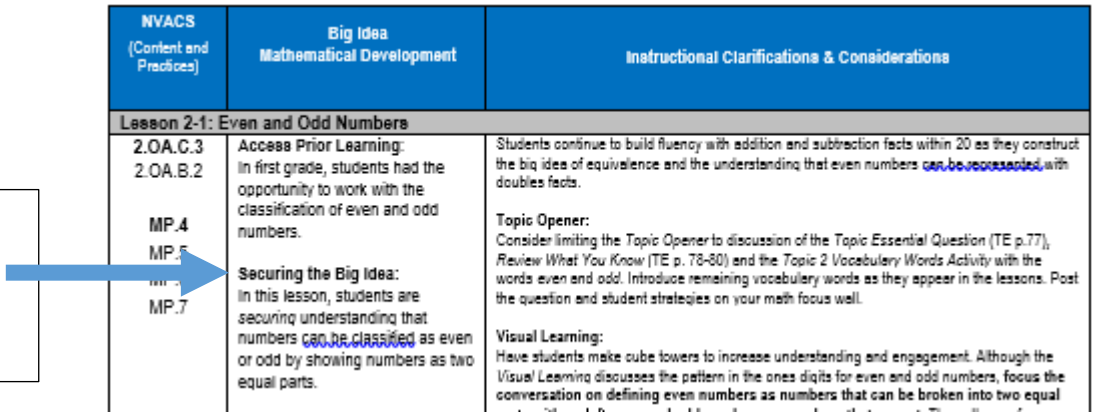
Throughout the guide the following language is used to describe the level of understanding expected at the lesson level. This language is found in the lesson-by-lesson section in the column labeled "Big Idea Mathematical Development".

Beginning: Indicates students' initial explorations with the mathematical idea(s) explored in the lesson. ***Instruction continues** to the next lesson*.

Developing: Students have worked with the mathematical ideas in previous grades or previously during the year. The focus of the lesson is to connect and build student understanding. Teachers provide intensified support to students who may exhibit misconceptions, partial understanding, no or limited understanding. ***Instruction continues** to the next lesson*.

Secure: Indicates that students have worked previously with these ideas and are expected to be at a level of secure understanding. Students with secure understanding are able to make connections and use the mathematics in a variety of situations; yet may still struggle expanding the understanding to non-routine situations. Students who are secure may still make mistakes at times; yet these students demonstrate that they have mathematical understanding with limited if any misconceptions. Students not secure in the understanding by the end of that Unit/Topic might benefit from small group intensification on these ideas. Teachers may choose to use an **A/D/E** (**A**ssessment, **D**ifferentiation or **E**xtension) day to provide additional instructional opportunity; yet should be cautious to not spend too long exploring these ideas to ensure students have ample opportunity for instruction to ALL of the Nevada Academic Content Standards (NVACS) for mathematics.

This lesson indicates a level of secure understanding.



NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 2-1: Even and Odd Numbers		
2.OA.C.3 2.OA.B.2 MP.4 MP.5 MP.7	**Access Prior Learning:** In first grade, students had the opportunity to work with the classification of even and odd numbers. **Securing the Big Idea:** In this lesson, students are securing understanding that numbers can be classified as even or odd by showing numbers as two equal parts.	Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be represented with doubles facts. **Topic Opener:** Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE p.77), *Review What You Know* (TE p. 78-80) and the *Topic 2 Vocabulary Words Activity* with the words even and odd. Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. **Visual Learning:** Have students make cube towers to increase understanding and engagement. Although the *Visual Learning* discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that can be broken into two equal

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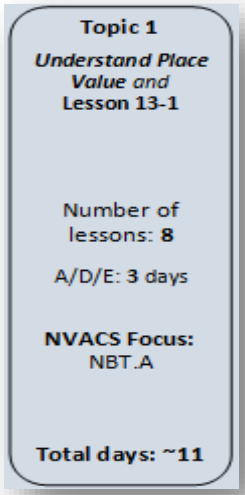
Curriculum Review Teams 2019/2020: Ben Beckam

Note:
Please e-mail Denise Trakas (dtrakas@washoeschools.net) with any questions, concerns or potential correction suggestions.

Grade 5 Topic 1: Understand Place Value; Lesson 13-1 embedded



Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 1 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 1A-1F), the Topic Planner (pp.1I-1K), all 8 lessons, and the Topic Assessments (pp. 51-54A). Consider including questions from the Topic 13 Assessment including problems 1-4, 11 and 14 (TE pp. 769-770).

Mathematical Background: Read Topic 1 Overview and Math Background (TE, pp. 1A-1F)	Topic Essential Question: How are whole numbers and decimals written, compared, and ordered? *Reference Answering the Topic Essential Questions (TE, pp. 51-52) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

1-1	1-2	1-3	13-1	1-4	1-5	1-6	1-7	Assessment

3 A/D/E days used strategically throughout the topic

5th grade Curriculum Pacing Framework: Balanced Calendar

Instructional note:

Topic 13 ideas are included in Topic 1 and will be reinforced throughout the 5th grade year. Students begin using the concepts and vocabulary introduced in Topic 13 during Topic 1 instruction. The vocabulary introduced (i.e., expression vs. equation) helps students develop the ability to write and interpret numerical expressions from a given context. A focus on understanding the relationships between values in a context before calculating will support mathematical problem solving and reasoning throughout the 5th grade. Consider displaying lesson 13-1 materials and problems if it is impractical to pull the unit pages from the Volume 2 student books.

Topic 1 focuses on deepening understanding of place value. Students will be exposed to the thousandths place for the first time. Focus instruction on 2010 Nevada Academic Content Standards (NVACS) 5.NBT.A.1, 5.NBT.A.2, 5.NBT.A.3a, 5.NBT.A.3b. and 5.NBT.A.4. These standards emphasize understanding the place value system and include:

- recognizing that the value of a digit changes to be 10x greater or 1/10 of the original digit when moved left or right
- explaining patterns in the number of zeros of a product when multiplying by a power of 10
- the placement of a decimal when multiplying or dividing by a power of 10
- using exponents to denote powers of 10
- reading, writing, and comparing decimals to thousandths
- using place value understanding to round decimals.

Students will work extensively with these concepts during instruction in Topics 2 through 6. Fifth graders will use place value understandings to add, subtract, multiply, and divide with whole numbers and decimals. Working with place value in context will build and strengthen understanding. Van de Walle, Karp, Lovin, and Bay-Williams (2014) assert, "…there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation" (p.176). Further, the National Council of Teachers of Mathematics (2000) claim, "It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value" (p.83).

In Topic 1 students will be asked to round decimals. Rounding is an estimation strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, Karp & Bay-Williams, 2010, p. 241). Number lines are useful tools for building a conceptual understanding of the value of decimals that will allow students to use rounding successfully. Students need to recognize words and phrases that signal estimation such as; about, approximately, close to, etc. Another important consideration for using estimation is given by Van de Walle et al.,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are reasonable for the situation or problem at hand (p. 242).

This topic has a total of 7 lessons with 3 additional days for assessment, differentiation and enrichment (A/D/E). Consider using the additional days added to this topic to establish classroom routines and expectations that will lay the groundwork for a classroom culture focused on a problem solving approach to mathematics. Ideas could include:

- Accessing of, strategic use, and clean-up of manipulatives
- Discussion norms
- Positive Mathematical mindsets and beliefs in a 'doing mathematics' problem based classroom
- Integrating ideas from the *Math Practices and Problem Solving Handbook* (TE, pp. F19-F36), *Problem Solving Guide* (TE, p. F29), and the Problem Solving Recording Sheet (TE, p. F30), and instructional strategies from the ELL Handbook

Solve and Share:
The Solve and Share begins each lesson in enVisionmath 2.0. It gives students the opportunity to engage in the mathematical thinking using their prior knowledge, making connections to new learning and strategies. The focus should not be about getting the correct answer but rather about building conceptual understanding, making connections and allowing students to practice explaining and sharing their thinking.

Possible guiding questions for Solve and Shares (for all Topics)

- What information does the problem give you?
- What do you still need to know?
- What is the problem asking you?
- Explain the strategy you used.
- Justify your answer to your partner/team.
- How do you know?
- What steps did you take to solve the problem?
- Do you agree? Why or why not?
- Can you prove your answer with a model?
- Have/Could you used another strategy to solve your problem?

It is possible that not all students will secure the place value concepts taught in this Topic. Students will continue to practice and apply place value understanding in future Topics as they work with operations involving multi-digit whole numbers and decimals. Topics 2, 3, 4, 5, and 6 begin with lessons integrating place value understandings with new content. It is not necessary for students to complete all practice problems during instruction and assessment to demonstrate understanding. Strategically choosing fewer items from the *Guided* and *Independent Practice* pages will allow students to explore this content at a deeper level.

Math Practice 7: Look for and make use of structure
Focus on opportunities for students to develop *Mathematical Practice 7* behaviors as this is the focus of the Math Practices and Problem Solving lesson, 1-7. Reference the Teacher's Edition (TE) and the NVACS (TE, pp. F27-F27A; NVACS, 2010, p.8).

- This topic has 3 additional days for assessment (A), differentiation (D) and enrichment (E) referenced in the pacing framework as (A/D/E) days. Use these days where appropriate to be learner responsive.

- Students do not need to do all of the *Independ Practice & Problem Solving* problems. Use graph/blank paper, sticky notes or other tools when appropriate to reduce scaffolding and provide additional work space.

▸ Grade 5 Topic 13: Write and Interpret Numerical Expressions

Big Conceptual Idea: K-5 Operations and Algebraic Thinking (pp. 32-35)
Prior to instruction, view the Topic 13 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 731A-731F), the Topic Planner (pp. 731I-731J), all 5 lessons, and the Topic Assessments (pp. 769-772A).

Mathematical Background: Read Topics 13 Cluster Overview/Math Background (TE, pp. 731A-731F)	Topic Essential Question: How is the value of a numerical expression found? *Reference Answering the Topic Essential Question (TE, pp. 769-770) for key elements of answers to the Essential Question.*

Topic 13 Instructional Note:
Instruction is focused on Nevada Academic Content Standards (NVACS) cluster 5.OA.A., "Write and interpret numerical expressions" (2010). This cluster is composed of two standards:

- 5.OA.A.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

- 5.OA.A.2: Write simple expressions that record calculations with number, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize the 3 x (18,392 + 921) is three times as large as (18,392 + 921) without having to calculate the indicated sum or product.*

Students have been working with numerical expressions and equations since kindergarten and with the Associative Property and parenthesis starting in 3rd grade. In fifth grade, the convention of order of operations is explicitly taught. Students may not initially see the need for this convention. In Lesson 13-1, students are given an expression to evaluate with little direction. It's possible that students may go left to right, do the easier pieces first (addition for many), or use order of operations. These differing strategies can produce many possible solutions for the same expression. If multiple solutions are seen, then students have discovered the reason for the convention. Using order of operations assures that every numerical expression or equation is unique and has only one possible value. Imagine the confusion of trying to represent a context using a number model if several interpretations were possible. Using order of operations will make it possible for students to interpret and represent more complicated real world contexts using the language of mathematics.

Students appreciate learning that while order of operations are a convention, they also make sense mathematically due to the meanings of these operations. An example such as the following can illustrate this point: Consider the expression *(4 + 5 + 5 + 5)*. Fifth graders know it is more efficient to show repeated addition as multiplication. In that case, the expression could be reinterpreted as *4 + (5 x 3)*. The two expressions are equivalent. Yet, if evaluated left to right so that the expression reads *(4 + 5) x 3*, the value does not match that of the original expression. However, when order of operations is followed, the same value is found for both expressions. Order of operations is necessary to keep the value of these two expressions equivalent and stable.

Teachers often incorporate acronyms to help students remember the correct order of operations. Many teacher find that the PEMDAS acronym creates misconceptions because it leads students to believe that multiplication must happen before division and that addition is calculated before subtraction. Using the acronym GEMS can help to avoid these misconceptions.

G Grouping symbols: NVACS state that parentheses, brackets and braces may be used as grouping symbols. The P in PEMDAS can be misleading because it does not include other grouping symbols.

E Exponents: Fifth graders have experience representing powers of 10 with exponents.

M Multiplication and it's inverse; division: Using only one letter to represent both operations may help students to think of them together as equal and inverse operations and calculate left to right.

S Subtraction and it's inverse; addition: Again, using one letter to represent two inverse operations may help students to consider the operations an equal priority and calculate left to right.

Topic 1 Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
exponent power base value expanded form thousandths equivalent decimals	

Additional terminology that students may need support with: annex, number name, standard form

Topic 13 Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
evaluate order of operations parentheses	*numerical expression* *equation* *expression*

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to represent powers of 10 values using multiple forms (ex. fraction notation, decimal notation, exponents, expanded form)?"
"Are students able to compare and round decimals while using multiple representations?"

Lesson	Evidence	Look for
1-3	*Homework and Practice* (student work samples) Items 22, 24 and 26	Focus CTC around the big idea: • use of place value understanding to determine value of a digit. • use of exponent notation for powers of 10. • use of fraction notation to represent powers of 10.
1-3	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
1-6	*Math Practices and Problem Solving* (student work samples) Items 23 and 26	Focus CTC around the big idea: • representations in explanation and comparison of digit value based on place value. • use of place value understanding to round decimals.
1-6	*Quick Check* (digital platform) Items 4 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 51-54	Use *Scoring Guide* TE pp. 51—54A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 1-1: Patterns with Exponents and Powers of 10		
5.NBT.A.2 MP.1 MP.2 MP.4 MP.5 MP.6 MP.7	Access Prior Learning: Students built and used place value understanding to perform operations with multi-digit numbers in 4th grade (4.NBT.A, 4.NBT.B) Beginning of the Big Idea: Students will use knowledge of single-digit multiplication facts to analyze patterns in products created by multiplying by different powers of 10. They will also see powers of 10 written with exponent notation.	Solve and Share: While solving this problem, students will benefit from using tools such as base ten blocks or place value charts to discover and analyze patterns in the products created. Facilitate a discussion about the patterns and connections seen between factors and products when multiplying by a power of 10. The *Look Back!* offers students a chance to practice modeling their ideas with numerical expressions. Visual Learning: Students will see powers of 10 written in exponent form. A common misconception for students is that 10^2 is 10 x 2 or 20. Students need to understand 10^2, 10 x 10, and 100 are equivalent methods for writing a power of 10. Consider asking students to write the exponent form and expanded form of powers of 10 to cement this understanding. Facilitate a discussion using the *Convince Me!* problem to draw attention to patterns in the number of zeros in the products. During the *Guided and Independent Practice*, consider rewriting some practice items varying the order of the factors or exponents to challenge students' thinking. All items on these pages will give the power of 10 as the second factor. Assess and Differentiate: The *Intervention activity* directs students to make flash cards that will reinforce the concept of equivalence for powers of 10 written in standard form, expanded form, and exponent form. Consider adding more time to explore the patterns of our base-ten system as students match and connect equivalent names.

Lesson 1-2: Understand Whole Number Place Value		
5.NBT.A.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In 4th grade students used place value understanding to recognize that a digit in one place represents ten times what it represents in the place to its right (4.NBT.A.1) Developing the Big Idea: Students will extend their knowledge of place value to include the 100 millions place. They will also develop the idea that each place has a value equal to 1/10 of what is represented in the place to its left.	Solve and Share: This problem will draw out students' varied understandings about place value and allow misconceptions to be tackled. Facilitate a discussion that highlights any student observations about place value. A common student misconception is to name the location of the digit when asked for the value. Remind students of the difference between place (where the digit is) and value (what the digit is worth in that place). Consider using a place value chart (Teaching Tool 3, *Teaching Resource Masters Volume 2)* as a visual scaffold. Visual Learning: The *Visual Learning* addresses how the standard, expanded, and exponent forms afford exploration of a digit's value based on its place. Students examine how a digit represents 1/10 the value of the digit to its left. Consider using the *Convince Me!* to facilitate a discussion that will extend students' understandings of place value to include this new concept. Check for understanding by asking students to compare digit values. *Another Example*, raises the question, "How many times as great is the 5 in the hundred thousand place compared to the 5 in the thousands place?" (pp. 13-14). Extend with probing questions, including "How many times LESS is the 5 in the hundreds place." These questions allow students to show understanding of the place value system while also drawing out misconceptions that need to be addressed. Assess and Differentiate: The *Intervention activity* asks students to create a place value chart and then enter different powers of 10. Use this to help students to see and analyze patterns in the numbers of zeros.
Lesson 1-3: Decimals to Thousandths		
5.NBT.A.1 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: In 4th grade, students used decimal notation to represent fractions with denominators of 10 and 100 (4.NF.C.6) and compared decimals to hundredths (4.NF.C.7). Developing the Big Idea: Students extend their knowledge of place value to include the thousandths place. They will also express decimals as fractions with denominators of 10, 100, and 1,000.	Solve and Share: Facilitate a discussion that builds upon what students already know and recognize about place value to include the thousandths place. Consider using decimal place charts (Teaching Tool 6, *Teacher's Resource Masters Volume 2*) to help students see the structure of decimal place value. Look for students who are able to use decimal and fractional notation to share examples. The *Look Back!* is a good tool to formatively assess whether students are able to read and interpret decimal place values to the thousandths. Visual Learning: Highlight that decimals can be expressed as fractions with denominators of 10, 100, or 1,000. Help students to verbalize comparisons of digits in different place values. Revisiting and facilitating a conversation based on the *Topic Essential Question* will help students access this content. The *Convince Me!* offers an excellent opportunity for students to practice using precise mathematical language. Assess and Differentiate: Using or creating a place value chart will benefit students struggling to use place value to read digits. Using the *Intervention Activity* and/or *Center Games* will help reinforce the decimal and fractional notation relationship. Additional engaging problems are found on the *Math Practices and Problem Solving* page or *Homework and Practice* (p. 22). *CTC: *Homework and Practice* (student work samples) Items 22, 24 and 26 *CTC: *Quick Check* (digital platform)
Lesson 13-1: Order of Operations		
5.OA.A.1 MP.2 MP.3 MP.4 MP.5	Access Prior Learning In previous grades and topics students evaluated expressions with parenthesis. Developing the Big Idea Students will evaluate expressions using the order of operations.	Solve and Share: This problem shows how performing correct math calculations can result in two different answers if the correct order of operations is not followed. The phrase "neither student made a mistake in the calculations" may lead students to believe that both answers are valid. If students struggle to determine more than one possible solution, consider giving them the solution and asking them to identify the steps performed to arrive at that solution. Look for students who put parentheses around their first step. It is possible students will not know which solution is the correct one. The *Look Back!* can be used to facilitate a discussion. Why is it important to follow the steps in an agreed upon order? Visual Learning: The *Visual Learning Bridge* models how to solve an expression with multiple operations using the correct order of operations. Students may be tempted to take shortcuts and perform steps mentally. Why is it important to rewrite the problem after each step is performed? The *Convince Me!* asks students to reason about an incorrect answer. This is an opportunity to introduce the GEMS acronym (see instructional note). *-continues on next page-*

		Assess and Differentiate: Consider deleting items 5, 10, 12, and 17 in the *Guided & Independent Practice* section. These work with rational numbers that are not secure at this time of the year. Topic 13 assessment problems such as 1-4, 11, and 14 (TE pp. 769-770) can be used during the Topic 1 assessment.
Lesson 1-4: Understand Decimal Place Value		
5.NBT.A.3a MP.1 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In previous lessons in this topic students have represented whole numbers and decimals using various forms. Students also used standard form, expanded forms and number names (word form) in 4th grade (4.NBT.A.2) Developing the Big Idea: Students will deepen their conceptual understanding of place value by representing equivalent decimals using various forms. This understanding lays the groundwork for comparing decimals in later lessons (5.NBT.A.3b).	Solve and Share: Consider providing a Decimal Place-Value Chart (Teaching Tool 6, *Teachers Resource Masters Volume 2*) to each student. Students will need to understand the value of each digit and extend that knowledge to include units of time. Scaffolding questions are located in the *Look Back!* and could be used to facilitate a discussion about this problem. Visual Learning: The standard, expanded, and number name forms are modeled using the same decimal. Draw out the idea that all three forms are equivalent because they represent the same value. The *Convince Me!* pushes thinking further by asking students to compare values of digits in different place values. Students comfortable moving between the various forms for representing a decimal value could move to the *Math Practices and Problem Solving* page for more challenge. Assess and Differentiate: The *Reteach* page contains visual models such as hundreds grids and a place value chart to help students represent decimals. These representations are appropriate for all students. Consider asking students to use concrete tools such as base 10 blocks if they need more time to build a conceptual understanding of decimal place value. Students may need more practice reading the names of decimal numbers correctly using place value language.
Lesson 1-5: Compare Decimals		
5.NBT.A.3b MP.1 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: Students built understandings of decimal place value in previous lessons. Developing the Big Idea: Students will deepen conceptual understanding of decimal place value while comparing decimals to the thousandths place.	Solve and Share: Consider providing Decimal Place-Value charts (Teaching Tool 6, *Teacher's Resource Masters Volume 2*). Encourage students to use what they know about the structure of place value to compare these decimals. This could lead to a discussion about students using the mathematical habits of MP.7. Use questioning to draw out ideas about how place value can be used to compare numbers including decimals. Visual Learning: A number line is an excellent visual to build conceptual understanding of decimal value and connects to whole number strategies. As a check for understanding, consider asking students to plot the cockroach lengths on the number line. The *Visual Learning Animation* will build on this and provide a step-by-step procedure for comparing decimals. Encourage students to explain why these steps work based on place value understanding. Ask students what happens if we start comparing at the left most digit without first thinking about the value of the digits. This will avoid a common misconception noted in the Teacher's Edition (TE, p. 30). The *Convince Me!* offers a chance for students to compare decimals and explain their thinking. Challenge students to use a number line to justify their solutions. A discussion based on this problem will reveal understanding; as well as, misconceptions. Address these misconceptions during the discussion.
Lesson 1-6: Round Decimals		
5.NBT.A.4 MP.1 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: Students rounded whole numbers in 4th grade (4.NBT.A.3) Students have worked with decimal place value in previous lessons in this topic. Developing the Big Idea: Rounding decimals requires a conceptual understanding of decimal place value. This knowledge will be connected to a procedural understanding of how to round numbers.	Solve and Share: A number line is used to help students compare decimal values to whole numbers (Teaching Tool 12). Consider using the *Look Back!* problem as a starting point for a discussion about how to decide where a decimal value will fall on the number line and how to determine the closest whole number. Ask students if they can explain why the number line is a useful tool for comparing and rounding decimals. Making connections to whole number and fraction benchmarks such as 0, $\frac{1}{4}$, $\frac{1}{2}$, and 1 will help students connect decimal ideas to content worked with in 4th grade and assists in solidifying place value understanding and number sense. Visual Learning: A connection is made between a number line and a step by step procedure for rounding. To ensure that students are building a conceptual understanding of rounding instead of memorizing a procedure, ask them to explain why the rules work to round decimals. This question will likely reveal misconceptions. Address student misconceptions while comparing the number line to the procedure for rounding. Teaching Tool 12 is a number line that could be used by students during the *Guided and Independent Practice*. *-continues on next page-*

		Assess and Differentiate: The *Reteach* page will show another example of using a number line to round decimals. *CTC: *Math Practices and Problem Solving* (student work samples) Items 23 and 26 *CTC: *Quick Check* (digital platform) Items 4 and 5
Lesson 1-7: Math Practices and Problem Solving- Look For and Make Use of Structure		
5.NBT.A.3a 5.NBT.A.3b MP.1 MP.6 MP.7 MP.8	Access Prior Learning: Students built content knowledge about decimal place value during the previous lessons in this topic. Securing the Big Idea: Students will need to apply knowledge of decimal place value and rounding given a real world context.	Solve and Share: Look for students making use of the structure of the decimal place value system to compare the decimal values. Ask these students to share their strategies and highlight that they are using the mathematical habits of MP.7. Consider facilitating a discussion about how students' strategies and explanations connect to the Thinking Habits (SE, p. 41). Visual Learning: The hundredths decimal chart used to visually represent decimal values offers an opportunity for students to make conjectures about decimal place value patterns and strategies for comparing. Have students analyze the chart to discover and explain patterns in place value. Use the chart in *Math Practices and Problem Solving* to have students generalize and prove their ideas (SE, p. 44). Consider using this chart to formatively assess, discover and address any misconceptions students may have about decimal place value. Assess and Differentiate: The *Reteach* page contains another blank hundredths decimal chart. Consider using to address misconceptions observed during the *Visual Learning* portion of this lesson. The varied structure of the *Homework and Practice* page can provide students with more challenge.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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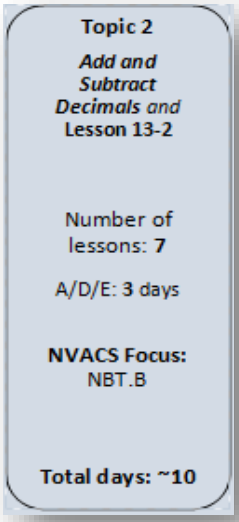
Grade 5 Topic 2: Add and Subtract Decimals to Hundredths; lesson 13-2 concepts



[5th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 55I-55J), all 7 lessons, and the Topic Assessments (pp. 107-108A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: How can sums and differences of decimals be estimated? What are the standard procedures for adding and subtracting decimals? How can sums and differences be found mentally? *Reference Answering the Topic Essential Questions (TE, pp. 105-106) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

2-1	2-2	2-3	2-4	2-5	2-6	2-7	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

Topics 2 through 6 address ideas in the Nevada Academic Content Standards (NVACS) cluster 5.NBT.B (2010). Topic 2 focuses specifically on standard 5.NBT.B.7, addition and subtraction with decimals to hundredths with an emphasis on "…using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used" (NVACS, 2010). Students have prior experience working with decimals and understanding the place value system. Students will use models and properties to extend knowledge of these concepts and build a conceptual understanding of decimal addition and subtraction. This work will also help students to strengthen their understanding of place value.

> *It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two-and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).*

Estimation plays a very important role in building understanding of decimal computation. It is tempting to begin instruction of addition and subtraction with decimals by focusing on the procedural rule of lining up the decimals and then performing the calculation. However, beginning with specific rules can deny students the opportunity to build a firm understanding of the connections between a digit's place and a digit's place value when computing with decimals. Instead, begin with estimation and encourage the use of multiple tools and models. Students gain practice determining the relative quantity created by adding and subtracting decimals using tools and models to support and explain their thinking. Van de Walle, Karp, Lovin, and Bay-Williams (2014) note, many students who are taught to focus on poorly understood rules for decimal computation do not even consider the actual values of the numbers, much less estimate, "Students should become adept at estimating decimal computations well before they learn to compute with pencil and paper" (p. 270). Topic 2 will allow students the time to build a conceptual understanding of computing with decimals by asking them to use mental math, estimation and models to add and subtract decimals before applying standard algorithms.

A caution on developing the meaning of addition and subtraction with decimals through keyword strategy instruction. Keyword strategy instruction is defined as assigning a mathematical operation to certain words such as; total, in all, remain, and left. Karp, Bush, and Dougherty (2014) state that, "reducing the meaning of an entire problem to a simple scan for key words has inherent challenges" (p. 21). Those challenges being:

- Using keywords often encourages student to strip numbers from the problem and use them to perform a computation outside of the context, which often times is misleading when working at the depth required by Smarter Balanced Assessment Consortium (SBAC- claims 1, 2, 3 and 4).
- Many keywords are common English words that can be used in many different ways thus denoting several different operations.

Justification for including 13-2 concepts: Concepts from Lesson 13-1 (order of operations) and 13-2 (evaluating expressions) can be embedded throughout Topic 2. Questions can be pulled explicitly from Lesson 13-2, or problems within Topic 2 can be manipulated (i.e. add a third term, change an operation, add grouping symbols) to include application of the concepts. Students are exposed to expressions with multiple grouping symbols in lesson 13-2 which include parenthesis, brackets and braces.

Math Practice 4: Model with mathematics:
Focus on opportunities for students to develop *Mathematical Practice 4* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 2-7. Reference the Teacher's Edition (TE, pp. F24-F24A) and the NVACS (2010, p.7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	compatible numbers *associative property of addition* *commutative property of addition* *compensation* *decimal* *tenths* *hundredths*

Additional terminology that students may need support with: *parentheses, brackets, braces*

Collaborative Team Conversations (CTC)
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to estimate sums and differences of decimals? Can they explain why their estimate is reasonable using place value understanding?
"Are students applying understanding of place value to calculate sums and differences of decimals?"

Lesson	Evidence	Look for
2-2	*Math Practices and Problem Solving* (student work samples) Items 19 and 22	Focus CTC around the big idea: • reasoning based on place value understanding to justify estimates. • reasonable estimates for the context of a situation.
2-2	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
2-6	*Math Practices and Problem Solving* (student work samples) Items 25 and 30	Focus CTC around the big idea: • student strategies and models used to add and subtract with decimals. • use of place value understanding to regroup.
2-6	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 105-108	Use *Scoring Guide* TE pp. 106A-108A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 2-1: Mental Math		
5.NBT.B.7 MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning: In 4th grade students used place value understanding and properties to add and subtract multi-digit whole numbers as well as 10th and 100th (4.NBT.B.4) (4.NF.C.5). Beginning of the Big Idea: Students will begin to extend their understanding of using properties to add and subtract whole numbers to include decimal values.	Solve and Share: This problem and lesson asks students to use mental math versus paper and pencil computation. Estimation will allow the use of whole number strategies to add and subtract with decimals. Once students have estimated a reasonable answer, they will be better able to apply properties and strategies learned through working with whole numbers (i.e. making wholes, tens, hundreds, friendly numbers). Look for students who use the Commutative and Associative Properties of Addition to facilitate a discussion. Draw out generalizations such as, 'there are multiple ways to add and subtract with decimals", "using the properties helps to make easier problems" and "whole number strategies will work with decimals". The *Look Back!* will help students to think about their own strategies. Consider building on this question by asking which numbers were challenging to add mentally. *-continues on next page-*

		Visual Learning: Using properties to add and subtract decimal values is demonstrated. Ask students to make connections between their own ideas shared during the *Solve and Share* and those modeled in the *Visual Learning Bridge*. The compensation strategy is modeled in *Another Example*, in the Student Edition (SE, p. 61). Assign a few items from the *Guided Practice* page and then ask students to explain and name their strategies. Assess and Differentiate: The *Intervention Activity* focuses on the Associative Property. The *Homework and Practice* page demonstrates use of the Commutative Property and compensation. Students should try using multiple methods solve these problems mentally.
Lesson 2-2: Estimate Sums and Differences		
5.NBT.B.7 5.NBT.A.4 MP.2 MP.6 MP.4	Access Prior Learning: Students added, subtracted and rounded multi-digit whole numbers in 4th grade (4.NBT.A.3) (4.NBT.B4). Students rounded decimals during Topic 1. Developing the Big Idea: Students will practice using two estimation strategies; rounding and compatible numbers, to build conceptual understanding and lay the groundwork for procedural skill with adding and subtracting decimals.	Solve and Share: Students will need to notice the language that indicates estimation versus that which indicates finding an exact answer (e.g. about, close to etc.). Look for students using different estimation strategies and ask them to share their thinking. Using student strategies, facilitate a discussion about different methods of estimation and which answers are likely to be over and under the true solution. Methods may include front-end methods, rounding and compatible numbers (Van de Walle, et. al., 2014, pp. 197-198). Visual Learning: Two estimation strategies; rounding and using compatible numbers, are demonstrated. Students may need additional clarification about the differences between these two strategies. Consider revisiting the shared student strategies to make connections between student's work and new content. Students are given an opportunity to practice estimating in the *Guided Practice*. Trying both strategies for estimation on several problems will help students to understand that there is more than one way to estimate. It will also provide an opportunity for students to construct the understanding that one strategy might be more efficient than the other based on the numbers in the problem. Assess and Differentiate: If students are struggling with estimation, consider revisiting other estimation strategies including the rounding strategy. Students have experience with several estimation strategies from previous grades. The *Reteach* page uses a number line to model rounding a decimal. *CTC: ***Math Practices and Problem Solving*** (student work samples) Items 19 and 22 *CTC: ***Quick Check*** (digital platform)
Lesson 2-3: Use Models to Add and Subtract Decimals		
5.NBT.B.7 MP.1 MP.3 MP.4 MP.5	Access Prior Learning: Students have worked with tools and models such as base ten blocks, number lines, place-value charts and grids in 4th grade and in previous lessons in Topic 1. Developing the Big Idea: Students will use tools and models to build a conceptual understanding of addition and subtraction with decimals.	Solve and Share: Estimating before solving will help students make sense of the problem and determine a reasonable answer. Encourage students to use tools and models to justify their solutions. Consider asking students to use a hundreds grid (Teaching Tool 8) to model the problem. Use their ideas to facilitate a discussion to draw out mathematical generalizations such as whole number strategies work with decimals and the importance of place value in determining the relative size (magnitude) of the solution. Including the *Look Back!* question in the discussion might help students to reach these big ideas. The idea that adding decimals means we may need to count past one grid can be confusing yet it represents the important understanding that decimals add to form whole numbers with pieces left over. Visual Learning: Grids are used to model the solution to an addition problem with decimals. The *Convince Me!* provides students an opportunity to reason with and demonstrate understanding about addition with decimals. Using hundreds grids to subtract decimals is modeled in *Another Example*. The *Guided and Independent Practice* has grids on items 8-11. Consider using these items first with students needing more practice, asking that they use the models to explain their solutions. Assess and Differentiate: The *Intervention Activity* gives students more practice with a concrete model (base ten blocks) while the *Reteach* and *On-Level Activities* use grids to model decimal addition and subtraction. Grid models are used in the *Homework and Practice* pages for students who need more practice with models to build a conceptual understanding of this content.

Lesson 2-4: Add Decimals		
5.NBT.B.7 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In 4th grade students used place value understanding and properties to add multi-digit whole numbers (4.NBT.B.4). Students built an understanding of decimal place value during Topic 1. Developing the Big Idea: Students will combine their knowledge of multi-digit addition and decimal place value to build a conceptual understanding and procedural skill of multi-digit addition with decimals.	Solve and Share: Estimating before solving will help students make connections between this problem and what they already know about multi-digit addition and decimals. Look for students using whole number strategies to solve this problem. Facilitate a discussion about why those strategies work and the role of place value. How can we use the decimal point to be sure place values are aligned? Visual Learning: Students might recognize the algorithm demonstrated to add decimals from work in previous grade levels. Encourage students to identify what is similar and different about using this method to add whole numbers versus decimals. Regrouping from decimal places can be modeled using tools such as base ten blocks or grids. Assess and Differentiate: The grid model is used to show students how the pieces in decimals are joining to form new numbers. This understanding will help students to use algorithms correctly.
Lesson 2-5: Subtract Decimals		
5.NBT.B.7 MP.1 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In 4th grade students subtracted multi-digit whole numbers (4.NBT.B.4). Students built an understanding of decimal place value in Topic 1. Developing the Big Idea: Students will use knowledge of adding decimals and subtracting multi-digit whole numbers to build a conceptual understanding and procedural skill to subtract decimals.	Solve and Share: Estimating before solving will help students to make connections between this problem and what they already know about multi-digit subtraction and decimals. Look for students using whole number strategies to solve this problem. Facilitate a discussion about why those strategies work and the role of place value. Ask, "How can we use the decimal point to be sure place values are aligned?" Visual Learning: Students might recognize the algorithm demonstrated to subtract decimals from their work in previous grade levels. Encourage students to identify what is similar and different about using this method to subtract whole numbers versus decimals. A part-part-whole bar diagram is used to model the relationships in this problem. Ask students to explain how this model represents the context of the problem. Encourage students to create a part-part-whole model of their own using either the *Solve and Share* or an item from the *Guided and Independent Practice* pages. Encourage students to make connections between this concrete representation and the algorithm demonstrated for subtracting decimals. Assess and Differentiate: More practice with tools and models such as base ten block, grids, and the part-part-whole bar diagram will help students to deepen their conceptual understanding of decimal subtraction. More examples of the part-part-whole representation are found on the *Homework and Practice* pages.
Lesson 2-6: Add and Subtract Decimals		
5.NBT.B.7 MP.1 MP.2 MP.3 MP.4 MP.5 MP.7 MP.8	Access Prior Learning: In previous lessons students added and subtracted decimals. Securing the Big Idea: Students will build procedural skill for adding and subtracting decimals.	Solve and Share: Students can demonstrate conceptual understanding of this problem by modeling the context. Decimal Grids (Teaching Tool 8), as well as bar diagrams allow students to represent the context visually. Students can mathematically model the context of this problem using a numerical expression. Encourage students to attempt multiple models, find connections between different models and explain how their models represent the given context. Ask students how models can help them to make sense of math problems and remind them that they are demonstrating the thinking habits of MP.4 (Model with Mathematics). Visual Learning: One standard algorithm is demonstrated for addition with decimals. Extra attention is given to regrouping with decimals and how it connects to what students already know about regrouping with whole numbers. Remind students that they can estimate before calculating to make sense of the problem and check if their answer is reasonable. Students might also benefit from continuing to use models as they practice using standard algorithms to help tie conceptual understanding to procedural skill. Students demonstrating proficiency with a standard algorithm can move to the items on *Math Practices and Problem Solving* for more challenge (SE, p. 92). Assess and Differentiate: The *Reteach* page focuses student attention to regrouping using the U.S. Traditional standard algorithm to add decimals and whole numbers. ***Homework and Practice*** page 94 will offer more challenge for students proficient with standard algorithms for adding and subtracting decimals. *CTC: *Math Practices and Problem Solving* (student work samples) Items 25 and 30 *CTC: *Quick Check* (digital platform)

Lesson 2-7: Model with Math		
5.NBT.B.7 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In previous lessons students added and subtracted decimals and used models to solve models. Securing the Big Idea: Students will apply knowledge of modeling addition and subtraction problems containing decimals in a real world context. Students will practice evaluating the reasonableness of their solution.	Solve and Share: Bar diagrams are given so that students can practice modeling a given context that includes decimals. There are multiple ways this multi-step problem can be represented and solved. Consider using student solutions to facilitate a conversation about how modeling multi-step problems can help to visualize relationships and determine the correct operations and strategies. Thinking Habits tied to MP.4 are listed on the Student Edition page and can be brought into the class conversation. Visual Learning: Students are applying strategies and using models practiced in previous lessons. Consider facilitating a discussion about how models are used to represent math problems. The *Convince Me!* gives students an opportunity to use estimation as a strategy to determine if their answer is reasonable. Assess and Differentiate: The *Intervention Activity*, *Reteach* page, and *Homework and Practice* page all offer more practice using bar diagrams and numerical expressions to model mathematical problems.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Karp, K., Bush, S., & Dougherty, B. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8* (2nd ed.). New York, NY: Pearson.

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Grade 5 Topic 3: Fluently Multiply Multi-Digit Whole Numbers; lesson 13-3 concepts

Topic 3
Fluently Multiply Multi-digit Whole Numbers and* Lesson 13-3 *concepts
Number of lessons: **7**
A/D/E: **3** days
NVACS Focus: NBT.B
Total days: ~10

[5th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-19)

Prior to instruction, view the Topic 3 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 109A-109C), all 7 lessons, and the Topic Assessments (pp. 161-162A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What are the standard procedures for estimating and finding products of multi-digit numbers? *Reference Answering the Topic Essential Questions (TE, pp. 159-160) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

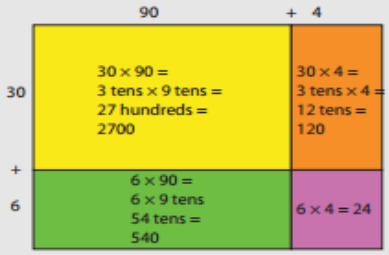
3-1	3-2	3-3	3-4	3-5	3-6	3-7	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:
This topic focuses on multiplying multi-digit whole numbers. Focus instruction on Nevada Academic Content Standard (NVAC) 5.NBT.B.5, "Fluently multiply multi-digit whole numbers using the standard algorithm" (2010). The Progression Documents add that students should also understand that "underlying this algorithm are the properties of operations and the base-ten system" (Common Core Standards Writing Team (CCSWT), 2015, p. 16).

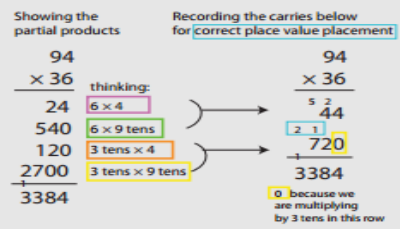
5.NBT.B.5 is the only standard in fifth grade that calls for fluency. As defined by the NVACS, fluency refers to "skill in carrying out procedures **flexibly, accurately, efficiently** and **appropriately**" (NVACS, 2010, p. 6). Development of fluency occurs in three phases: 1) Constructing meaning and counting strategies 2) Reasoning strategies and 3) Working toward quick recall (Van de Walle, Karp, Lovin & Bay-Williams, 2014). The foundations for fluency are built in the third and fourth grades as students apply multiplicative thinking and generalize about place value for multi-digit whole numbers (NVACS, 2010, 4.OA.A, 4.OA.B, 4.NBT.A). Additionally, standard 4.NBT.B.5 requires students to multiply multi-digit whole numbers by a one-digit whole number and to multiply two two-digit whole numbers using strategies based on place value and the properties of operations. Students must "**Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models**" (NVACS, 2010, p. 29). Maintain this focus on reasoning by discussing various strategies and algorithms and the connections between them.

Illustrating partial products with an area model



The products of base-ten units are shown as parts of a rectangular region. Such area models can support understanding and explaining of different ways to record multiplication. For students who struggle with the spatial demands of other methods, a useful helping step method is to make a quick sketch like this with the lengths labeled and just the partial products, then to add the partial products outside the rectangle.

Methods that compute partial products first



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from 30 × 4 = 120 is placed correctly in the hundreds place and the digit 2 from 30 × 90 = 2700 is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second row of the method on the right is there because the whole row of digits is produced by multiplying by 30 (not 3). Colors on the left correspond with the area model above.

The U.S. traditional algorithm for multiplication is based on the distributive property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand. **Instruction that includes the conceptual knowledge behind procedures is crucial**. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999).

Use of area models and the partial products algorithm rely on the distributive property and can be used to help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These standard algorithms allow students to see

how partial products are created while avoiding the errors that often occur when carrying and recording using the U.S. traditional algorithm. They also allow students to work horizontally or vertically and to multiply factors and add partial products in varying order. These "slight changes in recording will scaffold all students to fluency" (Bay-Williams, *Topic 3 PD video*, enVisionmath2.0).

Area models and the partial products algorithm can be very efficient. When using place value strategies and algorithms, students create separate values for each partial product. Often these strategies are just as efficient, and require minimal if any additional time as compared to other standard algorithms. The area model has many advantages over the U.S. traditional algorithm, especially as students begin using two-digit multipliers (Van de Walle, et al., 2014). Students should understand that multiple strategies can be used to solve these problems with some being more appropriate than others in certain instances.

Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014). Although the lessons of Topic 3 will quickly focus on use of the U.S. traditional algorithm, encourage students to use multiple procedures and connect what they are doing to place value understanding. Do not require students to work with only one algorithm. Instead, ask students to compare different algorithms using the same factors. They will see the connections and naturally move towards using more efficient strategies and algorithms. Students need time to build a conceptual understanding of multi-digit multiplication prior to using abstract procedures. Students experiences with the area model will also help build an understanding of division in later Topics. The area model can visually represent division contexts as "missing factor" problems and connect student understanding between these inverse operations.

Please note that the components of fluency do NOT include memorization. When instruction focuses on memorization, students are less willing to think about numbers and their relationships and to apply and develop their number sense (Boaler, 2009).

> *High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed testing stand in the way of number sense, giving students the impression that sense making is not important (Boaler, 2015, para.13).*

When assessing fluency, AVOID timed tests. Approximately one-third of students begin to experience math anxiety at the onset of timed testing. Brain research also concludes that stress blocks the working memory, preventing students from accessing math facts they know (Boaler, 2014). In addition, timed tests do not tell us which strategies a student used or their level of flexibility. It is important that our instruction and assessment focus on numbers and their relationships. Better options for assessment include student interviews, observations, journaling or quizzes based on strategies (Bay-Williams & Kling, 2014).

Justification for including 13-3 concepts: Concepts from Lesson 13-2 (grouping symbols) and 13-3 (writing numerical expressions) can be included in Topic 3 instruction. The focus of lesson 13-3 is to represent multi-step problems with numerical expressions. Students can analyze and interpret the Solve and Share as a numerical expression before solving. Multi-step problems that allow students to record the calculations needed to solve as a numerical expression may also be found in the *Visual Learning Bridge* and in the *Math Practices and Problem Solving* pages throughout Topic 3

Math Practice 3: Construct viable arguments and critique reasoning of others
Focus on opportunities for students to develop *Mathematical Practice 3* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 3-7. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
underestimate overestimate	*partial products* *variable* *numerical expression* *equation* *factor* *product*

Additional terminology that students may need support with: *exponent*

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to estimate products of multi-digit whole numbers and explain if their estimate is reasonable using place value understandings?"
"Are students using a partial products strategy to multiply multi-digit whole numbers?"

Lesson	Evidence	Look for
3-2	*Solve and Share* (student work samples)	Focus CTC around the big idea: • reasoning based on place value understanding to justify estimates • reasonable estimates for the context of a situation
3-2	*Quick Check* (digital platform) Items 1, 3 and 4	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
3-5	*Guided Practice* (student work samples) Item 2	Focus CTC around the big idea: • student strategies and models used to multiply • correct use of multiplication to model context
3-5	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 159-162	Use *Scoring Guide* TE pp. 159-162A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 3-1: Multiply Greater Numbers by Powers of 10		
5.NBT.A.2 MP.1 MP.3 MP.5 MP.6 MP.7	Access Prior Learning: Students practiced multiplying powers of 10 during Topic 1. Developing the Big Idea: Students extend place value understanding to multiply multi-digit numbers by powers of 10.	Solve and Share: Students will multiply a single digit by powers of 10 and analyze patterns of zeros in the products. They can model this problem using concrete tools such as base-ten blocks, representational tools (Teaching tools 4 and 5) and numerical expressions. The *Look Back!* can be used to help students extend knowledge of the patterns they describe to a similar context. Watch for students that use language such as "just add a zero" or "zero trick" to describe patterns seen in the products. Steering students towards precise language such as "a place value is added" will help to avoid misconceptions when students begin multiplying with decimal factors during the next topic. Visual Learning: Students will see how patterns of zeros in the products created by multiplying with powers of 10 extend to using two-digit factors. Powers of 10 are shown in exponent form. Students will need to recognize the equivalent powers of 10 written in standard form and exponent form. Both of these forms will be used to multiply two-digit factors on the *Guided and Independent Practice* pages. The *Convince Me!* can be used to facilitate a discussion that will help students connect the term annexing zeros to place value patterns. Assess and Differentiate: The *Homework and Practice* page is very similar to the *Guided practice*. Students showing proficiency with these item types will find more challenge on the *Math Practices and Problem Solving* and the "Common Core Performance Assessment" on *Homework and Practice* (SE, pp. 116 and 118). Consider using the "Common Core Assessment" problems (SE, p. 116) with students as part of the formative assessment process. Focus on using reasoning to explain solutions.

Lesson 3-2: Estimate Products		
5.NBT.B.5 MP.1 MP.2 MP.3	Access Prior Learning: Students have estimated in previous grades and topics. Make connections to the various types of estimation. Developing the Big Idea: Students will build number sense by estimating products and reasoning about how different estimation strategies produce products above or under the actual product.	Solve and Share: This lesson focuses on using estimation when multiplying in order to determine the relative size of the product. This practice will help students think about the reasonableness of their answers as they build procedural fluency. Students may need to be cued to the words in the problem that signal estimation. Consider asking students why shopping is a context that often requires estimation. As students share their estimates, draw out the idea that there are multiple ways to estimate. Different methods will give estimates that are above or below the actual answer. Visual Learning: The *Visual Learning* demonstrates rounding as one estimation strategy. *Another Example* uses compatible numbers to model an additional strategy (SE, p. 121). Ask students to think about whether an underestimate or an overestimate is created. Students may need clarification on what these terms mean and practice analyzing how rounding factors up or down changes the estimate. What happens when both factors are rounded down versus rounded up? How can we use this understanding to estimate more precisely? Consider using *Math Practice and Problem Solving* items 23 and 25 to formatively assess student reasoning. Assess and Differentiate: If needed, use the *Intervention Activity* in small groups or whole class if more practice with rounding and estimating. Use *Math Practice and Problem Solving* (SE, p. 122) or *Homework and Practice* (SE, p. 124) to challenge students to apply their understanding. *CTC*: Guided Practice* (student work samples) Item 2 *CTC*: Quick Check* (digital platform) Items 1, 3 and 4
Lesson 3-3: Multiply 3-Digit By 2 Digit-Numbers		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.7 MP.8	Access Prior Learning: In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5) Developing the Big Idea: Students learn to extend multiplication strategies to include larger multi-digit numbers. Emphasize the connections between place value and partial products using different multiplication algorithms/strategies.	Solve and Share: Students multiply a three-digit whole number by a two-digit whole number using a strategy or algorithm of their choosing. *Teacher's Edition* page 125 under *Analyze Student Work* shows an example of an area model and the partial products algorithm. These strategies are important for building a conceptual understanding of using the distributive property to solve multi-digit multiplication problems. Consider showing the same problem solved by an area model, the partial products algorithm and the U.S. traditional algorithm side by side. Ask students to look for connections between the strategies. Facilitate a discussion about what property is used in all three strategies and how the partial products are created in each. Visual Learning: The U.S. traditional algorithm for multiplication is modeled using multi-digit factors. Students ready for this strategy will find it to be an efficient way to multiply. Look for mistakes in "carrying" and placing the zero as a place holder. Students making these mistakes will greatly benefit from continued use of the area model or the partial products algorithm (see Topic 3 Instructional Note). These strategies can also become very efficient and will scaffold students towards fluency with standard algorithms. Consider providing a separate page to allow space for students to use visual models as they complete the *Guided and Independent Practic*e problems. Assess and Differentiate: The *Intervention Activity* and *Reteach* page focus only on the U.S. traditional algorithm. Students using other algorithms or strategies will benefit from completing the *Homework and Practice* items on a separate blank page that will allow room for visual models or alternative approaches. Although *Homework and Practice* items 1 and 2 reteach the standard algorithm, allow students to use multiple strategies (SE, p. 129). It is important to focus on understanding rather than using an algorithm.
Lesson 3-4: Multiply Whole Numbers with Zeros		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5). Developing the Big Idea: Students learn to extend multiplication strategies to include larger multi-digit numbers. Students will see how zeros within multi-digit whole numbers can create partial products when multiplying.	Solve and Share: Consider beginning with the *Look Back!* to focus on making sense of problems before solving. Students will multiply multi-digit numbers that contain a zero in one of the factors. *Analyze Student Work* shows an example of the Distributive Property used to decompose a problem into smaller simpler problems (TE, p. 131). Look for students using multiple strategies and facilitate a discussion noting how the partial products are created. What connections can be seen between the varying strategies? Look for the common misconception that the zeros in factors do not create partial products. *-continues on next page-*

		Visual Learning: Emphasize estimating before calculating to help students determine the reasonableness of their solutions. The U.S. traditional algorithm is modeled. Once again, look for mistakes in "carrying" and placing the zero as a place holder. Students making these mistakes will greatly benefit from continued use of the area model or the partial products algorithm (see Topic 3 Instructional Note). Students should use a strategy they understand to build understanding before moving to the more abstract standard algorithms. Assess and Differentiate: The *Intervention Activity* asks students to find the partial products created from a multi-digit multiplication problem. Have students use concrete, representational and/or abstract strategies. This helps students to understand the creation of partial products. The *Reteach* and *Homework and Practice* page focus on use of the U.S. traditional algorithm. Consider giving students a separate page to model their strategies. Use *Homework and Practice* item 25 to assist students in thinking about how partial products from the area model or partial products algorithm combine to form the larger partial products seen in other standard algorithms; including the U.S. traditional algorithm.
Lesson 3-5: Multiply Multi-Digit Numbers		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4	Access Prior Learning In 4th grade, students multiplied multi-digit whole numbers (4.NBT.B.5) Developing the Big Idea Students build procedural skill with multi-digit multiplication while deepening conceptual understanding through modeling contexts with multiplication.	Solve and Share: Students must create a context that matches a numerical equation with multiplication. Look for students who use a real world context versus restating the equation in words (e.g. multiply 36 by 208 and find the answer). Note the use of a variable in this problem. Facilitate a whole class discussion around student contexts and how these contexts represent the given equation. Draw out generalizations about the types of contexts that require multiplication. The *Look Back!* highlights how the Commutative Property allows us to multiply factors in any order, however doing so may change the context. Visual Learning: A bar diagram is used to model a multiplication problem. Consider building on the *Solve and Share* discussion to determine how this model represents the given context. The *Convince Me!* gives students a chance to practice mathematical reasoning with justification (MP.2). The *Guided and Independent Practice* page require use of the U.S. traditional algorithm for items 3 - 10. Students building understanding using other strategies can begin with items 11-22. Students demonstrating proficiency can move to *Math Practices and Problem Solving* for practice modeling contexts with multiplication (SE, p. 140). Assess and Differentiate: The *Intervention Activity* provides additional practice modeling situations with multiplication. The *Reteach* and *Homework and Practice* page scaffold using the distributive property to decompose multi-digit factors and create partial products. *CTC: *Guided Practice* (student work samples) Item 2 *CTC: *Quick Check* (digital platform)
Lesson 3-6: Solve Word Problems using Multiplication		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning In 4th grade, students used multiplication to solve word problems (4.OA.A.3). Securing the Big Idea Students will apply knowledge of multiplication to model and solve word problems.	Solve and Share: Challenge students to model a real world context as a math problem (mathematical model). Initially, students may use many strategies to make sense of this context. Challenge students to check their thinking by modeling and solving this problem more than one way (MP.4). Facilitate a discussion using student models and strategies. Focus on why multiplication can be used to model this situation. Quickly have students connect the factors and product of their mathematical model (equation) back to the context of the problem to assist with solidifying the understanding of what a mathematical model is. The *Look Back!* will remind students that they should always use estimation to determine whether their answer is reasonable. Visual Learning: Students will continue to extend their understanding that multiplication problems are modeled and solved in different ways. Draw out the idea that students could use the properties, multiple strategies and models, and estimation to check the accurateness of their answers. The *Guided and Independent Practice* problems ask students to apply understanding of multiplication to model and solve real world problems. Assess and Differentiate: The *Intervention Activity* teaches a keyword strategy which has been shown to be ineffective, consider omitting. The *Reteach* and *Homework and Practice* page items ask students to model word problems using a bar diagram.

Lesson 3-7: Math Practices and Problem Solving- Critique Reasoning		
5.NBT.B.5 MP.1 MP.2 MP.3 MP.6	Access Prior Learning: Students should have prior experience communicating their own reasoning and critiquing the reasoning of others. Make connections to prior class discussions. Securing the Big Idea: Students will apply understanding of multiplication to solve word problems and critique the mathematical reasoning of others.	Solve and Share: Challenge students to determine and justify their answer to this problem using estimation and reasoning before calculating. Facilitate a discussion about how students can determine if someone else's solution makes sense (MP.3). How can students make sure they are clearly communicating their own mathematical ideas? How can you decide which situations require estimation versus an exact answer? Visual Learning: Ideas on how to determine the reasonableness of other's thinking are given. Consider comparing/connecting these to the *Solve and Share* class discussion. Incorporate the thinking habits tied to MP.3 (SE, p. 149). The *Convince Me!* provides an opportunity to practice reasoning, explaining and critiquing the mathematical reasoning of others. The *Guided and Independent Practice* provide opportunities for students to critique reasoning with a focus on MP 3 (SE, pp. 151). Consider choosing a limited number of items for practice and encourage students to give complete and thoughtful answers. The *Math Practice and Problem Solving* provides an opportunity for students to work with multiple math practices (SE, p. 152). Assess and Differentiate: The *Intervention Activity* and *Reteach* page ask students to analyze and critique mathematical reasoning using the concepts of underestimate/overestimate. Although it is important to analyze and critique these written mathematical arguments, consider spending more time creating and critiquing arguments within your class discussions.

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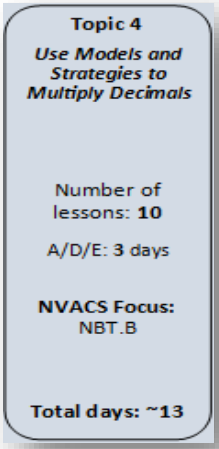
Grade 5 Topic 4: Use Models and Strategies to Multiply Decimals



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 4 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 163A-163D), all 10 lessons, and the Topic Assessments (pp. 235-236A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What are the standard procedures for estimating and finding products involving decimals? *Reference Answering the Topic Essential Questions (TE, pp. 231-232) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	4-9	4-10	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

The focus of this topic is multiplication with decimals. Instructional emphasis is on "…using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used." (NVACS, 2010, 5.NBT.B.7). Students have prior experience working with decimals, whole number multiplication, and understanding the place value system. Provide students opportunities to analyze patterns and build number sense using decimal factors in a multiplication problem. Students should notice the similarities between multiplication with whole numbers and decimals and how decimals change the size of a product. This conceptual understanding of decimal multiplication will allow students to determine the reasonableness of their solutions and will lay the groundwork for procedural understanding. Use estimation consistently during this topic as a tool to help students develop number sense and connect strategies for working with whole numbers to decimal multiplication (Van de Walle, Karp, & Bay-Williams, 2016).

A common misconception is that multiplication always results in bigger numbers. While this is true when multiplying *whole* numbers, when a number is multiplied by a decimal or fraction that is less than one, the product is *less* than the whole number factor. For example, 2x3 is 2 **"groups of"** 3 = 6. However, 0.2 x 3 is 0.2 **"of"** 3 = 0.6. A piece of the whole number factor instead of groups; or 6 groups of 0.2 (commutative property). The product is less than one of the factors. This concept will be addressed again in Topic 8 when students multiply fractions.

Throughout this topic, students will benefit from working with visual models while multiplying. Consistently use and be precise with language. Use the language "***of, pieces of***, and ***two tenths***" (instead of *point two*) when referring to multiplication and decimals. "This level of precision in language will provide your students the opportunity to *hear* the connections between decimals and fractions" (Van de Walle, et al., p 146).

Math Practice 4: Model with mathematics

Focus on opportunities for students to develop *Mathematical Practice 4* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 4-10. Reference the Teacher's Edition (TE, pp. F24-F24A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	factors *products* *partial products* *tenths* *hundredths*

Additional terminology that students may need support with: base, expanded form, exponential form, standard form, powers of ten

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to estimate products of whole numbers and decimals? Can they explain why their estimate is reasonable?"
"Are students using a partial products strategy to multiply factors with decimals?"

Lesson	Evidence	Look for
4-2	*Convince Me!* (student work samples)	Focus CTC around the big idea: • reasoning based on place value understanding to justify estimates. • reasonable estimates for the context of a situation.
4-2	*Quick Check* (digital platform) Items 1, 2 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
4-6	*Homework and Practice* (student work samples) Item 14	Focus CTC around the big idea: • appropriate strategies and models used to multiply. • correct placement of the decimal.
4-7	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 231-236	Use *Scoring Guide* TE pp. 235-236C

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 4-1: Multiply Decimals by Powers of 10		
5.NBT.A.2 MP.2 MP.3 MP.7	Access Prior Learning: In Topic 1, students worked with powers of ten and explored patterns when multiplying by powers of ten (5.NBT.A). Beginning of the Big Idea: Students will extend their understandings of place value and multiplication to develop procedural skills and mental strategies for multiplying decimals by powers of ten. Using **estimation** and procedures, students will build conceptual understanding of decimals as factors.	Solve and Share: Use questioning to help students connect place value understandings to multiplying with decimals. Students who use structure and mental math are discovering patterns in multiplying decimals by powers of 10. Ask students to share their solutions and explain their thinking. Use questioning to help students to see the patterns in their solutions. Draw out the idea that patterns can be used to multiply by 10, 100, and 1,000. Visual Learning: A common misconception is that when multiplying by powers of 10, zeros can be added to find the product. Consider writing whole numbers with the decimal point in places after the ones place. For example, 36 can also be written as 36.0, 36.00, 36.000 without changing the value. It is important that students understand the place value is changing as they explore the movement of the decimal when multiplying by powers of 10. Assess and Differentiate: Students who see patterns in the products when multiplying by powers of 10 will be able to complete the *Guided and Independent Practice* problems mentally. If students are still having difficulty seeing or applying these patterns mentally, encourage them to use different representations including charts and tables to draw their attention to the patterns. Students will encounter multiplying by powers of 10, with numbers less than 1 such as 0.1 and 0.01 in the practice problems. Students may again need to see the whole numbers written with zeros in place values that are not represented. Showing how the example of 3.63 can be written as 03.63 will help students to see that this number contains no tens but the place value is still there. This gives students a visual for moving the decimal when multiplying 3.63 by 0.1 or 0.01 and connects to the same patterns used to multiply by 10, 100, and 1,000. If students are able to use patterns to mentally calculate the problems in the *Guided and Independent Practice* (SE, p. 167) or *Homework and Practice* (SE, p. 167), move them to the more challenging problems in the *Math Practices and Problem Solving* (SE, 168) or *Homework and Practice* (SE, p. 170) to extend their thinking. Ask these students to explain and justify their thinking so that others can benefit from their insights.

Lesson 4-2: Estimate the product of a decimal and a whole number		
5.NBT.B.7 MP.2 MP.6 MP.8	Access Prior Learning: Students will use prior experiences with compatible numbers and rounding to estimate (5.NBT.A.4). Developing the Big Idea: Students will build conceptual understanding of decimal as factors through the use of estimation. These understandings will build students number sense and help them to accurately place the decimal point in future problems and to determine when exact products versus an estimate is required.	Solve and Share: Students will need to apply what they know about multiplying with whole numbers to multiplying with decimals. Ask students to explain how they arrived at their estimates; as well as, how they know their estimate is reasonable. Ask, "How do you know if your solution will be too high or too low?" Orchestrate a discussion by asking students who used differing estimation strategies to share. This will highlight several strategies and methods used when forming estimations. Consider using the *Look Back!* to extend student thinking. Allow students to provide explanations for the methods they used and for determining the reasonableness of their solutions. Visual Learning: Focus on the context of this problem and ask students, "What makes an estimate appropriate?" Two different methods are shown for estimating the product. Consider facilitating a discussion about which method is better for this situation and why. Ask students if they can use these ideas to determine whether their solutions are reasonable for future problems. The *Convince Me!* offers an opportunity for students to apply their ideas using the same context but with different numbers. Assess and Differentiate: As students complete the practice problems using estimation techniques, continue to challenge them to use multiple methods and to explain whether their estimation will be higher or lower than the exact answer. *CTC: *Convince Me!* (student work samples) *CTC: *Quick Check* (digital platform) Items 1, 2 and 5
Lesson 4-3: Use Models to Multiply a Decimal and a Whole Number		
5.NBT.B.7 MP.3 MP.4	Access Prior Learning: In Topic 2 students added decimals. In Topic 3, students multiplied whole numbers. Developing the Big Idea: Students will use a model that encourages repeated addition to multiply a whole number and a decimal. This will help them to extend previous understandings of multiplication to include decimals.	Solve and Share: Ask students to model the given problem using the hundredths grids or objects. Encourage students to explain how their models represent the context of the problem. Ask students if their model represents a problem that can be solved using more than one operation and which would be more efficient for this problem. Look for students who use a separate grid for each factor versus those who fit factors together continuously. Consider using multiple colors and a number line in addition to the hundredths grid as visuals to demonstrate how parts join to form wholes. Visual Learning: Students have another opportunity to model a multiplication problem using hundredths grids. Consider having students complete the multiplication problem without the decimals. By comparing the products created by 4 x 36, 4 x 360 and 4 x 3.6, students will connect to previous understanding of patterns and how the decimal is placed when using the U.S. traditional multiplication algorithm. Assess and Differentiate: Offer students the use of tools such as base ten blocks or blank hundredths grids as they practice modeling multiplying a whole number by a decimal.
Lesson 4-4: Multiply a Decimal by a Whole Number		
5.NBT.B.7 MP.2 MP.5 MP.6 MP.8	Access Prior Learning: In Topic 3 students estimated and multiplied whole numbers. During Topic 1 students used estimation to round decimals. They will use these concepts together to multiply a decimal by a whole number. Developing the Big Idea: Students will use understandings of multiplication and decimal place value to multiply a whole number and a decimal using a known algorithm and then correctly place the decimal in the product.	Solve and Share: This problem allows students to model with both repeated addition and multiplication. Look for students using multiplicative reasoning on this problem. Some students will be able to represent the different distances in a table and express the rule t x 1.15=d (km). Why is multiplication used to solve this problem? Visual Learning: Students are shown that it is possible to multiply the factors without a decimal and then place the decimal into the product. Placing the decimal is not given as a rule. The focus is on using number sense and place value understanding to place the decimal after multiplying with whole numbers. Consider having students generalize a rule based on the patterns in the given examples. Allow students to use different algorithms or strategies such as an area model for multi-digit multiplication. Ask students to use place value understanding and number sense to explain where the decimal is placed and why these methods work to efficiently multiply decimals. Ensure that students build a conceptual understanding prior to being given an abstract rule. Assess and Differentiate: Students should work to see the connection between multiplying whole numbers and decimals. If they are multiplying and placing the decimal with precision, indicating conceptual understanding, they might move to the *Math Practices and Problem Solving* page to practice with applying their understanding to contextualized problems.

Lesson 4-5: Use Models to Multiply a Decimal and a Decimal		
5.NBT.B.7 MP.3 MP.4 MP.5 MP.6 MP.8	Access Prior Learning: In fourth grade students used area models to multiply whole numbers (4.NBT.B.5) and in Topic 3 students used area models to multiply multi-digit whole numbers. Developing the Big Idea: Students will multiply two decimal factors for the first time building on the understanding that when one factor is less than one, the product is less than the larger factor. Students will work with two decimal factors to discover how the product is affected.	Solve and Share: Students will benefit from discussing the value one small square of this grid (1/100) represents before modeling this problem. Note that this problem asks for one possible solution. Consider having students share different plausible solutions and discuss why they are multiple solutions. Help students make connections to pervious understandings about factors. Visual Learning: The students will see how overlapping two decimals in the hundredths grid gives a solution to a decimal multiplication problem. The word "of" can help students to understand what this model is demonstrating and why it works. 2 x 3 is 2 groups "of" 3. Decimals are less than 1, so they take a piece "of" a whole number. Two decimals multiplied together finds a piece "of" a piece. The overlap created on the hundredths grid model visually shows the size of the piece created when two decimals are multiplied. This will later connect to multiplication of fractions. Part C will use two hundredths grids side by side because one of the factors is 1.5. Students often have trouble modeling numbers greater than one when using hundredths grids. Ask students if they can rename the 1.5 into an equivalent value to assist them with thinking about this. Ask students if renaming 1.5 to 15 tenths changes its value. This will help students understand the multiple ways they can reason about value and that either way enables them to use this model to multiply decimals, even when whole numbers are included. (Note: Continue to explore equivalent names for 100 hundredths or 10 tenths to solidify understanding that either of these will make a whole unit and thus 1 "whole" hundredths grid). Assess and Differentiate: Allow students to use a variety of tools such as blank hundredths grids or base 10 blocks to complete the *Guided and Independent Practice* problems (*Teaching Tools 8 and 9, Teacher's Resource Masters Volume 2*). Students will work with numbers greater than 1 on items 4, 7, 8, 10, 11, 14, and 16 (SE, pp. 191-192). Strong base-ten understanding will help students think fluidly about number. Help students make connections between numbers such as 2.1 as 21 tenths and 1.7 as 17 tenths. This understanding of equivalency will help students to see how these numbers can be represented on the hundredths grid models while supporting deeper number sense.
Lesson 4-6: Multiply Decimals Using Partial Products		
5.NBT.B.7 MP.1 MP.2 MP.4 MP.5	Access Prior Learning: In Topic 3 students used partial products strategies such as an area model to multiply two digit numbers. Developing the Big Idea: Students will develop conceptual understanding while building procedural skills for multiplying decimals using a partial products strategy.	Solve and Share: Students will use a visual model to multiply two decimal factors, which represents partial products created. Students should connect how strategies for multiplying decimals are similar to those used for whole numbers. Ask students to think about the *Look Back!* question and share their observations. Draw out the mathematical generalization that the product of two decimals less than one is less than either factor. Visual Learning: An area model is used to visually show the partial products created by multiplying two decimals. Consider asking students to complete the same multiplication problem without the decimals using an area model. Students will be able to see the similarities between the partial products created by both problems and use place value reasoning to think about how decimals are placed in the product created by the decimal factors. Ask students if the example seen here confirms, denies, or changes the generalizations made about the product when multiplying two decimals less than one. Guided and Independent Practice: Have students solve these problems on a blank page to allow use of visual model strategies such as the open area model demonstrated in the *Visual Learning Bridge*. Assess and Differentiate: The intervention activity models a partial products strategy that could increase conceptual understanding of multiplication for all students. This partial products algorithm does not hide place value. Instead, it shows all of the partial products and is procedurally similar to other standard algorithms for multiplication, including the U.S. traditional algorithm. *CTC: *Homework and Practice* (student work samples) Item 14

Lesson 4-7: Use Properties to Multiply Decimals		
5.NBT.B.7 MP.1 MP.2 MP.6 MP.7	Access Prior Learning: Students have worked with the associative and commutative properties in previous grades (4.NBT.B.5) and topics. Developing the Big Idea: Students will develop conceptual understanding of multiplication with decimals through use of the associative and commutative properties. Important conceptual understandings are built in preparation for use of the standard algorithm to multiply decimals. Connections are made between fractions and decimals.	Solve and Share: Estimating will help students to use the context of the problem and think about reasonable solutions. Providing a blank hundredths grid to students will remind them that they can use visual models to multiply decimals. Look for students who use properties to break apart the decimals and multiply. If needed, project and share the student sample titled "Javier's work" provided in *Analyze Student Work* (TE, p. 201). Visual Learning: Several understandings need to be used together to understand the example shown. Students need to understand that decimals can be written as equivalent fractions, rewritten as a whole number times a fraction using the associative property, and that the commutative property states you can change the order of the factors. Students can utilize these properties to decompose decimal factors and use number sense to multiply. Assess and Differentiate: The *Intervention Activity* provides steps for using properties to multiply decimals. Use these steps as a scaffold to help students connect the different concepts being used in this lesson. *CTC: *Quick Check* (digital platform)
Lesson 4-8: Use Number Sense to Multiply Decimals		
5.NBT.B.7 MP.1 MP.2 MP.3 MP.8	Access Prior Learning: In previous lessons students have multiplied whole numbers and decimals. They have also estimated products using decimals. Developing the Big Idea: Students will build conceptual understanding of decimal multiplication through using number sense and estimation to correctly place decimals in a product.	Solve and Share: The digits that make up the products of several decimal multiplication problems are given. Students will need to think about the size of the factors and reason to place the decimals in these products. Facilitate discussion around student reasoning about how factors affect the size of products. Visual Learning: Build on students' ideas from the *Solve and Share* to create generalizations about how the size of decimal factors can help us to place the decimal. Guided and Independent Practice: Ask students to connect how they might use whole number strategies to reason about decimal placement. This will build conceptual understanding of decimal multiplication and lay the groundwork for the standard multiplication algorithms. Students may choose to use whole number strategies and place the decimals using reasoning.
Lesson 4-9: Multiply Decimals		
5.NBT.B.7 MP.1 MP.3 MP.6 MP.7 MP.8	Access Prior Learning: In previous lessons students used models and partial products strategies to multiply decimals. They have used estimation and reasoning to place the decimal in products. Securing the Big Idea: Students will use a standard algorithm to build procedural knowledge of multiplying decimals.	Solve and Share: Consider setting parameters for this *Solve and Share* such as asking students to use 0.5 as a factor for a portion of their created problems. This will help to keep the discussion focused. Students should notice that there are several ways to find the product of two decimals which includes using whole numbers and then reasoning about the numbers or counting the number of decimal places in the factors. Visual Learning: The U.S. traditional algorithm is shown using decimal factors. Estimating a reasonable answer before calculating will help students to remember to use reasoning and number sense as they also learn the procedure of counting decimal places in the factors to place the decimal. In *Another Example*, the U.S. traditional algorithm is used to explicitly model the problem. Students are shown to count decimal places in the factors after using the same algorithm with whole numbers. Avoid showing any rules until you are confident that students have the conceptual understanding to know why they work.
Lesson 4-10: Math Practices and Problem Solving- Model with Math		
5.NBT.B.7 MP.1 MP.2 MP.4 MP.6	Access Prior Learning: Students have worked to represent real world contexts using mathematical expressions in previous grades. Securing the Big Idea: Students will select and use a model to practice the application of multiplying decimals given a real world context.	Solve and Share: Models used to represent this problem may be visual models or numerical expressions. Ask students to explain how their models represent the context of the problem. Share students' models and facilitate a discussion about how different models are useful for analyzing and solving problems. Facilitate a discussion about how using models promotes the habits that make a good problem solver. Visual Learning: Discuss how the numerical model represents the context of the problem. Students have a chance to work with a similar context in the *Convince Me!* where they will analyze how the steps in the equation match the problem (mathematical modeling). Focus on the habits of mind of problem solvers. Reinforce how to create models that represent the context of a problem during the *Guided Practice*. -continues on next page-

		Assess and Differentiate: The *Intervention* Activity calls for students to use play money as a scaffold to model a mathematical situation. Consider using a manipulative or tool throughout all components of this lesson to build conceptual understanding.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8* (2nd ed.). New York, NY: Pearson.

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Grade 5 Topic 5: Use Models and Strategies to Divide Whole Numbers

Topic 5
Use Models and Strategies to Divide Whole Numbers

Number of lessons: **8**
A/D/E: **3** days
NVACS Focus: NBT.B
Total Days: ~11

[5th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 5 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition. (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 237A-237C), all 8 lessons and the Topic Assessments (pp. 297-298A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What is the standard procedure for division and why does it work? *Reference Answering the Topic Essential Questions (TE, p. 293-294) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

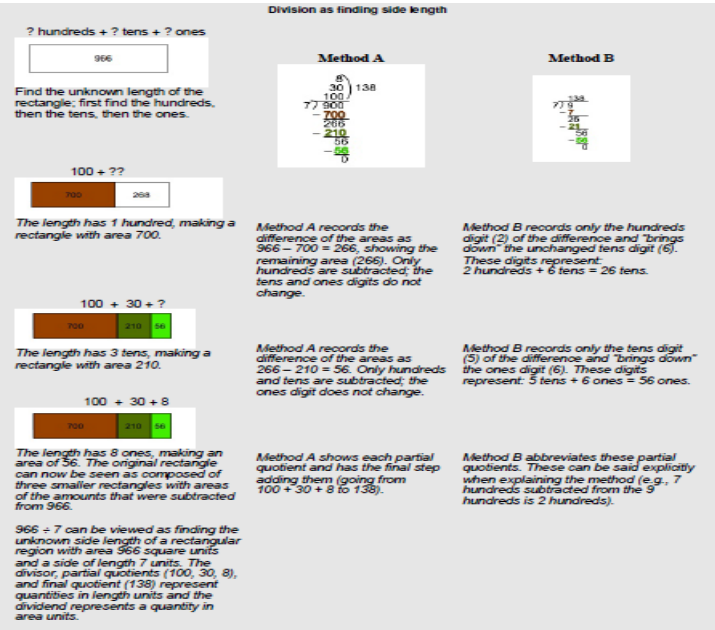
5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	Assessment

3 A/D/E days used strategically throughout the topic

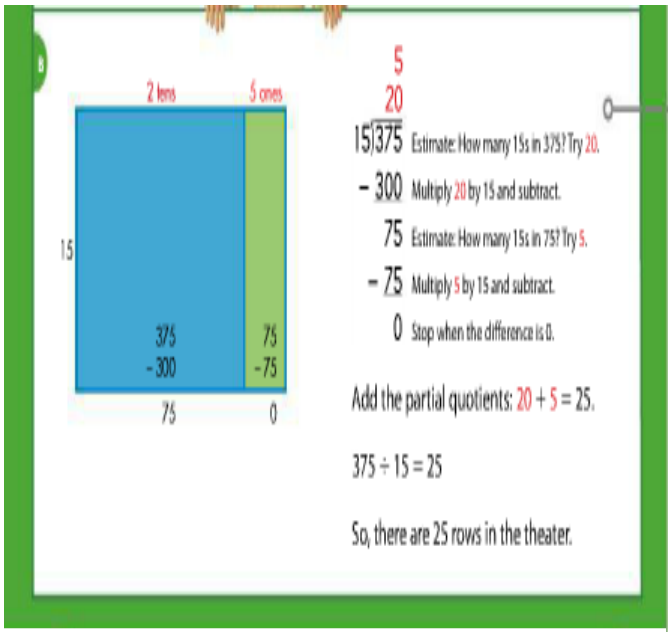
Instructional Note:

Instruction for Topic 5 is focused on Nevada Academic Content Standards (NVACS) 5.NBT.B.6 which ask students to "Find whole-number quotients of whole numbers with up to four-digit dividends and a two-digit divisor…" (2010). This standard continues with "…using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models" (NVACS, 2010, 5.NBT.B.6). This standard requires 5th graders to use multiple strategies flexibly and then explain why those strategies are appropriate. The Progression Documents for the Common Core Math Standards further explain that "Division in 5th grade involves breaking the dividend apart…applying the distributive property…and can also be viewed as finding an unknown factor…" (Common Core Standards Writing Team (CCSWT), 2015, p. 18).

This topic will rely heavily on using understandings students have built around place value and multiplication to solve division problems with whole numbers. To become procedurally fluent, students need to build deep and flexible knowledge about which strategies and procedures are appropriate and why (Pellegrino & Hilton, 2012). Thinking of **division as multiplication with a missing factor** will help students connect prior knowledge and learned strategies with new ideas. Students' experience with the area model allows them to use multiplication to solve division problems. The quotient of any division problem can be reinterpreted as a missing factor and therefore a missing side on the area model. The divisor takes the other side on the area model as the other factor and the dividend takes the place of the product. Students can begin with any underestimate, subtract the partial product created, and continue until the area model represents a true multiplication fact. Using the area model, students visually connect multiplication and division. Through practice they will naturally learn to become more efficient (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The area model connects to the partial products algorithm and builds conceptual knowledge needed to become procedurally fluent with division algorithms.



Progression Documents, Numbers and Operations in Base Ten, pg. 17



enVisionmath 2.0, Lesson 5-4 Visual Learning Bridge

The NVACS do not require fifth graders to become procedurally fluent with the long division algorithm. The long division algorithm is efficient and useful once students have built the appropriate conceptual understandings. Until they have, allowing students to choose the strategy they currently understand will allow them to build the skills needed to become procedurally fluent.

Online tools from enVision 2.0 are available which allow students to manipulate base ten blocks and visually observe regrouping while exploring division strategies.

Students will encounter and work with two different division problem types in Topic 5. Quotative division, also known as measurement or "chunking" division, gives the group size and students must find the number of groups needed. An entry point to this type of division problem can be repeated subtraction or addition. Partitive division, commonly referred to as "dealing" division, gives the number of groups and not the number within each group. An entry point for this type of division problem can be dealing out the whole to individual groups one by one or in small quantities. Exposure to both types of division is important for building understanding. However, when students are using entry level strategies to make sense of division concepts, allowing time to explore one of the problem types before switching to the other can help to reinforce strategies and understandings.

Math Practice 1: Make sense of problems and persevere in solving them

Focus on opportunities for students to develop *Mathematical Practice 1* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 5-8. Reference the Teacher's Edition (TE, pp. F21-F21A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	quotient *dividend* *divisor* *remainder*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using multiplication to help them solve division problems?
"Are students using a partial quotients strategy to divide with whole numbers?"

Lesson	Evidence	Look for
5-2	*Solve and Share* (student work samples)	Focus CTC around the big idea: • reasonable estimates for the context of the situation. • use of multiplication and number sense to make a reasonable estimate.
5-1	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
5-4	*Math Practices and Problem Solving* (student work samples) Item 22	Focus CTC around the big idea: • use of a partial quotients strategy. • use of multiplication and place value understanding to explain reasonableness of solution and strategy.
5-4	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 293-298	Use *Scoring Guide* TE pp. 293-298A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 5-1: Use Patterns and Mental Math to Divide		
5.NBT.B.6 MP.2 MP.3 MP.6 MP.7 MP.8	Access Prior Learning: In 4th grade students applied division to build place value concepts and to solve word problems (4.NBT.A.1) (4.OA.A.2). Students have used place value patterns in previous topics. Developing the Big Idea: Students extend place value understandings to build conceptual knowledge of dividing by multiples of 10.	Solve and Share: The *Solve and Share* problem is an example of measurement division. This problem type allows students to use the known group size as chunks to find the unknown number of groups. Repeated subtraction can be used as an entry point strategy while repeated addition and multiplication can be used to build up to the solution. Place value patterns can be used to extend this thinking to larger numbers and help move students towards more efficient strategies. Students look for patterns that will help them to divide by multiples of 10. This problem connects to lessons in previous topics using operations and place value to discover helpful number patterns (structure of number). The *Look Back!* can be used to facilitate a discussion that uses the observed patterns to connect multiplication and division. How can students use these patterns to determine whether an answer to a division problem is reasonable? Visual Learning: The *Visual Learning Bridge* problem is an example of partitive division. This provides the number of groups and asks students to determine the group size. Repeated subtraction or building up is more difficult with this problem type. Students might choose to equal share to divide the set into groups. Other students might estimate the group size and use multiplication to check their guess. Patterns in the zeros of quotients are shown for dividing by a multiple of 10. Students will benefit from seeing the inverse multiplication and division facts together. Some students may use a multiplication strategy to solve the *Convince Me!* which can be compared to the inverse division fact. Ask, "Why are both strategies useful for solving a division problem? How does each represent the context of the problem?" Students will be able to demonstrate understanding with a small sample of problems from the ***Independent Practice***. The *Math Practice and Problem Solving* provides practice and application for students proficient in dividing by multiples of 10 (SE, p. 232). Have students explain their thinking for item 30. Assess and Differentiate: Student's proficient using place value patterns to divide by powers of 10 should move to *Homework and Practice* (SE, p. 244). *CTC: *Quick Check* (digital platform)
Lesson 5-2: Estimate Quotients with 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students used rounding and estimation in previous topics. In 4th grade, students estimated quotients with 1-digit divisors (4.OA.A.3). Developing the Big Idea: Students extend knowledge of estimating and rounding to include division with 2-digit divisors. This conceptual understanding is the foundation for building procedural skills with division algorithms.	Solve and Share: Division algorithms require students to underestimate and then adjust their estimates before finding an exact quotient. This problem allows students to practice using rounding and compatible numbers to estimate an answer to a division problem. Facilitate a discussion about which numbers and methods for estimation will produce the closest results. Students can use multiplication to check whether their estimated quotients are reasonable and strengthen their understanding of the connection between the two operations. Visual Learning: The example shown offers opportunities to discuss how to fine tune estimates and how doing so assists in solving division problems. This understanding will help students use division algorithms in later topics. Student answers to the *Guided and Independent Practice* problems will vary. Students should be able to explain how they created their estimate and whether it is an underestimate or an overestimate. Assign a small number of problems and ask students to explain their reasoning to build conceptual understanding. Math Practices and Problem Solving problem may be used formatively to check student understanding and/or to facilitate a class discussion. Assess and Differentiate: The *Reteach* page focuses on using compatible numbers. Strategies and answers to the *Homework and Practice* pg. 249 will vary. Consider asking students to explain their estimating strategies. *CTC: *Solve and Share* (student work samples

Lesson 5-3: Use Models to Divide with 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.4 MP.5 MP.6	Access Prior Learning Students used an area model in 4th grade (4.NBT.B.5) and in Topic 3. Developing the Big Idea Students extend knowledge of the area model to include using two-digit divisors.	Solve and Share: A grid is provided to scaffold student use of the area model to solve a division problem. Look for students using the distributive property to break apart the two-digit divisor and model this problem. Use their work/ideas to facilitate a discussion drawing connections between using an area model to solve multiplication and division problems. Visual Learning: An area model is demonstrated to solve a division problem. The *Convince Me!* introduces a variable. Ask students to compare an area model used for multiplication versus one for division. Where are the factors and the product? Where is the quotient, divisor and dividend found? Provide students with a blank sheet of paper to enable them to show their strategies and reasoning when working on the *Independent Practice* page. Caution: Using a grid here may create misconceptions since the larger numbers in the problems will not match the number of squares available. If you choose to use a grid, make sure to discuss this choice and the potential challenges. Assess and Differentiate: The *reteach* page models using base ten blocks to solve a partitive (fair/equal share) division problem. This is a different understanding than the area model practiced in the beginning of the lesson.
Lesson 5-4: Use Partial Quotients to Divide		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning Students used an area model to solve a division problem in lesson 5-3. Developing the Big Idea Students connect the area model to partial quotients to build conceptual understanding of division and procedural skill.	Solve and Share: Look for students using different strategies to solve this problem. Facilitate a discussion to draw out ideas about how underestimates are used to create partial quotients. The *Look Back!* can be used to remind students that estimation and multiplication should be used while working with division problems. Visual Learning: An area model is demonstrated alongside the partial quotients strategy. Connect the partial quotients strategy to student ideas used during the *Solve and Share*. Partial quotients can be interpreted as underestimates of the answer to a division problem. How can we create more accurate underestimates while finding partial quotients? What similarities and differences can be found between partial quotients and other strategies? Consider using the *Convince Me!* to give students an opportunity to analyze the mathematical thinking of others (MP.3). Assess and Differentiate: The *Intervention Activity* reminds students they should use estimation and multiplication to solve division problems. The *Reteach* page reviews division vocabulary and models using multiplication to create partial quotients. *CTC: *Math Practices and Problem Solving* (student work samples) Item 22 *CTC: *Quick Check* (digital platform)
Lesson 5-5: Divide by Multiples of 10		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: Students have solved division problems using various strategies in previous lessons. Developing the Big Idea: The standard algorithm for division is introduced using multiples of 10 as the divisor. Instructional Note: The remainder created by the Solve and Share (money) and the Visual Learning Bridge (students) must be handled differently and change the quotient. Consider using these differences to facilitate a discussion about how context	Solve and Share: Encourage students to use various strategies such as repeated subtraction, skip counting, area models and partial quotients. Consider asking students to display their strategies and facilitate a discussion focused on connections between the different methods. Seeing the connection between an entry-level strategy such as repeated subtraction and a more efficient algorithm such as partial quotients will help to push students towards more sophisticated strategies and algorithms. The remainder in the *Solve and Share* problem is ignored due to the context of the problem. However, the *Visual Learning Bridge* problem will have a remainder requiring students to create one more group (all students need to go on the field trip). Consider facilitating a discussion about how context can change the solution to a division problem. Visual Learning: The U.S. traditional algorithm is modeled. Draw connections between this algorithm and other strategies students are currently using and practicing. *-continues on next page-*

	changes the way remainders and answers are reported. *See instructional note for lesson 5-6.*	The long division algorithm is shown again in the *Guided Practice* items 5-7 (SE, p. 265). Facilitate making connections between partial quotients and the algorithm shown. The focus of the standards in 5th grade are to "use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models" Consider asking students not ready for the long division algorithm to move to items 8-13 to continue practicing and working with other strategies or algorithms. Assess and Differentiate: Students practice placing the first digit using the long division algorithm during the *Intervention Activity.* This activity could be connected to estimation as a tool for determining a reasonable answer to a division problem. The U.S. traditional algorithm for division is shown in the steps on the *Reteach* page.
Lesson 5-6: Use Estimation to Place the First Digit of the Quotient		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.7	Access Prior Learning: Students have used estimation and partial quotients to solve division problems in previous lessons. Developing the Big Idea: Students use estimation to think about the magnitude of quotients created by different sized dividends and divisors. Instructional Note: Consider using this lesson before 5-5. This lesson requires students to use number sense, reasoning and estimation with powers of 10 to determine the magnitude of a quotient. This could help students become more proficient with all division algorithms.	Solve and Share: Estimation strategies such as rounding and compatible numbers will help students to think about the size of quotients for the different division problems. Use strategic questioning to assist students in noticing patterns in the magnitude of quotients and connect this to place value and multiplication understandings. Visual Learning: The *Visual Learning* models using powers of 10 to determine a range for the quotient. Orchestrate a whole class discussion employing additional experiences, explorations and explanations to bridge the connection between determining the range of a quotient and placing the first digit using the algorithm. Consider solving the same problem with an area model and the partial products or partial quotients algorithms to maintain focus to the requirements of the standard. The intent of the standards is to provide students experiences with "use(ing) strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models". What other strategies or algorithms might we use to solve this problem? How do these connect with what was shown? Help students to notice that all division strategies are more efficient when we use the largest possible underestimate for each place value. This helps to determine the size of the quotient and the first digit. Assess and Differentiate: The connection between multiplying by powers of 10 and determining the first digit in a quotient are made explicit in the *Intervention Activity* and the *Reteach* page. Consider focusing on other strategies or algorithms to meet the intent of the standard and build strong conceptual understanding.
Lesson 5-7: Divide by 2-Digit Divisors		
5.NBT.B.6 MP.1 MP.2 MP.4	Access Prior Learning: Students have used estimation and a variety of strategies to solve division problems in previous lessons. Securing the Big Idea: Students will practice solving division problems to build procedural skill in dividing multi-digit numbers by two-digit divisors.	Solve and Share: Encourage students to solve this problem using more than one strategy. Facilitate a discussion to make connections between the strategies. Which are more efficient? Do some strategies work better for this problem? What would happen if there were a remainder for this problem? Visual Learning: The U.S. traditional algorithm is modeled again. In the *Convince Me*! students are asked to explain how they can determine that the original estimate of 40 is too high. Can students use partial quotients or an area model to justify their thinking? Why can't we use an overestimate to solve a division problem? Assess and Differentiate: In the ***Intervention Activity*** and ***Reteach*** page, students use estimation with powers of 10 and multiplication to think about the size of an appropriate quotient given different division problems. The ***Homework and Practice*** provides application and practice for students demonstrating conceptual understanding and procedural fluency (SE, p. 280). Item 18 asks students to explain their reasoning and item 19 asks students to create a division context.
Lesson 5-8: Math Practices and Problem Solving- Make Sense and Persevere		
5.NBT.B.6 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students have solved division problems and used MP.1 in previous lessons. Securing the Big Idea: Students will apply knowledge of division to a real world context.	Solve and Share: Encourage students to analyze the context and information to make sense of this question before answering. Look for students modeling the problem and creating a plan. Connect student ideas and work to the Thinking Habits (SE, p. 281). *-continues on next page-*

MP.6 MP.7		Visual Learning: Thinking habits of MP.1 are shown in the *Visual Learning Bridge*. How do these habits connect to the ideas shared during the *Solve and Share*? How do these habits help us to become better mathematicians? *Independent Practice*: Items 6-10 are multi-step problems and push students to think deeply applying what they have learned (SE, p. 284). Consider asking students to explain their thinking for a single item. How did students make sense of the problem before answering these questions? Assess and Differentiate: The *Intervention Activity*, *Reteach* page, and *Homework and Practice* pages use a bar diagram to model division problems. Students have previous experience with this model. How can this model help make sense of new problems? How might this support explaining our own thinking or help us understand the reasoning of others?

References

Common Core Standards Writing Team. (2015). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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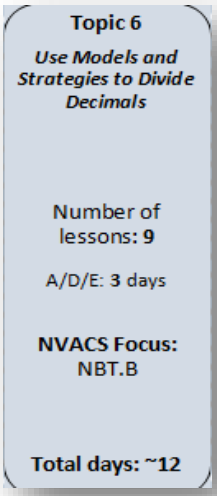
Grade 5 Topic 6: Use Models and Strategies to Divide Decimals



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations in Base Ten (pp. 18-21)

Prior to instruction, view the Topic 6 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 55A-55F), the Topic Planner (pp. 299A-299C), all 9 lessons, and the Topic Assessments (pp. 365-366A).

Mathematical Background: Read Topics 2-6 Cluster Overview/Math Background (TE, pp. 55A-55F)	Topic Essential Question: What are the standard procedures for estimating and finding quotients involving decimals? *Reference Answering the Topic Essential Questions (TE, pp. 361-362) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

6-1	6-2	6-3	6-4	6-5	6-6	6-7	6-8	6-9	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:
Instruction is focused on standard 5.NBT.B.7 requiring students to perform operations with multi-digit whole numbers and with decimals to hundredths. Students will "…divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used" (Nevada Academic Content Standards (NVACS), 2010, 5.NBT.B.7).

Topic 6 will follow a similar trajectory to previous topics in the 2 through 6, 5.NBT cluster (NVACS, 2010). One cluster heading of this domain requires students to understand the place value system. To this end, Topic 6 will begin by asking students to use mental math and place-value patterns to divide a decimal by a power of 10. The use of place value patterns and reasoning will help students extend their knowledge of multiplication and division with whole numbers to include division with decimals. "General methods used for computing quotients of whole numbers extend to decimals…Students can summarize the results of their reasoning as specific numerical patterns and then as one general overall pattern such as "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places" (Common Core Standards Writing Team (CCSWT), 2015, p. 18). Estimation plays a very important role in connecting division with decimals to computing with whole numbers. In fact, the best approach to division estimation generally comes from thinking about multiplication rather than division (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Several models are used in lessons 6-3 through 6-7. The long division algorithm is also shown and can be compared to other algorithms, strategies and models such as the area model (included in lesson 6-5). Since the NVACS do not require 5th grade students to use the long division algorithm, students can choose models, strategies and algorithms that work best for them at this point in their learning. Standard 5.NBT.B.7 also asks that students "relate their strategies to a written method and explain the reasoning used" (NVACS, 2010). This will be the focus of lessons 6-6 and 6-9 which ask students to place a decimal or choose a solution using their number sense and ability to reason mathematically. In 6th grade, students will use the long division algorithm to divide decimals. Students showing understanding of division concepts can begin to use the long division algorithm in preparation. Several of the lessons include practice problems requiring use of this algorithm.

Students will encounter and work with two different division problem types in Topic 6. Quotative division, also known as Measurement or "chunking" division, gives the group size and students must find the number of groups needed. An entry point to this type of division problem can be repeated subtraction or addition. Partitive division, also known as "dealing" division, gives the number of groups and not the number within each group. An entry point for this type of division problem can be dealing out the whole to individual groups one by one or in small quantities. Exposure to both types of division is important for building understanding. However, when students are using entry level strategies to make sense of division concepts, allowing time to explore one of the problem types before switching to the other can help to reinforce strategies and understandings.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop *Mathematical Practice 2* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 6-9. Reference the Teacher's Edition (TE, pp. F22-F22A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	dividend *divisor* *quotient* *remainder* *tenths* *hundredths*

Additional terminology that students may need support with: *annex zeros*

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using estimation to place the decimal in quotients?"
"Are students using multiplication to decide if a quotient is reasonable and/or correct?"

Lesson	Evidence	Look for
6-2	*Solve and Share* (student work samples)	Focus CTC around the big idea: • reasonable estimates for the context of the situation. • use of multiplication and number sense to make a reasonable estimate.
6-1	*Quick Check* (digital platform) Items 2, 3, 4 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
6-4	*Solve and Share* (student work samples)	Focus CTC around the big idea: • student strategies and models, sense-making. • use of multiplication and place value understanding to explain reasonableness of solution and strategy.
6-4	*Quick Check* (digital platform) Items 2, 3, 4, and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 361-366	Use *Scoring Guide* TE pp. 361-366A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 6-1: Patterns for Dividing with Decimals		
5.NBT.A.2 MP.2 MP.3 MP.7	Access Prior Learning: Students multiplied decimals by powers of 10 in Topic 4. In Topic 5 students divided using powers of 10. Developing the Big Idea: Students extend understanding of division and multiplication with powers of 10 to include decimals.	Solve and Share: The Partitive division problem identifies the number of groups and asks students to find the size of each group. Concrete strategies such as moving base ten blocks into 10 equal groups are very effective for modeling how to equal/fair share decimal pieces. Allow students opportunities to explore ways to divide the set into these named groups. Estimating will help students think about place value patterns and using multiplication understandings. For example, some students may choose to think about this problem as multiplication with a missing factor. What factor can be multiplied by 10 and make 279.4? Look for a range of student strategies and facilitate a discussion to draw out the idea that multiplication can be used to solve division problems with decimals. Visual Learning: A very similar problem to the *Solve and Share* is modeled and the movement of the decimal point is discussed. Help students connect these ideas back to the strategies and ideas discussed during the *Solve and Share*. *-continues on next page-*

		Guided and Independent Practice: These items ask students to use mental math. Consider allowing students to check their ideas by writing related multiplication facts as they work to cement an understanding of how patterns in place value can be used to determine decimal placement. Assess and Differentiate: The *Intervention Activity* explicitly teaches the movement of the decimal when working with powers of 10. *Homework and Practice* items 25 and 25 ask students to solve an equation using a given value for a variable *n* (SE, *p*. 306). *CTC: *Quick Check* (digital platform) Items 2, 3, 4 and 5
Lesson 6-2: Estimate Decimal Quotients		
5.NBT.B.7 MP.1 MP.2 MP.3	Access Prior Learning: Students have estimated in previous topics. Developing the Big Idea: Estimation helps students build conceptual understanding by connecting division with decimals to multiplication and computing with whole numbers.	Solve and Share: The word "about" in the problem signals that an exact answer is not expected. Look for students using rounding and compatible numbers to create reasonable estimates. Facilitate a discussion using student ideas to support understanding that whole number strategies for estimating work to estimate quotients with decimals. Visual Learning: Estimation is modeled using rounding and compatible numbers. How do these strategies compare to student strategies used during the *Solve and Share*? Consider revisiting the *Topic 6 Essential Question* to help students generalize rules about using whole number strategies and estimation to determine quotients with decimals. The *Convince Me!* can be used to facilitate a discussion about how students can create more precise estimates. Answers to the *Guided and Independent Practice* problems will vary as students are making estimations. Consider assigning fewer problems and asking students to allow more time for students to explain their thinking. Ask students questions such as, "How can you determine if your estimate is too high or too low?" Assess and Differentiate: The ***Intervention Activity*** can be used as a part of the formative assessment process. If students are having trouble creating an appropriate whole number division problem using the samples, they may not understand how to use decimal place value to find the nearest whole number. *CTC: *Solve and Share* (student work samples)
Lesson 6-3: Use Models to Divide by a 1-Digit Whole Number		
5.NBT.B.7 MP.1 MP.2 MP.4 MP.5 MP.6	Access Prior Learning: Students used models to multiply with decimals in Topic 4. Developing the Big Idea: Students build conceptual understanding of dividing with decimals using models to compute.	Solve and Share: Estimation will help students determine a reasonable answer and connect division with decimals to using whole number strategies. If students understand that each pen will cost about $1, they can use reasoning to place the decimal point in the quotient even if they ignore the decimal and compute with whole numbers. Models such as base ten blocks and hundreds grids also help students to see and understand how the decimal is placed in quotients. Share these strategies to help all students build conceptual understanding and deepening their number sense. Visual Learning: The U.S. traditional long division algorithm is modeled for a problem using similar numbers. Help students connect their strategies to this algorithm. Students ready to use this algorithm should be able to explain how this strategy is similar to dividing with whole numbers. Why does it work? Many students will prefer to continue with strategies, models or algorithms based on place value such as, area models and partial quotients. How do we place the decimal when working with these strategies, models or algorithms? How can estimation help to improve precision with each? *Independent Practice*: Items 4-7 force use of the long division algorithm. Consider moving students to items 8-11 to practice solving division problems using an approach of their choice. Assess and Differentiate: Base ten blocks and the long division algorithm are modeled to solve a division problem with decimals on the *Reteach* page and the *Homework and Practice* page.

Lesson 6-4: Divide by a 1-Digit Whole Number		
5.NBT.B.7 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: Students have divided decimals by 1-digit whole numbers in previous lessons. Developing the Big Idea: Students continue to practice dividing decimals by 1-digit whole numbers to build conceptual understanding and procedural skill.	Solve and Share: A Partitive division problem is given. Since students know the number of groups, they can think about strategies to create equal sized groups. The bar diagram modeled can help students to use concrete and representational strategies to equal/fair share the 107.25 pounds of sand. Encourage students to try multiple strategies and have them explain their thinking. Help students to make connections between the strategies and the context of the problem. How do they know their strategies are appropriate and their solutions are reasonable? The *Look Back!* reminds students to use estimation. Visual Learning: A bar diagram is used to model the problem and the U.S. traditional long division algorithm is used to find the quotient. Help students make connections between these mathematical models. What strengths does each have? How can we decide which strategy to use? Why do we need to annex a zero when using the long division algorithm? The *Convince Me!* will make students think about what contexts call for division with decimals. The *Guided and Independent Practice* page items 3-8 force use of the long division algorithm. Students working with other strategies can move to items 9-16 to practice and build procedural fluency with a different division algorithm. Assess and Differentiate: *Homework and Practice* page items 1-4 force use of the long division algorithm. Consider moving students to items 5-12 to allow practice with multiple division algorithms. *CTC: ***Solve and Share*** (student work samples) *CTC: ***Quick Check*** (digital platform) Items 2, 3, 4, and 5
Lesson 6-5: Divide by a 2-Digit Whole Number		
5.NBT.B.7 MP.1 MP.2 MP.4 MP.7	Access Prior Learning: In Topic 5 students divided whole numbers by two-digit divisors. In previous lessons students divided decimals. Developing the Big Idea: Students build procedural skill with dividing decimals by two-digit whole number divisors.	Solve and Share: The context of this problem promotes use of an area model and multiplicative thinking. The missing factor can be found through creating partial products inside the area model until the total of 23.4 m is reached. Look for students able to model this problem as multiplication with a missing factor and ask them to share their thinking. How can the area model be useful when working with two-digit divisors? Visual Learning: The area model and long division algorithm are shown side by side. Help students draw connections between these strategies. Where do they see the same partial quotients and how are they similar to partial products? Consider revisiting the *Topic 6 Essential Question* to facilitate a discussion about how dividing with whole numbers and decimals use similar strategies. The *Convince Me!* reminds students to use estimation. Consider providing a blank page to allow and encourage use of the area model while solving the ***Independent Practice*** problems. Assess and Differentiate: Area models and the long division algorithm are shown side by side in the *Intervention Activity*, the *Reteach* page, and on the *Homework and Practice* page. Items 5-8 on the *Homework and Practice* page force use of the long division algorithm. Consider moving students to items 9-12 to encourage practice with multiple strategies.
Lesson 6-6: Use Number Sense to Divide Decimals		
5.NBT.B.7 MP.2 MP.3	Access Prior Learning: Students have used multiplicative thinking to determine the size of products and worked with division in previous topics. Developing the Big Idea: Students will build conceptual understanding and connect to procedural skill through use of reasoning and number sense.	Solve and Share: Students must use number sense to place the decimal correctly into the digits of the quotient for several given division problems. Ask students to share their strategies and decide if they were successful. How does the size of the divisor effect the size of the quotient? The *Look Back!* can be used to further this discussion. Select students to share who have used a strategy based on number sense or place value understanding. Have these students explain how they used their strategy to correctly place the decimal. Visual Learning: An example of using reasoning to place the decimal is shown. Students can offer their own ideas or critique those given. The *Convince Me!* specifically asks students to use their own number sense and place value understanding to critique the reasoning of someone else. *-continues on next page-*

		Assess and Differentiate: A table modeling the relationship between the size of the divisor and quotient is provided on the *Another Look!* and on the *Homework and Practice* (SE, p. 335). Consider using this table to create an anchor chart to display in the room.
Lesson 6-7: Divide by a Decimal		
5.NBT.B.7 MP.2 MP.3 MP.4 MP.6 MP.7 MP.8	Access Prior Learning: Students have previously multiplied decimals and divided decimals. Developing the Big Idea: Students divide by a decimal for the first time. They will extend knowledge of division and working with powers of 10 to build conceptual understanding and procedural skill of this new content.	Solve and Share: This is a Measurement division problem. Since students know the size of the group, they are able to use strategies such as repeated subtraction/addition and skip counting/multiplication to solve this problem. These strategies are important for building conceptual understanding of how dividing a decimal by less than 1 results in a quotient larger than the dividend. Visual Learning: Hundredths grid models are shown and connected to using the long division algorithm. Why do we multiply the dividend and the divisor by a multiple of 10? This could be connected to creating equivalent fractions as well. If $a \div b = a / b$, then when we multiply the dividend and divisor by 10, we have not changed the problem. Instead we created an equivalent division problem that can be solved using whole number strategies. The *Convince Me!* asks students to think more about this concept. Teaching Tool 8 (hundredths grids) will allow students to model their thinking. Items 30-31 on *Math Practices and Problem Solving* remind students that number sense and place value understanding can be used to determine quotients (SE, p. 340). Assess and Differentiate: The *Reteach* page reinforces using powers of 10 to create equivalent division problems with whole numbers. These division problems can also be modeled as equivalent fractions.
Lesson 6-8: Continue to Divide with Decimals		
5.NBT.B.7 MP.1 MP.3 MP.6 MP.8	Access Prior Learning: Students divided decimals by decimals in the previous lesson. Securing the Big Idea: Students will practice dividing decimals by decimals to build procedural skill with a division algorithm or strategy.	Solve and Share: This is a Measurement division problem. Students can use the same strategies they practiced during the *Solve and Share* in lesson 6-7 to apply to solving this problem. Some students may take chunks of 2.4 until the 8.4 lbs. of food is used up while others may skip count or multiply by 2.4 until they reach the correct total. Encourage students to attempt multiple strategies to build conceptual understanding. Help students make connections between the different strategies and discuss which strategies are more efficient or are appropriate to build procedural fluency. Visual Learning: Using estimation, number sense and place value understanding are modeled as strategies to determine if a quotient is reasonable. The U.S. long division algorithm is shown to divide a decimal by a decimal. The *Convince Me!* encourages students to use multiplication to check that the quotient is correct. Item 1 on *Guided Practice* can be used to facilitate a discussion clarifying when to annex zeros. Assess and Differentiate: Estimation and number sense are emphasized on the *Intervention Activity* and *Reteach* page.
Lesson 6-9: Math Practices and Problem Solving- Reasoning		
5.NBT.B.7 MP.1 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: Students have used MP.2 in previous lessons and topics. Securing the Big Idea: Students will apply understandings built during Topic 6 to a real world context.	Solve and Share: Encourage students to model this problem using pictures, numbers and/or equations before solving. Focus discussion on how students made sense of the problem and quantities before solving. This is a Partitive division problem; students know the number of groups but not the size of the group. Students can experiment with multiple strategies for decomposing the quantities to create equal/fair shares. Facilitate a discussion on how students made sense of the problem and worked with the given quantities. How are they modeling the thinking habits of MP.2 when they explain their reasoning? (p. 349). Visual Learning: The thinking habits of MP.2 are modeled for a Measurement division problem. The extra step of adding to find the total quantity is needed to solve this problem. Help students connect their strategies and thinking to those being modeled. The *Convince Me!* uses the same context as the *Visual Learning Bridge* with different numbers. However, this problem will have a remainder. Can students decide what the remainder represents in this context? Do they know what they need to do with the remainder? Will they drop the remainder, is the remainder the answer, will they have to make one more group? Assess and Differentiate: On the *Homework and Practice* page consider assigning items 1-3 or items 4-8 and asking students to explain their thinking.

References

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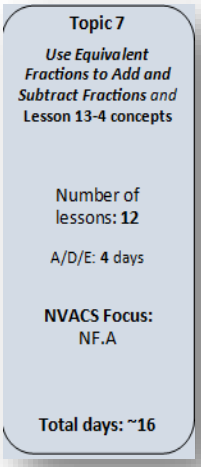
Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8* (2nd ed.). New York, NY: Pearson.

Grade 5 Topic 7: Use Equivalent Fractions to Add and Subtract Fractions; lesson 13-4 concepts



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 367A-367F), the Topic Planner (pp. 367I-367L), all 12 lessons, and the Topic Assessments (pp. 453-453A).

Mathematical Background: Read Topic 7 Cluster Overview/Math Background (TE, pp. 367A-367F)	Topic Essential Question: How can sums and differences of fractions and mixed numbers be estimated? What are standard procedures for adding and subtracting fractions and mixed numbers? *Reference Answering the Topic Essential Questions (TE, pp. 449-450) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

7-1	7-2	7-3	7-4	7-5	7-6	7-7	7-8	7-9	7-10	7-11	7-12	Assessment

4 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on Nevada Academic Content Standards (NVACS) cluster 5.NF.A, "Use equivalent fractions as a strategy to add and subtract fractions". Standards in this cluster include:

- 5.NF.A.1- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. (NVACS, 2010).

- 5NF.A.2- Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (NVACS, 2010).

Understanding equivalence is critical; in order to successfully work with fractions. It is a central idea for which students must have sound understanding and skill (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). In 4th grade, students added and subtracted fractions with like denominators to determine that these operations work when fractions have equal sized pieces. Fourth graders also expressed a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and used this technique to add the fractions. In 5th grade, students learn to add and subtract fractions (including mixed numbers) with unlike denominators by finding equivalent fractions. Connecting visual representations and concrete models is an important part of this process, which helps students understand equivalence and apply their knowledge to new situations (Van de Walle et al., 2014). These understandings take time to build but are important to students' future success. As equivalence understandings grow and emerge, students foster a sense of unity and connectedness in the study of mathematics (NCTM, 2003). It is important to start with concrete and semi-concrete experiences to build conceptual understanding before completing abstract work.

Students who understand the concept of fraction equivalence and the need for equal-sized pieces should also understand why algorithms for addition and subtraction of fractions work. Students who memorize a procedure before understanding equivalence are not likely to apply it successfully. Research suggests that students who have memorized and practiced procedures they do not understand have less motivation to understand their meaning or the reasoning behind them (Hiebert,1999). The focus of the NVACS is understanding how equivalence allows fractional pieces to be added and subtracted. In fact, "It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding fractions," (Common Core Standards Writing Team, 2013, p. 11). Initially, students will want to use the operations of addition and subtraction without paying attention to the size of the pieces. Students need time to think and reason about fractions and determine the need for equivalency.

> *However, about half of students will write* $\frac{3}{8}+\frac{2}{8}=\frac{5}{16}$, *even after drawing the model correctly. And they won't seem to be bothered that the two answers* ($\frac{5}{8}$ and $\frac{5}{16}$) *are different. In such a case, ask students to decide whether both answers can be right. Ask them to defend which is right and why the other answer is not right. You cannot just tell students which is right-the key is for them to overcome their misconceptions (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).*

Procedures alone do not help students to think conceptually about the operations and what they mean. Understanding equivalency will help students to apply understanding even when they encounter changes in fractions or problem types. Research summarizing successful teaching for fractions suggests four steps: (1) use contexts, (2) use a variety of models, (3) include estimation and informal methods, and (4) address misconceptions (Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, & Wray, 2010). Conceptual and procedural explanations from students look and sound different.

> *Concept: Two fractions are equivalent if they are representations for the same amount or quantity-if they are the same number.*
>
> *Procedure: To get an equivalent fraction, multiply (or divide) the top and bottom numbers by the same nonzero number (Van de Wall et al., 2014).*

Have students use tools, visual models and mathematical reasoning to find equivalent fractions until they are able to generalize or understand more abstract procedures. Poorly understood procedures will quickly be forgotten or confused, especially as students move to multiplication and division with fractions.

Justification for including 13-4 concepts: Lesson 13-4 focuses on interpreting expressions without calculating. How can you tell if an expression matches the context of the problem? How can you determine which expression is greater without calculating? If parentheses are added or removed how does that change the value of the expression? Students should realize that all expressions can have only one stable value. Consider using the Solve and Share, Visual Learning Bridge, or items from the Guided Practice pages from Lesson 13-4 to help students build understanding of these concepts.

Math Practice 4: Model with mathematics
Focus on opportunities for students to develop *Mathematical Practice 4* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 7-12. Reference the Teacher's Edition (TE, pp. F24-F24A) and the NVACS (2010, p. 7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
mixed number	*equivalent fractions* *benchmark fraction* *common denominator* *numerator* *denominator*

Additional terminology that students may need support with: *annex zeros*

Collaborative Team Conversations (CTC)
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using benchmarks (ex. 0, ½, 1) to estimate sums and differences of fractions?"
"Are students replacing given fractions with equivalent fractions to find a common denominator when finding sums and differences of fractions with unlike denominators?"

Lesson	Evidence	Look for
7-1	*Math Practices and Problem and Solving* (student work samples) Item 14	Focus CTC around the big idea: • use of benchmarking to make a reasonable estimate. • appropriate estimate for the situation.
7-1	*Quick Check* (digital platform) Items 2, 3, 4 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
7-5	*Homework and Practice* (student work samples) Item 15	Focus CTC around the big idea: • student strategies and models used to find a common denominator. • understanding and use of equivalent fractions.
7-5	*Quick Check* (digital platform) Items 2, 3, 4, and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 449-454	Use *Scoring Guide* TE pp. 449-454A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 7-1: Estimate Sums and Differences of Fractions		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.8	Access Prior Learning: In 4th grade, students worked with benchmark fractions and learned to add and subtract fractions with like denominators (4.NF.A) (4.NF.B). Developing the Big Idea: Students extend knowledge of benchmark fractions to estimate sums of fractions with unlike denominators.	Solve and Share: Look for students able to use number sense to estimate the sums of the fractions and ask these students to explain their thinking. Important benchmarks such as 0, $\frac{1}{2}$, $\frac{1}{4}$, and 1 can be used to help with estimation. Have students use concrete tools such as fraction strips and representations such as number lines to support their thinking. How do these strategies help us to work with fractions with unlike denominators? Visual Learning: Benchmark fractions are modeled on a number line and then used to estimate a sum for two fractions with unlike denominators. The *Convince Me!* asks students to think about the relationship between the numerator and denominator and explain their reasoning. Consider revisiting the *Topic 7 Essential Question*. A number line can be used to help place benchmark fractions and estimate sums and differences of fractions with unlike denominators. Students may confuse rounding versus estimating with benchmark fractions. How are these different? Help students connect that rounding is one type of estimation. Teaching Tool 12 (blank number lines) can be used to assist students estimating on *Independent Practice* page questions. The *Intervention Activity, Reteach* page and *Homework and Practice* page offer practice using benchmark fractions and number lines. Pages 374 and 376 offer application problems. Remember that students who seem to be struggling without context are successful with engaging real-world problems. **CTC: Math Practices and Problem and Solving* (student work samples) Item 14 **CTC: Quick Check* (digital platform) Items 2, 3, 4 and 5
Lesson 7-2: Find Common Denominators		
5.NF.A.1 5.NF.A.2 MP.1 MP.3 MP.4 MP.7	Access Prior Learning: In 4th grade students generated equivalent fractions and compared fractions with unlike numerators and denominators (4.NF.A.1) (4.NF.A.2). Developing the Big Idea: Students generate equivalent fractions using visual models to build conceptual understanding and lay the foundation for adding and subtracting fractions with unlike denominators.	Solve and Share: Students are asked to cut a pan of cornbread so that two people get the size they requested. This context leads students to use an area model to visually represent the fractions. Look for students using two separate pans and remind them that there is only one pan of cornbread available. Some students may create $\frac{1}{2}$ and $\frac{1}{3}$ on different sides of the pan. Challenge these students to find a way to know how much cornbread is left in the pan. Ask students who find a common denominator to share their strategies and explain their thinking. Can their models be used to find out how much cornbread was eaten altogether? How was the common denominator created? Why are equal sized pieces needed to solve this problem? Visual Learning: Area models are used to create equivalent fractions. The *Convince Me!* gives two new fractions and asks students to use an area model to find a common denominator. Students will begin to observe that the common denominator created by the area model is the product of the two denominators. Will this always work? Is this the only way to find a common denominator? Multiplying denominators is explicitly modeled on *Another Example* (SE, p. 379). Consider holding off on introducing this strategy until you are confident that students have the necessary conceptual understanding to know how it worked. Many students may still need a tool or a visual model to confirm their thinking on *Independent Practice* questions 4-11 (SE, p. 379). Questions 16-17 can be used to challenge students to think beyond multiplying the denominators to understand that common denominators can be any common multiples (SE, p. 381). Assess and Differentiate: Finding common multiples as a strategy to find common denominators is shown on the *Homework and Practice* (SE, p. 381). For students who are struggling, consider spending more time with the area model building conceptual understanding before using a procedure.

Lesson 7-3: Add Fractions with Unlike Denominators		
5.NF.A.1 5.NF.A.2 MP.1 MP.3 MP.4 MP.5	Access Prior Learning: In 4th grade students added fractions with like denominators (4.NF.B). Students found common denominators in the previous lesson. Developing the Big Idea: Students connect visual models with procedures for finding common denominators and adding fractions with unlike denominators.	Solve and Share: Look for a range of strategies (concrete, representational, abstract) used to find a common denominator and add the fractions. Students should support their thinking with a visual model. Ask students to share their ideas and facilitate a discussion comparing strategies. What connections are seen between the visual models and more abstract strategies for finding a common denominator? Do the fractions need a common denominator to find a sum? Visual Learning: Fraction strips and finding common multiplies are modeled together as strategies for finding a common denominator. Replacing with equivalent fractions is shown as a strategy for adding fractions with unlike denominators. Consider having students use their own fraction strips rather than only looking at the pictures. Making fraction strips is a valuable experience for students. Consider using the *Convince Me!* to facilitate a discussion about equivalent fractions. Is 5/6 the only possible solution? Fraction strip manipulatives or Teaching Tool 13 (fraction bars) can be used to help students model their solutions for the *Guided and Independent Practice* page. Assess and Differentiate: *Homework and Practice* questions 1-4 ask students to find the least common multiple. This extra demand can impede learning for students still building understanding of how and why equivalent fractions are used to add. Consider moving students to question 5, which requires using fraction bars to model the context a problem.
Lesson 7-4: Subtract Fractions with Unlike Denominators		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In 4th grade students subtracted fractions with like denominators (4.NF.B). Students added fractions with unlike denominators in previous lessons. Developing the Big Idea: Students build conceptual understanding of how equivalent fractions are used to subtract fractions with unlike denominators.	Solve and Share: Look for a range of strategies (concrete, representational, abstract) used to find a common denominator and subtract the fractions. Students can support their thinking with a visual model. Ask students to share their ideas and facilitate a discussion comparing strategies. What connections exist between the visual models and more abstract strategies for finding a common denominator? Do the fractions need a common denominator to find a difference? Visual Learning: : Fraction strips and finding common multiples are used to model subtracting fractions with unlike denominators. Notice that while using fraction strips, the whole is used as a comparison to remind students about benchmarks. Students draw connections between the methods for adding and subtracting fractions with unlike denominators. Consider revisiting the *Topic 7 Essential Question* as part of the formative assessment process. What have students discovered about the procedures for adding and subtracting fractions? Question 18 on *Math Practices and Problem Solving* helps students model fraction addition using a bar diagram and a variable (SE, p. 392). How can modeling problems this way be helpful?
Lesson 7-5: Add and Subtract Fractions		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: Students added and subtracted fractions with unlike denominators in previous lessons. Securing the Big Idea: Students will build procedural fluency by applying strategies for adding and subtracting fractions with unlike denominators.	Solve and Share: Consider providing Teaching Tool 14 (circle fractions) as a visual model for this problem. Students need to make sense of the context of this problem before computing. Look for students who mistakenly subtract the amounts eaten. Unfortunately, this will result in a correct solution due to the numbers used. Refer students back to the context of the problem to make sure their strategy matches what is happening in the problem. Look for and use a range of student strategies to facilitate a discussion. What connection is seen in all strategies? Is a common denominator generated by all successful strategies? Visual Learning: A multi-step problem is modeled. Consider asking students what they would do next before it is shown in the video. Why are the denominators 15 and then 30 chosen while computing this problem? Question 29 on *Math Practices and Problem Solving* connects adding fractions to geometry (SE, p. 398). Assess and Differentiate: *Homework and Practice* requires students to add three fractions with unlike denominators (SE, p. 399). Can this be done more than one way? Students can explore how the Commutative Property of Addition applies to fractions. Take the time to discuss the various strategies that students used. **CTC: Homework and Practice* (student work samples) Item 15 **CTC: Quick Check* (digital platform) Items 2, 3, 4, and 5

Lesson 7-6: Estimate Sums and Differences of Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.6 MP.8	Access Prior Learning: In 4th grade students added and subtracted mixed numbers with like denominators (4.NF.B) Students found sums and differences of fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use benchmarks to build number sense by estimating sums and differences of mixed numbers with unlike denominators.	Solve and Share: Students need to extend understandings of benchmark fractions to include mixed numbers. Students will need to see that although a mixed number is written using a whole number and a fraction, it represents one quantity. This means that $1\frac{3}{4}$ is close to 2 wholes. Number lines and visual models will help students to round mixed numbers to the nearest whole. Connecting mixed numbers to working with decimals can also enhance students' ability to visualize mixed numbers. Visual Learning: A number line is used to round mixed numbers to the nearest whole number. Estimation is a key strategy for determining whether answers are reasonable. How can using benchmark fractions help us to estimate when working with mixed numbers? Consider using the *Convince Me!* to facilitate a discussion on considering context when estimating. Questions 25 and 26 on the *Math Practices and Problem Solving* page ask which estimate is best for a given problem (SE, p. 404). Consider using these items as an opportunity for students to justify their strategies and the solution choice as "best".
Lesson 7-7: Use Models to Add Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In 4th grade students added mixed numbers with like denominators (4.NF.B) Students found sums and differences of fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use visual models to build conceptual understanding of adding mixed numbers with unlike denominators.	Solve and Share: Fraction strips and visual models will help students see that although mixed numbers are greater than 1, they are still composed of fractional pieces as denoted by the denominator. These tools also help students see that mixed numbers can be decomposed similarly to whole numbers. For example, in the same way that 5 can be rewritten as (4 + 1), a mixed number such as $1\frac{3}{4}$ can be rewritten as $(1 + \frac{3}{4})$, $(\frac{4}{4} + \frac{3}{4})$ or $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{3}{4})$. This flexible thinking helps students build procedural fluency working with fractions and mixed numbers. Visual Learning: Fraction strips are used to model adding mixed numbers with unlike denominators. In the *Visual Learning Bridge*, the wholes are added separately from the fractions. Being able to decompose a mixed number as described above is a crucial stepping stone to using this strategy. Students are encouraged to use fraction strips or another visual model while completing the *Independent Practice* problems (SE, p.409). Problems 22 and 23 use larger numbers and can be used to formatively assess students to see which are moving to more abstract procedures (SE, p. 410). Assess and Differentiate: Fractions strip representations are used in the *Intervention Activity*, the *Reteach* page, and on the *Homework and Practice* page. These models let students visualize how mixed numbers join to form a new whole. Manipulatives are appropriate for all students during this learning process.
Lesson 7-8: Add Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.7	Access Prior Learning: Students added mixed numbers with unlike denominators in the previous lesson. Securing the Big Idea: Students continue using models to build conceptual understanding of adding mixed numbers with unlike denominators and connect ideas to procedures for adding fractions with unlike denominators.	Solve and Share: Look for a wide range of student strategies (manipulatives, visual models, abstract procedures) used to solve this problem. Orchestrate a discussion to assist students in connecting these strategies. As a class, formulate a generalization of a mathematical rule for computing with fractions and mixed numbers with unlike denominators. Students may observe that no matter what strategy is used; a common denominator must be found before the equal sized pieces can be joined. Visual Learning: Adding mixed numbers with unlike denominators is shown and modeled with fractions strips. Why are the fractions added before the whole numbers? Is this the only way this type of problem can be solved? The *Convince Me!* can be used to help students think and talk about what makes an estimate reasonable. The *Independent Practice* problems remind students to estimate before computing. Can they explain why their estimates are reasonable?

Lesson 7-9: Use Models to Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.5 MP.8	Access Prior Learning: In 4th grade students subtracted mixed numbers with like denominators (4.NF.B). In 4th grade students decomposed fractions into a sum of fractions with like denominators (4.NF.B.3). Students subtracted fractions with unlike denominators in previous lessons. Developing the Big Idea: Students use visual models to build conceptual understanding of subtracting mixed numbers with unlike denominators.	Solve and Share: Fraction strips and visual models will help students see that although mixed numbers are greater than 1, they are still composed of fractional pieces as denoted by the denominator. These tools also help students see that mixed numbers can be decomposed similarly to whole numbers. The ability to work with these numbers flexibly is key to understanding when and how it is necessary to rename the whole before subtracting mixed numbers. Renaming the whole is modeled in the *Visual Learning Bridge*. Visual Learning: Students see that subtracting wholes and fractions separately will not always work as it does with addition. If the fractional piece of the subtrahend is larger than that of the minuend, a negative number would be created. An example of renaming the whole is modeled. This creates a larger fraction before subtracting on the minuend. The example used shows that 2 6/12 is renamed as (1 + 12/12 + 6/12) and then as 1 18/12. Understanding these different forms as equivalent is key to students building procedural fluency. Fraction strips and visual models help students see the equivalence and explore why these steps are sometimes necessary to subtract mixed numbers with unlike denominators. Focus on building understanding with manipulative tools. The *Convince Me!* gives students another chance to practice renaming the whole. Consider facilitating a discussion comparing the similarities and differences in the processes used for adding and subtracting mixed numbers. Assess and Differentiate: The *Intervention Activity*, *Reteach* page, and *Homework and Practice* page encourage students to use tools to model and compute subtraction problems with mixed numbers. Tools help students build conceptual understanding and lay the groundwork for procedural fluency and application. Use the additional practice subtracting a fraction from a whole number formatively to identify students needing support with decomposing.
Lesson 7-10: Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: Students subtracted mixed numbers with unlike denominators in the previous lesson. Securing the Big Idea: Students use visual models and written methods to build conceptual understanding and procedural skill for subtracting mixed numbers.	Solve and Share: The part-part-whole diagram is used to mathematically model the context of the problem. Can students identify what the numbers in this model represent? Look for a wide range of student strategies (manipulatives, visual models, abstract procedures) used to solve this problem. A discussion will help students connect these strategies and generalize a mathematical rule for computing with fractions and mixed numbers with unlike denominators. Visual Learning: Subtracting mixed numbers with unlike denominators is shown and modeled with fraction strips. How did $4\frac{3}{12}$ become $3\frac{15}{12}$? Is this the only way this type of problem can be solved? The *Convince Me!* can be used to help students think and talk about what makes an estimate reasonable? The *Independent Practice* problems also remind students to estimate before computing. Can they explain why their estimates are reasonable?
Lesson 7-11: Add and Subtract Mixed Numbers		
5.NF.A.1 5.NF.A.2 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students added and subtracted mixed numbers with unlike denominators in previous lessons. Securing the Big Idea: Students apply strategies and procedures for adding and subtracting mixed numbers with unlike denominators to build procedural skill.	Solve and Share: Look for students who make sense of this multi-step problem through estimation and modeling. Facilitate a discussion using their ideas to help students generalize that multiple operations may be necessary to solve a single problem. Have students share their strategies and thinking. Can students explain why their strategies are appropriate for this problem? Visual Learning: Would it be better to overestimate or underestimate the amount of fabric needed? Use this problem to discuss why considering the context when creating estimates is important. Consider revisiting the *Topic 7 Essential Question*. Consider using the problem shown in the *Visual Learning* as a jumping off point to discuss both questions. Students practice computing expressions with mixed operations on the *Guided and Independent Practice* pages. Remind students to use estimation to check the reasonableness of their solutions. Assess and Differentiate: Estimation is used as a tool check for reasonableness and build conceptual understanding in the *Intervention Activity*. This activity could be used whole class or in small groups to explore different estimation techniques and improve precision.

Lesson 7-12: Math Practices and Problem Solving- Model with Math		
5.NF.A.2 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Students have used models in previous lessons and topics. Students have added and subtracted mixed numbers with unlike denominators in previous lessons. Securing the Big Idea: Students build conceptual understanding and procedural skill through application.	Solve and Share: Encourage students to use tools and sense making skills to model this problem before computing. How do their models represent the context of the problem? Consider changing the problem to ask "Which is longer, the Scotch Bonnet sea shell or the combined lengths of two Alphabet Cone Seashells? How much longer? How do you know? Focus class discussion on the thinking habits of MP.4 and connect back to student thinking and strategies. How did they model the thinking habits of MP.4? Visual Learning: A multi-step problem is solved. Consider, how do bar diagrams and equations help us to model and solve problems? Students are asked to use equations and bar diagrams to model similar problems in the *Convince Me!* and the *Guided and Independent Practice* page. Nudge students to model and explain their thinking using the multi-step problem on the *Math Practices and Problem Solving* page (SE, P. 440). Assess and Differentiate: Students practice modeling context with a part-part-whole bar diagram in the *Intervention Activity*. Observe students who struggle with transferring between a model and mathematical notations.

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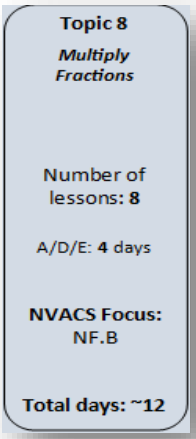
Grade 5 Topic 8: Apply Understanding of Multiplication to Multiply Fractions



Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 8 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 455A-455F), the Topic Planner (pp. 455I-455K), all 8 lessons, and the Topic Assessments (pp. 521-521A).

Mathematical Background: Read Topic 8-9 Cluster Overview/Math Background (TE, pp. 455A-455F)	Topic Essential Question: What does it mean to multiply whole numbers and fractions? How can multiplication with whole numbers and fractions be shown using models and symbols? *Reference Answering the Topic Essential Questions (TE, pp. 517-518) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	8-8	Assessment

4 A/D/E days used strategically throughout the topic

5th grade Curriculum Pacing Framework: Balanced Calendar

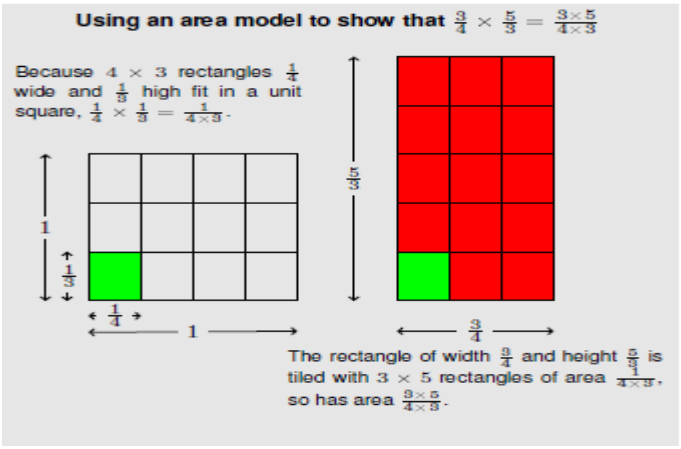
Instructional Note:

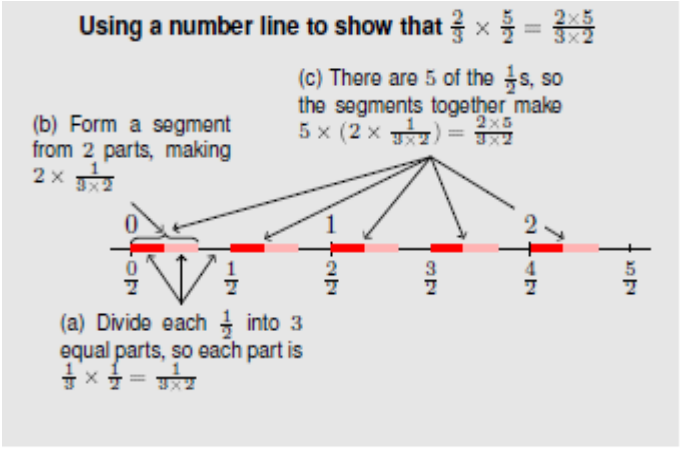
This topic focuses on 2010 Nevada Academic Content Standards (NVACS) cluster 5.NF.B, "Apply and extend previous understandings of multiplication and division to multiply and divide fractions". This cluster consists of four standards.

- 5.NF.B.4a- Interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b*. For example, use a visual fraction model to show *(2/3) x 4 = 8/3*, and create a story context for this equation. Do the same with *(2/3) x (4/5) = 8/15*. In general, *(a/b) x (c/d) = ac/bd* (NVACS, 2010).
- 5.NF.B.4b- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fractions products as rectangular areas (NVACS, 2010).
- 5.NF.B.5a- Interpret multiplication as scaling (resizing) by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication (NVACS, 2010).
- 5.NF.B.5b- Interpret multiplication as scaling (resizing) by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (n x a)/(n x b)* to the effect of multiplying *a/b* by 1 (NVACS, 2010).

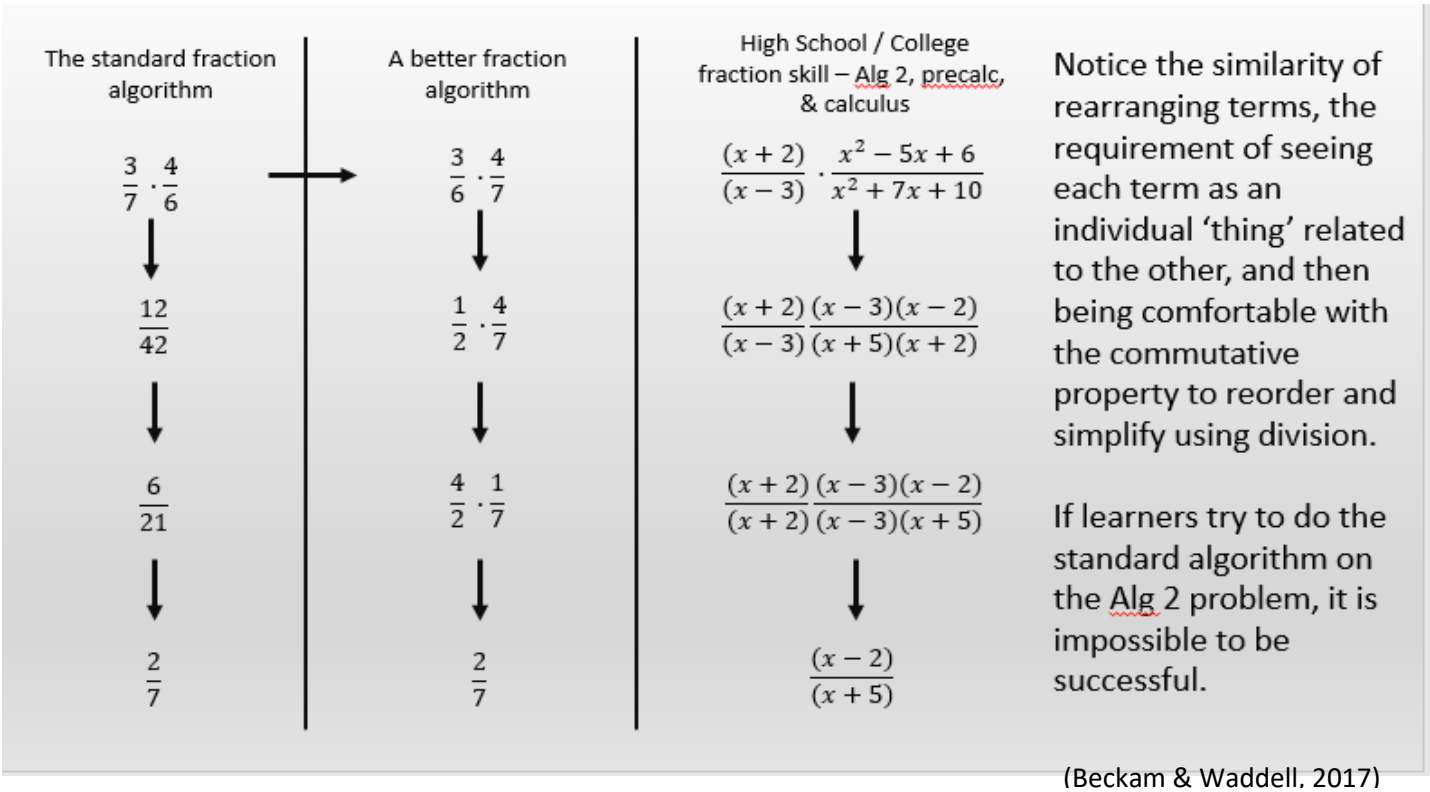
Students extend understanding of multiplication with whole numbers to include multiplication with fractions. A common misconception that requires immediate attention is that multiplication always creates greater numbers (Karp, Bush & Dougherty, 2014). Working with whole numbers, we would describe a multiplication problem such as 3 x 4 as "3 groups or sets of 4" (set models), "3 rows of 4" (area model or array), or "3 four times" (number line, linear model, skip count). When fractions are included in these models, products can be smaller than one ***or*** both factors. Thinking of multiplication as scaling connects working with whole numbers to multiplication with fractions. When we multiply 3 x 4, the 3 becomes 4 times bigger. However, when we multiply 3 by $\frac{1}{2}$, the 3 becomes half of its original size. Building on student understanding of whole number multiplication, we can say that 3 x $\frac{1}{2}$, is "3 groups of $\frac{1}{2}$," "3 rows of $\frac{1}{2}$," or even "3 half of a time" (jump halfway to 3 on a number line). Helping students to interpret multiplication as scaling will address the common misconception that multiplication always makes greater numbers and increase student's ability to determine whether solutions are reasonable.

Students initial experiences with fraction multiplication should rely heavily on using visual models. These models allow students to build a conceptual understanding of how and why multiplication with fractions results in resizes through partitioning. Students gained experience with set models, number lines and area models while working with whole numbers. Students in grades 3 worked with linear and area models to explore fraction understandings. In grade 4, students worked with all three models with denominators of 2, 4, 5, 6, 8, 10, 12 and 100. These same models are used to illustrate fraction multiplication. For more challenging problems, the area model is particularly useful. Students are able to create a rectangle with fractional side lengths or visually demonstrate using the Distributive Property to multiply mixed numbers (Common Core Standards Writing Team (CCSWT), 2013).





Connecting visuals with operations will be important to help students understand why the U.S. traditional algorithm for multiplying fractions works. Memorized procedures are likely to be forgotten or confused with other operations. Students must be able to compute with fractions flexibly and accurately. Success with fractions in particular is closely related to success in Algebra (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). An example of flexible thinking to solve a fraction multiplication problem is shown below. In 5th grade this student is using the Commutative Property to switch the numerators on the fractions and create a much easier problem. In Algebra 2, use of the U.S. traditional algorithm would result in a harder problem and incorrect solution. Student's conceptual understanding of this content is crucial to their future success in higher levels of mathematics.



(Beckam & Waddell, 2017)

Math Practice 1: Make sense of problems and persevere in solving them

Focus on opportunities for students to develop *Mathematical Practice 1* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 8-8. Reference the Teacher's Edition (TE, pp. F21-F21A) and the NVACS (2010, p.6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	Associative Property *Iteration* *Unit Fraction*

Additional terminology that students may need support with: annex zeros

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Can students use a model to demonstrate and explain how a denominator partitions both factors when multiplying with a fraction?"
"Are students applying knowledge of multiplication to estimate products when multiplying with fractions?"

Lesson	Evidence	Look for
8-2	*Math Practices and Problem Solving* (student work samples) Item 11	Focus CTC around the big idea: • reasonable estimate for the context of the situation. • understanding of the connection between multiplying with a fraction and finding parts of a set.
8-2	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
8-4	*Homework and Practice* (student work samples) Item 16	Focus CTC around the big idea: • ability to estimate a product based on the sizes of factors.
8-4	*Convince Me!* (student work samples)	Focus CTC around the big idea: • appropriate use of an area model to multiply two fractions. • ability to demonstrate and explain how denominators partition the whole.

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 517-522	Use *Scoring Guide* TE pp. 517-522C

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 8-1: Use Models to Multiply a Whole Number by a Fraction		
5.NF.B.4a 5.NF.B.6 MP.2 MP.3 MP.4 MP.6 MP.7	Access Prior Learning: In 4th grade students multiplied a whole number by a fraction using visual models and equations (4.NF.B.4). Developing the Big Idea: Students will build conceptual understanding of multiplying with a fraction. A visual model is connected to the repeated addition interpretation of multiplication.	Solve and Share: In 4th grade students iterated or repeatedly added with unit fractions to find that a fraction such as $\frac{3}{4}$, could be decomposed to ($\frac{1}{4}$, $+\frac{1}{4}+\frac{1}{4}$). The number line model used in this problem can represent the context as 5 jumps of $\frac{1}{2}$. This model helps students connect multiplying with fractions to repeated addition and iterating with unit fractions. Visual Learning: The *Visual Learning Bridge* uses bar diagrams to model repeated addition as multiplication with a whole number and a unit fraction The example above ($\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$) can be reinterpreted as 3 x $\frac{1}{4}$. This understanding allows students to use the Associative Property to reinterpret whole number by fraction multiplication. If $\frac{3}{4}$ is written as (3 x $\frac{1}{4}$), then 4 x $\frac{3}{4}$ can be reinterpreted as 4 x (3 x $\frac{1}{4}$). Now the whole numbers can be multiplied, (4 x 3) x $\frac{1}{4}$, to create 12 x $\frac{1}{4}$. Since the fraction is a unit fraction, students will see that it partitions the whole into 4 equal parts. This creates the fraction 12/4 or 12 ÷ 4. Assess and Differentiate: Use fraction strips to model repeated addition as fraction multiplication in *Another Look,* the *Homework and Practice* page, the *Intervention Activity* and the *Reteach* page. Students can try using this model with concrete tools; as well as, representations such as Teaching Tool 13 (fraction strips).

Lesson 8-2: Use Models to Multiply a Fraction by a Whole Number		
5.NF.B.4a 5.NF.B.6 MP.2 MP.3 MP.4	Access Prior Learning: Students multiplied a whole number by a fraction in the previous lesson. Developing the Big Idea: Students build conceptual understanding of multiplication with fractions. They will multiply a fraction by a whole number using models and equations.	Solve and Share: Students view that 5 x $\frac{1}{2}$ could be interpreted as adding $\frac{1}{2}$ five times in the previous lesson. Multiplying a fraction by a whole number is similar because of the Commutative Property, yet the meaning changes. $\frac{2}{3}$x 6 means to take $\frac{2}{3}$ of a set of 6. Look for students representing the context of this problem correctly using visual models and equations. Facilitate a discussion connecting the different strategies. What happens to the numbers in this problem? The *Look Back*! asks why multiplying a fraction and a whole number results in a product smaller than the whole number. Visual Learning: Bar diagrams are used to visually demonstrate multiplying a fraction by a whole number. Draw out the idea that the denominator partitions the whole amount while the numerator determines how many parts should be taken. The *Convince Me*! gives students a chance to practice using a set model. These models will also be used on the *Guided and Independent Practice* pages. Fraction strips or other concrete tools can also help students complete these problems with understanding. Assess and Differentiate: The *Reteach* page uses an area model instead of set models. Consider bringing this model into the class discussion earlier in the lesson to allow comparison and analysis by all students. Does this accurately represent all contexts? How is it similar to a set model? Some students will prefer the area model as they become more efficient. *CTC: *Math Practices and Problem Solving* (student work samples) Item 11 *CTC: *Quick Check* (digital platform)
Lesson 8-3: Multiply Fractions and Whole Numbers		
5.NF.B.4a MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students multiplied fractions and whole numbers in previous lessons. Developing the Big Idea: Students will build procedural skill through practice. They will be shown two abstract strategies for multiplying a fraction and a whole number.	Solve and Share: Consider starting this lesson with the *Look Back!* to give students the opportunity to practice reasoning and sense making with this relatively new content. Look for students modeling and solving this problem using a range of strategies. Facilitate a class discussion using student work to connect visual models and abstract strategies. Why do we need both multiplication and division when multiplying fractions and whole numbers? Visual Learning: The *Visual Learning Animation* shows two methods of multiplying a whole number by a fraction. The first example builds on understanding that fractions can be interpreted as a whole number times a unit fraction and incorporates use of the Associate Property. The second method demonstrated is the U.S. traditional algorithm for multiplying fractions. Students should justify why it works with visual models. Placing a 1 under a whole number to create a fraction can create misconceptions. Students may need clarification with the idea that a/b = a ÷b. This means that 7/1 is equivalent to 7 ÷ 1 which is still a whole 7. Consider having students work on a problem such as problem 9 on the *Independent Practice* (SE, p. 471). Use strategic questioning to draw out student understanding around the idea that it is possible to use the denominator to partition the whole number before multiplying with the numerator. Ask, "When is this an appropriate strategy for this problem type?"
Lesson 8-4: Use Models to Multiply Two Fractions		
5.NF.B.4a MP.2 MP.4 MP.6	Access Prior Learning: Students used models to multiply two decimals in Topic 4. Developing the Big Idea: Students build conceptual understanding using two visual models to multiply fractions.	Solve and Share: Students can literally model the context of this problem if they fold a piece of paper in half, color $\frac{1}{4}$ of one side and then determine how much of the paper is colored. Other students will represent the problem using fractions strips or visual representations. Help students connect the meaning of multiplication to the numbers in this problem. Why is the solution smaller than either of the factors? This question is also explored in the *Look Back!.* Visual Learning: Students may need clarification on why the overlap created on the area model is the product. This is an excellent opportunity to think more deeply about the meaning of multiplication. If multiplying 3 x 4 means to take 3 sets of 4, then $\frac{1}{2}$ x $\frac{3}{4}$ means to take $\frac{1}{2}$ of a set $\frac{3}{4}$. We are taking a "part of a part" which is shown by the overlap on the area model. Students are given opportunity to practice using an area model with the *Convince Me!* problem. Encourage students to experiment with various models while completing the *Guided and Independent* practice problems to build conceptual understanding. *CTC: *Homework and Practice* (student work samples) Item 16 *CTC: *Convince Me!* (student work samples)

Lesson 8-5: Multiply Two Fractions		
5.NF.B.4a MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students multiplied two fractions in the previous lesson. Securing the Big Idea: Students build procedural skill through practicing use of strategies to multiply two fractions.	Solve and Share: Look for students using a range of strategies including models and equations to solve this problem. Facilitate a class discussion to help students connect the models to the equations. How do the models represent the equations and vice versa? How do both represent the context of the problem? Visual Learning: This revisits the idea of estimation to determine reasonableness of solutions. The U.S. traditional algorithm for multiplying two fractions is shown. How does this algorithm connect to other models and strategies? Giving answers in simplest form is not required by the 5th grade standards. This allows students to focus on building conceptual understanding and procedural fluency. Although, it might be appealing to present solutions in simplest form, there is no mathematical imperative to do so. Problems 31-34 on *Math Practices and Problem Solving* offer opportunity for students to demonstrate understanding of multiplication with fractions (SE, p. 484). Assess and Differentiate: *Homework and Practice* page problems 1-6 require use of the U.S. traditional algorithm. Consider moving students using other strategies to problems 7-24. Do not rush students to use the U.S. traditional algorithm, focus on building conceptual understanding. Use a small sample of problems to check for understanding before moving students to more challenging problems. Watch for students using subtraction. Consider scaffolding problem context by replacing fractions with whole numbers.
Lesson 8-6: Area of a Rectangle		
5.NF.B.4b MP.1 MP.2 MP.3 MP.5 MP.6	Access Prior Learning: Students used area models to multiply whole numbers, decimals and fractions in previous topics. Developing the Big Idea: Students build conceptual understanding and procedural skill by connecting use of an area model to the standard algorithm for multiplying two fractions.	Solve and Share: A hundredths grid is provided in the Student Edition. Students may prefer to draw their own area model to represent this problem. However, important understandings about multiplication can be built through use of the hundredths grid. For example, some students may realize that they will need to create a 4 x 4 array (representing 1 whole yard) within the grid to solve this problem. This connects to multiplying the denominators when using the U.S. traditional algorithm. Visual Learning: Using area models with fractional side lengths is modeled. Draw out the idea that the denominators help to determine the size of a single unit (a tile) and the numerators how many units (or tiles) can be used to fill in the total area. These visual models demonstrate how the U.S. traditional algorithm works to multiply fractions. Students are given models to practice with on the *Guided and Independent Practice* page. Help students connect the models to the procedures used in the U.S. traditional algorithm. Assess and Differentiate: Review of finding area using whole numbers is given on the *Reteach* page.
Lesson 8-7: Multiply Mixed Numbers		
5.NF.B.6 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: In previous topics students used area models to multiply multi-digit whole numbers and decimals. In previous lessons students used area models to multiply fractions. Developing the Big Idea: Students build conceptual understanding and connect previous learning to multiplying mixed numbers through use of an area model.	Solve and Share: Students may initially have difficulty deciding how to multiply mixed numbers. Watch for students that only multiply the whole numbers and only multiply the fractions. Help these students to think about what making a quantity 2 and 3 times bigger means. Asking students if there is more than one way to represent a mixed number may lead some to rename the mixed number as a fraction greater than one, and multiply. Some students may use repeated addition to make the quantity greater. Facilitate a discussion to share student ideas and celebrate how they were able to apply what they know about multiplying fractions to this new content. Visual Learning: An area model is used to multiply two mixed numbers. Students have seen this model used with whole numbers and decimals. It allows use of the Distributive Property to multiply mixed numbers and create partial products. It is an effective model for building conceptual understanding and procedural skill for these problem types. Renaming the mixed numbers as fractions greater than one is also demonstrated. Do not rush to proceduralize this understanding, focus on building understanding of what this means and why this works. *Independent Practice* page problems 2-9 require use of this strategy (SE, p.495). Assess and Differentiate: *Homework and Practice* page problems 1-4 require students to rename mixed numbers. Problems 5-12 remind students to estimate and allow use of multiple strategies (SE, p. 497).

Lesson 8-8: Multiplication as Scaling		
5.NF.B.5a 5.NF.B.5b MP.2 MP.7	Access Prior Learning: In previous lessons students have multiplied with whole numbers, decimals, fractions and mixed numbers. Securing the Big Idea: Students strengthen conceptual understanding of multiplication and increase procedural precision through reasoning about the size of a product based on the sizes of the factors.	Solve and Share: Students will likely want to compute the problems to find the correct solution. However, the goal of this lesson is to strengthen student's number sense and understanding of multiplication as scaling. Building on what they know about multiplication, students can use reasoning to determine the size of each product based on the size of the factors. Can students justify their solutions? Orchestrate a discussion to draw out the idea that in a multiplication problem, the first factor can be interpreted as a quantity while the second factor is a scaler that changes the size of the quantity to create a product. Visual Learning: Multiplication by fractions less than 1 and greater than 1 are discussed. These problems reveal an important understanding for students about the relationship between the size of the factors and the product. Can students use a model to prove these ideas? If students are having difficulty using reasoning to complete the *Independent Practice* problems, model estimating the size of a product using whole number factors and then easier fractions such as $\frac{1}{2}$. How do these same ideas carry over to working with mixed numbers? Assess and Differentiate: The *Intervention Activity* gives students a chance to discuss and justify their thinking about the sizes of products. Use of number lines is modeled on the *Reteach* page and *Homework and Practice* page.
Lesson 8-9: Math Practices and Problem Solving- Make Sense and Persevere		
5.NF.B.6 5.NF.B.5a 5.NF.B.5b MP.1 MP.3 MP.4 MP.6	Access Prior Learning: Students have practiced the thinking habits of MP.1 in previous lessons and grades. Students have made sense and persevered solving word problems in previous lessons and grades. Securing the Big Idea: Students apply knowledge of multiplication to solve a real world multi-step problem.	Solve and Share: Consider starting with the *Look Back!* to remind students to estimate before calculating. Look for students making sense of this problem and modeling the context. Can they explain how their model represents the context of the problem? Is there more than one way to solve this problem? How can estimation help to determine the reasonableness of their solutions? Facilitate a class discussion focused on how students are using the thinking habits of MP.1 to solve this problem. Celebrate students' growth and success as problem solvers. Visual Learning: A multi-step problem is modeled and solved. How are the thinking habits modeled in the *Visual Learning Bridge* similar to those students celebrated during the *Solve and Share*? Multi-step real world problems are given on the *Independent Practice* and *the Math Practices and Problem Solving* page. Can students explain why their strategy choices are appropriate for these problems? How did they create a plan before solving?

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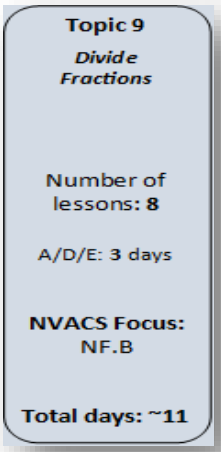
Grade 5 Topic 9: Apply Understanding of Division to Divide Fractions



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Numbers and Operations-Fractions (pp. 11-14)

Prior to instruction, view the Topic 9 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 455A-455F), the Topic Planner (pp. 523A-523C), all 8 lessons, and the Topic Assessments (pp. 581-581A).

Mathematical Background: Read Topics 8-9 Cluster Overview/Math Background (TE, pp. 455A-455F)	Topic Essential Question: How are fractions related to division? How can you divide with whole numbers and unit fractions? *Reference Answering the Topic Essential Questions (TE, pp. 579-580) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

9-1	9-2	9-3	9-4	9-5	9-6	9-7	9-8	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on Nevada Academic Content Standards (NVACS) cluster 5.NF.B; "Apply and extend previous understandings of multiplication and division to multiply and divide fractions" (2010). Topic 9 will specifically target:

- 5.NF.B.3; Interpret a fraction as division of the numerator by the denominator *(a/b) = a ÷ b*. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fractions models or equations to represent the problem (NVACS, 2010).

- 5.NF.B.7a; Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3 (NVACS, 2010)*.

- 5.NF.B.7b; Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4 (NVACS, 2010)*.

- 5.NF.B.7c; Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? (NVACS, 2010).*

Topic 9 moves students from multiplying with fractions to dividing. In 5th grade, the focus is on strengthening students' understanding of division and its connection to multiplication. It is critical that students engage in the actual modeling and/or representation of the division process. Representational models are critical tools for extending understandings related to the division of whole numbers to this new content, division of whole numbers and unit fractions (Fennell, Topic 9 PD video enVisionmath2.0).

In fifth grade, students only work with whole numbers and unit fractions. They will not be asked to divide a fraction by a fraction until sixth grade. This boundary allows students the time to build conceptual understanding before they are asked to apply abstract algorithms. At this point students do not need to learn the U.S traditional algorithm for dividing fractions. Moving students too quickly to that level of abstraction can be detrimental to their learning. Division of fractions is one of the most misunderstood and mysterious algorithms in mathematics. To avoid this mystery, we need to help students really understand when and how to divide fractions (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The importance of teaching this content for understanding rather than focusing on procedures first is reiterated by Francis Fennell, "You may have heard, ours is not to wonder why, just invert and multiply. Our challenge, and our responsibility, is to ensure that all students understand how and why these procedures work. And that is so important" (Topic 9 PD video enVisionmath2.0).

A common misconception is that division will always make numbers smaller (Karp, Bush & Dougherty, 2014). Students tackled a related misconception about multiplication in Topic 8 (see Topic 8 instructional note).

Students need to think about the roles of the dividend and the divisor in relation to context to determine if their answers make sense. They should be able to determine what is being shared and how many equal shares are needed. Modeling the context of a problem can help students decide if their answer is reasonable and understand why.

Math Practice 8: Look for and express regularity in repeated reasoning

Focus on opportunities for students to develop *Mathematical Practice 8* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 9-8. Reference the Teacher's Edition (TE, pp. F28-F28A) and the NVACS (2010, p.8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	unit fraction *dividend* *divisor* *quotient* *inverse operation*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students using models to represent division situations containing whole numbers and fractions?"
"Are students using multiplication to estimate and determine if quotients are reasonable?"

Lesson	Evidence	Look for
9-3	*Homework and Practice* (student work samples) Item 13	Focus CTC around the big idea: • ability to connect visual model with a division expression. • use of multiplication to check reasonableness of a quotient.
9-3	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
9-6	*Solve and Share* (student work samples)	Focus CTC around the big idea: • student strategies and models used to represent the context. • use of multiplication to check reasonableness of a quotient.
9-6	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 579-582	Use *Scoring Guide* TE pp. 579-582A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 9-1: Fractions and Division		
5.NF.B.3 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In 4th grade students learned that a multiple of a/b is 1/b (4.NF.B) Students divided whole numbers and decimals to find quotients and remainders in previous topics. Developing the Big Idea: Students build conceptual understanding of division through use of tools, models and procedures to explore dividing two whole numbers where the quotient is a whole number and a fraction.	Solve and Share: A wide range of strategies will be used to solve this problem. Some students will partition every waffle into fourths while others will share 1 whole waffle and only partition the remaining two. Even here, students may partition by halves or fourths. Yet, the share of 1 person is always 6/4. Use students' models and thinking to analyze this pattern and discuss about how division is related to fractions. Draw out the idea that when equal/fair sharing, the total amount is partitioned by the number of shares. Students are building towards the idea that fractions and division are the same, or that $a \div b = a/b$. Students can test their strategies and conjectures using the *Look Back!* problem which eliminates one of the waffles. Visual Learning: An equal/fair sharing problem is modeled using visual representations including a number line. How do visual representations help to solve these problems? Division problems interpreted as fractions are modeled. Can students generalize this idea to create a mathematical rule? Students can test a new model/strategy using the *Convince Me!* problem.

		The *Independent Practice* problems are practice rewriting division problems as fractions and vice versa. Items 18-21 provide context, which can help students make sense of division and its connection to fractions. Assess and Differentiate: The third activity in the *Intervention* Activity uses whole number Partitive division problems to help students think about the relationship between division and fractions. These problem types are an excellent jumping off point for this concept and could be used whole class, in small groups, or to formatively assess student's current understanding. Students can use the context of these equal/fair sharing problems to think about the quantity being shared and the number of shares needed.
Lesson 9-2: Fractions and Mixed Numbers as Quotients		
5.NF.B.3 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students worked with mixed numbers in Topics 7 and 8 and whole number division in Topic 5. Students explored dividing whole numbers when the quotient is a fraction in the previous lesson. Developing the Big Idea: Students continue to build conceptual understanding by dividing whole numbers when the quotient is a whole number and a fraction.	Solve and Share: Look for students using a wide range of strategies and facilitate a discussion using students thinking and explanations. Using visual models will build conceptual understanding since students can see how a whole number is partitioned to create a quotient with a fraction. Some students may solve this problem by setting up a division problem (8 ÷ 5) and then reinterpreting this as a fraction ($\frac{8}{5}$). Can students explain how this model is appropriate for this context? What does an answer of $\frac{8}{5}$. mean in this context? Visual Learning: Students will see a problem modeled with division and then reinterpreted as a fraction. Can students explain why this rule is appropriate? What does a solution with a fraction mean? Students can practice reasoning with this new content using the *Convince Me!* problem which asks for a solution range instead of an exact answer. Students will need to understand how to rename fractions as mixed numbers and fractions greater than one, in order to complete the *Independent Practice* items. Modeling this change with fraction strips will help to build understanding. Assess and Differentiate: Students may need assistance using the model provided on the *Reteach* page as it includes several steps. As an extension, consider having students create a context for questions 2-9 on the *Homework and Practice* page (SE, p. 537).
Lesson 9-3: Use Multiplication to Divide		
5.NF.B.7b 5.NF.B.7c MP.2 MP.4 MP.7	Access Prior Learning: Students have connected multiplication and division in previous grades and topics. Developing the Big Idea: Student build conceptual understanding of division with fractions by connecting to multiplication as a means to check a quotient.	Solve and Share: The context of this problem allows students to use multiplication understandings to model and solve. However, students will find that division is also used, as each sandwich is partitioned into fourths. This means that the quotient will also be partitioned into fourths. Look for students utilizing models and use their strategies to facilitate a class discussion. Draw out the idea and that when you divide a number of things (*a*) into equal parts (*b*), you get *a x b* parts. Therefore, *a ÷ (1/b) = a x b*. Use the *Look Back!* to challenge students to model this problem with an equation. Visual Learning: Students see that when dividing wholes by a fraction, more parts are created because each whole is partitioned. This understanding allows students to use multiplication to solve these types of problems. Challenge students to analyze the patterns visible between multiplication and division and create a rule for dividing a whole number by a fraction. Have students test their rule using the models provided on questions 6-9 on the *Independent Practice* page (SE, p. 541). Item 18 on the *Math Practices and Problem Solving* page can be used to formatively assess students. This item asks students to represent a context using both a multiplication and a division equation. Assess and Differentiate: The *Intervention Activity* uses fraction strips to help students visualize the connection between multiplication and division. These tools can be used to great effect during all parts of this lesson. *CTC: *Homework and Practice* (student work samples) *Item 13* *CTC: *Quick Check* (digital platform)

Lesson 9-4: Divide Whole Numbers by Unit Fractions		
5.NF.B.7b 5.NF.B.7c MP.1 MP.2 MP.4 MP.5	Access Prior Learning: Students divided a whole number by a fraction in the previous lesson. Developing the Big Idea: Students use models and procedures to build a conceptual understanding of division with fractions by connecting to multiplication.	Solve and Share: A grouping problem is given with a context that leads to using circle fractions. Consider providing Teaching Tool 14 (circle fraction models) to encourage use of a visual model to solve this problem. Can the circle models represent a division and multiplication equation? How does the division equation also represent the context of this problem? Visual Learning: A Measurement division problem is modeled with circle fractions and a number line. This type of problem helps students see how fractional chunks can be subtracted or added up to form a larger total. Students see why the quotient becomes larger than the dividend or divisor. Can this problem be solved with multiplication? Students can test their ideas using the *Convince Me!* problem. Use of a number line or visual model is encouraged for solving the *Guided and Independent Practice problems.*
Lesson 9-5: Divide Unit Fractions by Non-Zero Whole Numbers		
5.NF.B.7a 5.NF.B.7c MP.2 MP.3 MP.4 MP.5 MP.8	Access Prior Learning: Students have learned about the meanings of division as well as how fractions can partition wholes in previous grades and topics. Developing the Big Idea: Students use models to build conceptual understanding of dividing a unit fraction by a whole number.	Solve and Share: Ask students to estimate before solving. If 4 people are sharing a half, will the quotient be larger or smaller than $\frac{1}{2}$? Students can use concrete tools and visual models to represent the context of this problem. Facilitate a discussion to compare students' strategies and solutions. What happens to the quotient when a unit fraction is partitioned into equal shares? Can this problem be represented using a multiplication fact? Visual Learning: Dividing a unit fraction by a whole number is shown using an area model (pan of cornbread) and a number line. Help students connect these strategies to using multiplication. The *Convince Me*! can be used to further that discussion. Consider assigning a small number of items from the *Independent Practice* to enable students have time to try various models. Encourage students to connect their models to a related multiplication fact. Assess and Differentiate: Partitioning a rectangular area model is show on *Another Look!* Students should explore using this model, as well as, the number line to solve the *Homework and Practice* problems.
Lesson 9-6: Divide Whole Numbers and Unit Fractions		
5.NF.B.7a 5.NF.B.7b 5.NF.B.7c MP.1 MP.2 MP.4 MP.8	Access Prior Learning: Students divided unit fractions and whole numbers in previous lessons. Developing the Big Idea: Students connect procedures to models and drawings to strengthen conceptual knowledge and build procedural skill with dividing unit fractions and whole numbers.	Solve and Share: Look for students using visual models and numerical expressions to represent this problem. Facilitate a discussion to help students connect the different representations. Are students using representations to make sense of the problem before calculating? How can a numerical expression and a visual model represent the same context mathematically? Visual Learning: The *Visual Learning Bridge* demonstrates use of an area model and a number line. How is this similar or different to strategies shared by students during the *Solve and Share*? Help students to make connections between the models and the numerical expression representing the problem. Can students generalize about the size of a quotient based on whether the dividend is whole number or a fraction? Assess and Differentiate: On the *Homework and Practice* pages, students practice using models to solve problems with unit fractions divided by a whole number and a whole number divided by a unit fraction. *CTC: ***Solve and Share*** (student work samples) *CTC: ***Quick Check*** (digital platform)
Lesson 9-7: Solve Problems Using Division		
5.NF.B.7c MP.1 MP.2 MP.4 MP.6	Access Prior Learning: Students divided unit fractions and whole numbers in previous lessons. Securing the Big Idea: Students apply knowledge of dividing unit fractions and whole numbers to solve word problems.	Solve and Share: Students should work to model and create a plan before solving this multi-step problem. Share student strategies and explanations to draw out ideas about how good problem solvers make sense of a problem. Visual Learning: Students might need additional time and strategic questioning support to help them make sense of this problem. Are students able to create a model of their own that matches the context of this problem? *-continues on next page-*

		The remainder is reported as R6 in the Visual Learning Bridge. Students in 5th grade will understand that a remainder is actually a fractional piece. Can they write this remainder correctly as a fractional piece? Watch for students who represent all fractions as tenths. They might have misconceptions of fractional pieces due to over generalizing the base-ten system. Assess and Differentiate: The *Reteach* page breaks a word problem into smaller pieces. It may be difficult for students to complete this independently because the context of the problem can be lost. Consider skipping the reteach section and instead assigning a single (or small number) of *Homework and Practice* items so that students have time to explain their strategies and solutions. Writing their thinking and sharing can help to strengthen mathematical reasoning.
Lesson 9-8: Math Practices and Problem Solving- Repeated Reasoning		
5.NF.B.7a 5.NF.B.7b 5.NF.B.7c MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning: Students have practiced the thinking habits of MP.8 in previous topics. Students have divided unit fractions and whole numbers in previous lessons. Securing the Big Idea: Students apply conceptual understanding of division and multiplication to analyze equations and generalize a mathematical rule.	Solve and Share: Students look for patterns in the computations when dividing a unit fraction by a whole number and then dividing the whole number by the same unit fraction. Have students create and test mathematical conjectures leading to a rule, using division and multiplication understandings. What do students discover about the relationship between these computations? Can these ideas also be connected to multiplication? Visual Learning: The discussion started in the *Solve and Share* is carried into the *Visual Learning Bridge*. Hold off showing the last part of the video until students have had a chance to make their own generalizations about the patterns. The relationship between dividing and multiplying whole numbers and unit fractions is explained and shown numerically. The *Guided Practice, Independent Practice*, and *Math Practices* and *Problem Solving* pages ask students to apply the relationships modeled in the *Visual Learning Bridge* and discovered in the *Solve and Share*. Assess and Differentiate: *Another Look!* shows equations with division and multiplication facts. Students are asked to make a generalization about each set. Consider using this table earlier in the lesson to help students connect multiplication to division as part of their generalizations.

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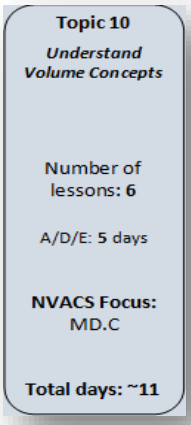
Grade 5 Topic 10: Understand Volume Concepts



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Geometric Measurement (pp. 26-28)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 583A-583F), the Topic Planner (pp. 583I-583J), all 6 lessons, and the Topic Assessments (pp. 629-630A).

Mathematical Background: Read Topics 10 Cluster Overview/Math Background (TE, pp. 583A-583F)	Topic Essential Question: What is the meaning of volume of a solid? How can the volume of a rectangular prism be found? *Reference Answering the Topic Essential Questions (TE, pp. 627-628) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	10-5	10-6	Assessment

5 A/D/E days used strategically throughout the topic

Instructional Note:

Instruction is focused on Nevada Academic Content Standards (NVACS) cluster 5.MD.C; "Geometric measurement; understand concepts of volume and relate volume to multiplication and to addition," (2010). This cluster is composed of eight individual standards:

- 5.MD.C.3- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- 5.MD.C.3.a- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- 5.MD.C.3.b- A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
- 5.MD.C.4- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.C.5- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 5.MD.C.5.a- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- 5.MD.C.5.b- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- 5.MD.C.5.c- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Volume can be a very challenging concept for fifth graders and much is asked by the eight standards for this cluster. As stated in the Progression Documents for the Common Core Math Standards;

> "Volume not only introduces a third dimension and thus a significant challenge to students' spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container," (Common Core Standards Writing Team (CCSWT), 2012 p. 26).

How can we provide the opportunities students need to form a conceptual understanding of volume that will transfer to real world problem solving? The Progression Documents and researchers agree that students will need experiences with concrete tools that allow them to extend their spatial understanding to include a third dimension. Caldwell claims, "Building prisms with unit cubes is an important step in building an understanding of volume as an attribute of a three dimensional shape that describes the space inside it," *(Topic 10 PD video enVisionmath2.0).* From these experiences, students "learn to mentally decompose and recompose a right rectangular prism built from cubes into layers…given the prism, they have to be able to decompose it, understanding that it can be partitioned into layers, and each layer partitioned into rows, and each row into cubes" (CCSWT, 2012, p.26).

The *Solve and Share* problem in the first lesson of Topic 10 shows a picture of a 4x1x1 rectangular prism. Will students connect this two dimensional representation to a real world three dimensional figure? Building the shape allows students the experience that will push their thinking into three dimensions. The *Visual Learning Bridge* in the same lesson will add a layer to this prism. This can also

be modeled with physical tools to help students observe the connection between surface area and volume. Understanding that volume can be decomposed into layers is very important to understanding the additive nature of volume; as well as, applying volume formulas (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Conceptual understanding of volume builds towards understanding volume formulas. This occurs as student reasoning develops and increases in sophistication and efficiency. Students will increasingly apply multiplicative reasoning to determine volumes by looking for and making use of structure such as the number of cubes in a single layer. Then, students will be ready to learn the formulas for computing volume for right rectangular prisms (CCSWT, 2012). Students may view the formulas as efficient methods connected to their previous work and experiences with finding the number of unit cubes that pack into a right rectangular prism. Having developed the understandings behind formulas in meaningful ways, students come to abstract their understandings and thus are no longer required to memorize these as isolated pieces of mathematical fact. Instead, students are able derive formulas from what they already know (Van de Walle, Karp, & Bay-Williams, 2010).

Consider switching the *Solve and Shares* from *Lessons 10-2* and *10-3* so that your students can use manipulatives with the *Solve and Share* for *Lesson 10-3* and build multiple rectangular prisms with the same volume. This hands-on experience allows students to see how the dimensions of length, width, and height combine to create volume using a visual model. They can bring this conceptual understanding to the *Solve and Share* for 10-2 which deals with a liquid and pushes students to apply multiplication to find the volume.

Math Practice 5: Use appropriate tools strategically

Focus on opportunities for students to develop *Mathematical Practice 5* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 10-6. Reference the Teacher's Edition (TE, pp. F25-F25A) and the NVACS (2010, p.7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
volume cubic unit rectangular prism unit cube formula Base	*cube* *area* *perimeter*

Additional terminology that students may need support with: *layers, additive*

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Do students recognize the attributes of a solid figure that can be used to find volume? Do they recognize that volume can be decomposed into layers (connecting to understanding of area)?"
"Are students able to find the volume of non-overlapping right rectangular prisms by decomposing the shape into right rectangular prisms and adding the volumes of the parts?"

Lesson	Evidence	Look for
10-1	*Math Practices and Problem Solving* (student work samples) Items 14 through 18	Focus CTC around the big idea: • student strategies used to find volume. • understanding that volume can be composed/decomposed as layers.
10-1	*Quick Check* (digital platform) Items 1, 2 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
10-4	*Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10	Focus CTC around the big idea: • student strategies and models used to find the volume of the figure. • understanding that volume of individual right rectangular prisms can be added together to find volume of a larger shape.

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 627-630	Use *Scoring Guide* TE pp. 627-630A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 10-1: Model Volume		
5.MD.C.3a 5.MD.C.3b 5.MD.C.4 MP.2 MP.5 MP.7	Access Prior Learning: Students worked with measurement concepts including perimeter and area in previous grades. This is the first time they will work with the concept of volume. Beginning the Big Idea: Students work with rectangular prisms composed of unit cubes to form a conceptual understanding of volume.	Solve and Share: Students may not realize the two-dimensional drawing represents a three-dimensional figure. **Building the prism with unit cubes or similar tools** will help push spatial thinking to include a third dimension. How are students figuring out the dimensions of the prism such as height, width and length? Ask students to look at their rectangular prisms from different viewpoints such as side view, top view, and front view. Can they connect these viewpoints to dimensions such as length, width, and height? Students may need clarification that the volume of single unit cube is 1. How many unit cubes compose the larger rectangular prism? What does this mean about its volume? Watch for students answering 24 cubes. Why do we not count all the cubes shown in the picture (the same cube may be shown more than once from multiple perspectives)? Visual Learning: A layer is added to the rectangular prism. Students may realize it is possible to find the new volume using multiplication. They can test their thinking by counting the unit cubes. Consider using the essential question to draw out student thinking about what volume means and how it can be determined. The *Guided and Independent Practice* items show pictures of rectangular prisms built with unit cubes. Some students may have trouble visualizing the unit cubes on the unseen side of the prisms. Using tools to build the prisms will allow students to view the prisms from all sides. Assess and Differentiate: The *Homework and Practice* page includes two questions, 8 and 9, composed of more than one rectangular prism. Have students use tools to build these shapes and explore the additive nature of volume. The table shown for questions 10-12, ask students to find different configurations for a rectangular prism with a volume of 12. Have students use the given numbers to build the shapes and observe how the sides change. What relationships can they discover between volume and the dimensions of length, width and height? *CTC: *Math Practices and Problem Solving* (student work samples) Items 14 through 18 *CTC: *Quick Check* (digital platform) Items 1, 2 and 5
Lesson 10-2: Develop a Volume Formula		
5.MD.C.5b 5.MD.C.4 5.MD.C.5a MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students learned about the connection between area and multiplication in previous grades and topics. In the previous lesson students explored the concept of volume. Developing the Big Idea: Students build conceptual understanding of volume and begin to build procedural skill through exploring the connection between multiplication and volume.	Solve and Share: The context of this problem may need clarification. In the previous lesson students worked with unit cubes (solid) while this problem asks about capacity. What are we trying to find out about the fish tank? How can this be modeled using tools? If the water is emptied out of the tank, could unit cubes be used to find the tank's volume? Consider using student strategies to facilitate a discussion about meanings of volume and how it can be measured. Why do we record volume measurements as cubic units? Visual Learning: The formula for volume *l x w x h* is demonstrated. The first representation shown has the unit cubes drawn. How would the volume of this shape change if a layer were taken off? If one was added? Such questions might help students see that volume is composed of layers and connected to area. The *Guided and Independent Practice* items move to larger numbers making use of tools difficult. Question 7, uses smaller numbers and can be built with unit cubes. This will allow students to test the formula with a hands on experience. Assess and Differentiate: Consider providing additional opportunities to build rectangular prisms with unit cubes and identify the dimensions used to find volume.

Lesson 10-3: Volume of Prisms		
5.MD.C.5a 5.MD.C.5b MP.1 MP.2 MP.4 MP.7 MP.8	Access Prior Learning: Students learned about the connection between area and multiplication in previous grades and topics. In the previous lesson students explored the connection between volume and multiplication. Developing the Big Idea: Students build procedural skill through exploring the connection between surface area and volume.	Solve and Share: Students are asked to find 5 possible configurations of a rectangular prism with a volume of 24 cubic units. Consider asking students to build the different models. How can shapes have the same volume if they look different and have different measurements? Facilitate a discussion to draw out observations that connect finding volume to surface area and multiplication. The ***Look Back!*** fixes the height at 2 and asks students to determine possible lengths and widths. Use this question to draw out generalizations connecting area to volume. Visual Learning: The formula for volume *V= B x h* is shown. How does this formula connect to generalizations made by students during the *Solve and Share*? When a layer is removed or added, which of the dimensions of a rectangular prism are changed? Item 15 on the *Math Practices and Problem Solving* page can be used as part of the formative assessment process to check for understanding of the volume formula *V= B x h.* Assess and Differentiate: The *Homework and Practice* page models using the *V= B x h* formula using a representations showing the unit cubes. The *Intervention Activity* gives students a hands on experience with unit cubes to explore why the volume formula works.
Lesson 10-4: Combine Volume of Prisms		
5.MD.C.5c MP.1 MP.2 MP.4 MP.7	Access Prior Learning: Students learned that area is additive in previous grades. Students worked to find the volume of rectangular prisms in previous lessons. Developing the Big Idea: Students apply understanding to find the volume of non-overlapping rectangular prisms.	Solve and Share: Consider providing Teaching Tool 19 (combining volumes) for student use. Students are asked to find the volume of two non-overlapping rectangular prisms. The word "combining" used in the problem may help students to create a plan. Concrete tools can also be used to model two smaller rectangular prisms. What happens to the volume of the individual prisms when they are moved together and then pulled apart? How will students decide which dimensions can be used to create two individual prisms from this shape? Does it make sense to use all of the measurements given? Why or why not? Is there more than one way to find the volume of these two combined prisms? The *Look Back!* asks students to give the dimensions they used for each of the shapes. Use of geometric language to describe attributes of the rectangular prisms will help students identify the appropriate dimensions needed to find volume. Visual Learning: An example of splitting two non-overlapping rectangular prisms into two separate rectangular prisms is shown. The *Convince Me!* asks students if they can find a different way to separate the shape and calculate volume. *Guided Practice* and *Independent Practice*: The items give more measurements than students need to find the volume one way. Consider asking students to make a plan and justify their thinking for one or more shapes before calculating. Assess and Differentiate: *Another Look!* on the *Homework and Practice* models pulling two non-overlapping rectangular prisms apart calculating a volume for each before adding the volumes together. *CTC: *Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10
Lesson 10-5: Solve Word Problems Using Volume		
5.MD.C.5c MP.2 MP.3 MP.4 MP.8	Access Prior Learning: Students found the volume of rectangular prisms in previous lessons. Securing the Big Idea: Students build procedural skill through application to real world problems.	Solve and Share: Look for students modeling the context of the problem and applying knowledge of volume to compute a solution. A class discussion using student ideas and strategies may draw out the idea that modeling problems and making a plan helps us to apply our knowledge to new problems. Visual Learning: A problem with a similar context to the *Solve and Share* is modeled. Is there more than one way to solve this type of problem? Encourage students to try various strategies when working on the questions that are similar to this on the *Independent Practice* page. Question 7 shows three non-overlapping rectangular prisms pushed together. Why is the remainder ignored? Assess and Differentiate: *Another Look!* gives a volume problem with a missing dimension. Students must use knowledge of the volume formula to find the height. However, the context uses "depth" instead of height so clarification may be needed.

Lesson 10-6: Math Practices and Problem Solving- Use Appropriate Tools		
5.MD.C.3a 5.MD.C.3b 5.MD.C.4 MP.1 MP.2 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: Students have used tools throughout Topic 10. Students found volume measurements for rectangular prisms in previous lessons. Securing the Big Idea: Students use knowledge of volume concepts to create a plan and choose an appropriate tool to solve a real world problem.	Solve and Share: Students can be very creative with this problem and test their knowledge of volumes additive nature. There are many possible solutions. Consider asking students to share not only their solutions but their choice of mathematical tools and how it helped them solve this problem. Visual Learning: Do students agree with the tool choices modeled to solve the problem in the *Visual Learning Bridge*? Are there other possibilities that could work? The *Guided Practice* problem only shows two steps but asks students to find the volume for 5 steps. How will students find the volume of the missing steps? Look for a wide range of strategies including use of concrete tools, representations, additive/multiplicative thinking and algebraic expressions. Assess and Differentiate: The *Intervention Activity* poses a design challenge and assigns tools to groups. This task could be modified (consider reducing the cubic units needed based on available tools) and used whole class as well.

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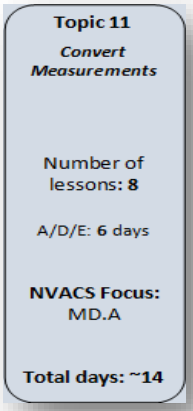
Grade 5 Topic 11: Convert Measurements



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Geometric Measurement (pp. 26-28)

Prior to instruction, view the Topic 11 Professional Development Video located in Pearson Realize online. Read the Teacher Topic Edition (TE): Cluster Overview/Math Background (pp. 631A-631F), the Topic Planner (pp. 631I-631K), all 8 lessons, and the Topic Assessments (pp. 693-694A).

Mathematical Background: Read Topics 11 Cluster Overview/Math Background (TE, p. 631A-631F)	Topic Essential Question: What are customary measurement units and how are they related? What are metric measurement units and how are they related? *Reference Answering the Topic Essential Questions (TE, pp. 691-692) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

11-1	11-2	11-3	11-4	11-5	11-6	11-7	11-8	Assessment

6 A/D/E days used strategically throughout the topic

Instructional Note

Instruction is focused on Nevada Academic Content Standards (NVACS) cluster 5.MD.A; "Convert like measurement units within a given measurement system" (2010). This cluster contains one standard:

- 5.MD.A.1- Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi step real world problems.

Topic 11 will also rely on application of understandings built from working with standards 5.MD.A.2, 5.NBT.B.5 and 5.NBT.B.6 (NVACS, 2010).

Topic 11 focuses on converting measurements of length, capacity and mass/weight of the customary system and the metric system. A big idea for this topic is equivalence. Students need to understand that any measure can be represented in multiple ways and have the same value. Students also need to have understanding of the attributes of the units used for measurement. Understanding attributes of measurement units includes knowing that length is used for linear two dimensional measurements, capacity measures three dimensions, mass measures the amount of matter in an object, while weight measures how heavy something is (using Earths gravitational pull). The differences between length, capacity, mass and weight determine how they are used. Confusion around these attributes can prevent students from sense making while converting measurements.

In addition to the attributes, students will also need to know the relative sizes of the units used for measurement within the metric and customary systems. The difference between an inch and a foot is much smaller than the difference when comparing a yard to a mile. Knowing the relative size of each unit is crucial for deciding which units to use and if an answer to a conversion problem is reasonable. Then, to convert units of measurements students will need to choose the appropriate operation, set up the problem and calculate. Read more about how these understandings fit together in Topic 11, refer to Teacher's Edition (pp. 631A-631E) in the *Volume 2 Teacher's Edition* of enVisionmath2.0.

In the *Topic 11 Professional Development video*, Caldwell offers insight on helping students build and apply the multiple understandings required. A question to ask students could be "If you measure something in centimeters and then in meters, which measure will be a greater number?" *(enVision*math2.0). This simple question raises many more possible questions for students and can be used to gauge current understandings about measurement. To begin, students need to know the relative sizes of meters and centimeters. What attributes are meters and centimeters measuring? What does centi- tell us about units in the metric system? How does this relate to place value? Which operation would be appropriate to make this conversion? Many students will want to know why we would measure something in different units. How can we use context to support understanding of this problem?

Estimation is a powerful tool that helps students build conceptual understanding of measurement units and their comparisons. Estimation allows students to focus on the attribute measured while building familiarity with the units. Using estimation also provides intrinsic motivation because students like to see how close they can get to the actual measurement. Estimation is highly recommended as an ongoing activity to promote understanding (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Once students have an understanding of the units of measurement and equivalency, they still need to plan and perform calculations to convert the units. Which operations are used? This will be an important question to help students understand early in this topic.

Visual representations are an excellent tool for students to see why and how multiplication and division are used for making measurement conversions. A bar diagram allows students to visually represent the relationship between customary units with differing

conversion rates such as yards, feet, and inches. When working in the metric system, it will benefit students to appreciate the connections between converting metric units and our base-10 place value system. Connecting the prefixes used in the metric system to the base 10 place value system will allow students to apply previously built understandings and strategies to convert metric units.

Topic 11 brings in new mathematical content and requires the transfer and application of previous understandings. If students initially struggle with these concepts, consider spending more time on fewer problems. Taking time to build a classroom chart listing visible benchmarks of measurement units will enhance students' ability to compare units and make sense of conversion problems. Scaffolds such as physical tools, benchmarks, and visual representations will help students build the conceptual understanding of measurement conversions needed to make sense of problems and apply appropriate strategies and methods.

Math Practice 6: Attend to precision

Focus on opportunities for students to develop *Mathematical Practice 6* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 11-8. Reference the Teacher's Edition (TE, pp. F26-F26A) and the NVACS (2010, p. 7). This topic relies heavily on knowledge of unit vocabulary, reference the vocabulary cards found in the *Teachers Edition Volume 2 (pp. 633-638)*.

Essential Academic Vocabulary Use these words consistently during instruction.		
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)	
Ton (t)	*foot (ft.)* *inch (in.)* *yard (yd.)* *mile (mi.)* *capacity* *gallon(gal)* *quart (qt)* *pint (pt)* *cup (c)* *fluid ounce (fl. oz)* *weight* *pound (lb.)*	*ounce (oz)* *kilometer (km)* *meter (m)* *centimeter (cm)* *millimeter (mm)* *liter (L)* *milliliter (ml)* *mass* *milligram (mg)* *gram (g)* *kilogram (kg)*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding questions: "Do students recognize customary units of measure, the relative sizes and relationships between them? Do students recognize metric units of measure, the relative sizes and relationships between them?
"Are students using multiplication to convert from larger units to smaller units? Are students using division to convert from smaller units to larger units?"

Lesson	Evidence	Look for
Topic 11	*Review What You Know* (student work samples) Items 17 through 21	Focus CTC around the big idea: • understanding of the relative sizes of given units. • understanding that different units are used to measure specific attributes such as length, capacity, and mass.
11-1	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
11-2	*Homework and Practice* (student work samples) Items 19 and 20	Focus CTC around the big idea: • student strategies and models used to convert measurements. • use of the correct operation to convert measurements.
11-4	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 691-694	Use *Scoring Guide* TE pp. 691-694A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 11-1: Convert Customary Units of Length		
5.MD.A.1 5.NBT.B.5 5.NBT.B.6 MP.2 MP.4 MP.6 MP.8	Access Prior Learning: Students learned about size of customary units of length. Students also converted larger to smaller units using multiplication (4.MD.A.1). Developing the Big Idea: Students extend understanding of converting units of measurement to include converting smaller to larger units. Students build conceptual understanding through expressing relationships of equivalence between units of measurement.	Solve and Share: Students may use vocabulary cards, rulers, yardsticks, or Teaching Tool 17 to answer and support their answer. Expressing relationships between units to show equivalence is an important strategy to draw out. This method will help students throughout Topic 11. The *Look Back!* Introduces the new concept of dividing to convert measurements. How can we determine whether to multiply or divide when converting measurements? Visual Learning: The *Visual Learning Bridge* models both multiplication and division. The video includes reasoning about remainders found when converting units. How can the remainder be represented? (fraction, decimal, etc.). The *Convince Me!* also models representing a remainder. Are students able to generalize about how the sizes of the units can be used to determine whether to multiply or divide when making conversions? Is it more important to pay attention to the numbers ***or*** the sizes of the units to make conversions? Assess and Differentiate: The *Intervention Activity* has students use Teaching Tool 17 or a ruler to manipulate measurements and conceptualize conversion (SE, p. 643A). *CTC: *Quick Check* (digital platform)
Lesson 11-2: Convert Customary Units of Capacity		
5.MD.A.1 5.NBT.B.5 5.NBT.B.6 MP.2 MP.8	Access Prior Learning: Students in 4th grade converted larger to smaller units (4.MD.A.1). In Topic 9, students divided two whole numbers and got a fraction of a mixed number quotient. Developing the Big Idea: Students will multiply and divide to convert customary units of measurements.	Solve and Share: Students use context to make sense of the problem. Would dividing or multiplying be more appropriate to create an equivalent expression? Use the *Look Back!* to facilitate a discussion about why moving to smaller units means larger numbers of the unit are needed. Students may make a connection to creating smaller products when multiplying a fraction. Visual Learning: Converting smaller to larger units and larger to smaller units is shown. The problems model how to convert with a fraction and having a fraction as part of the answer. Students work to create expressions of equivalence on the *Guided and Independent Practice* items. Assess and Differentiate: The *Intervention Activity* assists students to develop conceptual understanding of the relative size of the units (SE, p. 649A). In addition, students also will have the ability to physically move, fill, and remove the containers. The "Toss and Talk" *Center Activity* allows students to practice conversions in a game setting. The 2-star version of the game offers more of a challenge for those students who demonstrate an understanding of converting. *CTC: *Homework and Practice* (student work samples) Items 19 and 20
Lesson 11-3: Convert Customary Units of Weight		
5.MD.A.1 5.NBT.B.5 5.NBT.B.6 MP.4 MP.5 MP.6 MP.8	Access Prior Learning: Students converted customary units by multiplying and dividing in previous lessons. Developing the Big Idea: Students will continue to practice converting customary units.	Solve and Share: Students need to convert a mixed number. Look for a variety of strategies used to model converting the fraction. Are students able to connect these models and written methods to those learned while working with fractions during earlier topics? Are students able to explain how they decided whether to multiply or divide and why their solution is reasonable? Consider focusing on mixed numbers as a review. How else can you represent this number? Visual Learning: The *Visual Learning Bridge* discusses converting a smaller unit to a larger unit. Two whole numbers are being divided but the quotient results in a fraction. A possible student misconception is that the greater number must be divided by the smaller number. Why is this not always true? Can students find different strategies to represent and solve this conversion?
Lesson 11-4: Convert Metric Units of Length		
5.MD.A.1 5.NBT.A.2 MP.2 MP.3	Access Prior Learning: In previous topics, students multiplied and divided by powers of 10. In previous lessons students converted units of measurements.	Solve and Share: The problem asks students to use a ruler and discover patterns between the units of measurements. The *Look Back!* extends the pattern seen in the *Solve and Share*. Can students describe the relationships between these units using mathematical reasoning? How can place value understanding help to solve this problem? *-continues on next page-*

MP.5 MP.7	Developing the Big Idea: Students will multiply and divide to convert metric length measurements.	Visual Learning: The metric system uses different units, yet deciding when to multiply ***versus*** divide still applies. How do the prefixes used in the metric system help to determine size of the units (kilo = thousand)? What connections can be made between metric prefixes and place value? The *Convince Me!* provides an opportunity to analyze mathematical thinking and support a claim. Assess and Differentiate: Students demonstrating understanding will find more challenge moving to problems on the *Math Practices and Problem Solving* (SE, p. 660) or *Homework and Practice* (SE, p. 662). *CTC: *Quick Check* (digital platform)
Lesson 11-5: Convert Metric Units of Capacity		
5.MD.A.1 5.NBT.A.2 MP.2 MP.3 MP.7 MP.8	Access Prior Learning: In previous lessons students converted units of measurements. Developing the Big Idea: Students will convert metric units of capacity.	Solve and Share: Look for different strategies and written methods used to solve this problem. Orchestrate a discussion focusing on specifically selected student strategies. Can students make connections between each of the different strategies and determine if they are appropriate for the given context? The *Look Back!* problem is Partitive division. The number of groups is given, so students need to use strategies to create equal sized groups. Visual Learning: Students think more about how to determine whether to multiply or divide. The movement of the decimal is discussed when multiplying or dividing by powers of 10. The *Convince Me!* asks students to think about relative size of measurements and order from greatest to least. *Independent Practice* items 9-22 focus on use of procedural skills. Problems on the *Math Practices and Problem Solving* instead provide students' opportunity to apply understanding to real world contexts (SE, p. 666). Assess and Differentiate: The *Another Look!* page demonstrates converting metric units using base-ten blocks.
Lesson 11-6: Convert Metric Units of Mass		
5.MD.A.1 5.NBT.A.2 MP.1 MP.2 MP.7	Access Prior Learning: Students converted units of measurement in previous lessons. Developing the Big Idea: Students will convert metric units of mass.	Solve and Share: Look for a range of strategies and written methods. How are students modeling the problem? Can they explain how they determined an appropriate operation? Consider facilitating a discussion allowing students to share and explain their thinking. Visual Learning: Students think more about how to determine whether to multiply or divide. The movement of the decimal is discussed when multiplying or dividing by powers of 10. Assess and Differentiate: The *Reteach* page helps students create a reference chart used to help set up metric conversions and solve problems with metric units of mass. Use *Math Practice and Problem Solving* (SE, p. 672) and *Homework and Practice* page (SE, p. 674) for students who need an additional challenge.
Lesson 11-7: Solve Word Problems Using Measurement Conversions		
5.MD.A.1 5.NBT.B.5 MP.1 MP.2 MP.3 MP.6 MP.8	Access Prior Learning: In grade 3 students solved problems relating to perimeter (3.MD.D.8). Students in previous lessons have converted units of measurements. Securing the Big Idea: Students solve real word problems by converting units of measurements.	Solve and Share: This problem asks students to convert measurements and give the perimeter of a picture frame in inches. Use the *Look Back!* to facilitate a discussion about which measurement students converted. Have students justify their reasoning. Can the problem be solved successfully if students convert to the other measurement? Visual Learning: A very similar problem to the *Solve and Share* is modeled. The *Visual Learning Bridge* guides students through solving the multi-step word problem. Can students determine what they do and do not know and use this knowledge to develop a plan before they solve the multi-step problem? Assess and Differentiate: The problems in this lesson are multi-step word problems, formatted in a similar way to the Smarter Balanced Assessment Performance Task. Consider asking students to complete fewer problems in order to focus on showing and explaining thinking. A separate piece of paper will be needed for students to model the problems and show their strategies.
Lesson 11-8: Math Practices and Problem Solving- Precision		
5.MD.A.1 5.NBT.B.5 MP.1	Access Prior Learning: Students have attended to precision in previous lessons and grade levels. Consider facilitating a discussion about.	Solve and Share: Students use a ruler to measure the frame precisely in millimeters. Students apply knowledge learned in previous lessons to answer the problem. Why it is important to be precise when using mathematical tools? When is it important to be precise? *-continues on next page-*

MP.2 MP.4 MP.6	Securing the Big Idea: Students apply understanding to a real world context.	Visual Learning: The problem extends the discussion about the importance of precision when measuring. The *Visual Learning Bridge* attaches mathematical language to the thinking habits of MP.6. The *Convince Me!* extends students thinking to fix a mistake and solve the given problem more accurately.

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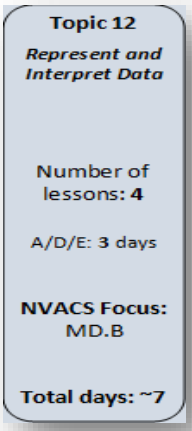
Grade 5 Topic 12: Represent and Interpret Data



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Grades 2-5 Measurement Data (pp. 11-13)

Prior to instruction, view the Topic 12 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp.695A-695F), the Topic Planner (pp. 695I-695J), all 4 lessons, and the Topic Assessments (pp. 729-730A).

Mathematical Background: Read Topics 12 Cluster Overview/Math Background (TE, pp. 695A-695F)	Topic Essential Question: How can line plots be used to represent data and answer questions? *Reference Answering the Topic Essential Question (TE, pp. 727-728) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

12-1	12-2	12-3	12-4	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

Instruction is focused on Nevada Academic Content Standards (NVACS) cluster 5.MD.B, "Represent and Interpret Data" (2010). This cluster contains one standard:

- 5.MD.B.2- Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were distributed equally.*

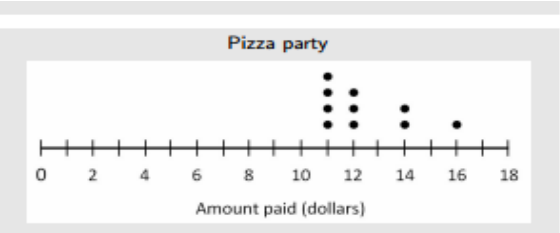
Topic 12 will rely on application of understandings built from working with standards 5.NF.A.2 and 5.NBT.B.6 (NVACS, 2010). Students have prior experience representing and interpreting data going back to first grade and with creating and analyzing line plots since the second grade. Using fractions of a unit is also not unique to fifth grade; it is part of the third and fourth grade standards. Fifth graders have lots of experience with the line plot and are now ready to analyze and work with data at a deeper level.
Karen Karp helps to clarify the intent of standard 5.MD.B.2 by explaining;

> Line plots are a way to organize and represent numerical data collected in a survey. You can use line plots to answer questions about a data set. Students must develop strategies and experience with examining data and unraveling the messages contained within. The interpretation of data is the most important part of the process. (Topic 12 PD video, enVisionmath2.0).

Topic 12 is four lessons in total and the first two lessons (12-1 and 12-2) will review previous grade level standards. Looking for opportunities to push student thinking with questioning will enhance instruction and get to the intent of the standards. After solving the initial problem given, questions such as *"Where is the data clustered? Do more students live close to school or far away?"* and *"How much farther is the longest walk than the shortest walk?"* push students to think more deeply about the data represented on a line plot. Students initial responses to questions such as these are likely to be a reiteration of the data such as "six blocks." Yet, in fifth grade students are capable of making multiplicative comparisons. The longest walk then could be "6 times larger than the shortest walk." Or the difference may represent a whole and a fractional piece such as $2\frac{1}{2}$ times larger. This situation gives students an opportunity to use multiplicative reasoning with fractional pieces. Exploring the mathematical relationships contained within the data set and drawing inferences should be a main focus of instruction during Topic 12. Students are building strategies to effectively interpret data and make decisions.

Standard 5.MD.B.2 also contains something new and unique to fifth grade: *"…For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally"* (NVACS, 2010). The concept of equal sharing is included in the standard. The enVisionmath2.0 lessons for Topic 12 will not cover this part of the standard in a *Solve and Share* or *Visual Learning Bridge*. It is touched upon in question 8 on the *Math Practices and Problem Solving* page for lesson 12-3.

An example of a question to build toward the understanding required by standard 5.MD.B.2 is given in the Grade 2-5 Measurement Data Progression Documents for the Common Core Math Standards on p. 13. This task could be modified and used during instruction to deepen understanding around equal sharing and analyzing data.



Example 3. Fair share averaging

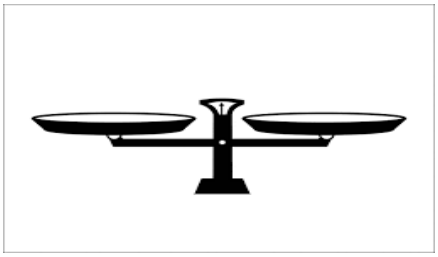
Ten students decide to have a pizza party and each is asked to bring his or her favorite pizza. The amount paid (in dollars) for each pizza is shown in the plot to the right.

Each of the ten is asked to contribute an equal amount (his or her fair share) to the cost of the pizza. Where does that fair share amount lie on the plot? Is it closer to the smaller values or the large one? Now, two more students show up for the party and they have contributed no pizza. Plot their values on the graph and calculate a new fair share. Where does it lie on the plot? How many more students without pizza would have to show up to bring the fair share cost below $8.00?

This same concept could be applied to most of the line plot examples used in Topic 12 by changing the context to contain the idea of equal sharing. Using concrete tools is highly recommended for this task! This task can be very challenging for fifth graders. Some students may pull the data off of the number line to add it up and then fair share the pieces. Challenge students to keep the data on the number line and begin by moving the outer pieces closer to each other. As students become more comfortable with this concept they will naturally begin to use multiplicative reasoning to find a central point for the data. As they progress, they may find the need to create smaller and smaller fractional units on the number line. Questioning can draw out big mathematical ideas learned through working with the data. For example; questions such as "C*an you find where the central point is by looking at how data is clustered? How do outliers change the shape of the data? Do they change where we find the central point? Why?"* ask students to think about data as a whole versus individual pieces of information. This task allows students to begin thinking about data as having a balance point. This interpretation is of data is explained by Van de Walle, Karp and Bay-Williams (2014),

> Balance Point Interpretation: Statisticians think about the mean as a point on a number line where the data on either side of the point are balanced. To help think about the mean in this way, it is useful to think about the data placed on a line plot. What is important is not how many pieces of data are on either side of the mean or balance point but the distances from the mean (p. 451).

Through this work, students are discovering, experiencing and working towards the idea of measures of central tendency (i.e., mean) in a way that will build conceptual understanding and prepare them for later grades.



Math Practice 3: Construct viable arguments and critique the reasoning of others

Focus on opportunities for students to develop *Mathematical Practice 3* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 12-4. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	data *line plot* *outlier*

Additional terminology that students may need support with: *analyze, frequency table*

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Can students use the information in a data set to create a line plot?"
"Are students able to use the information given in a table and/or line plot to solve problems?"

Lesson	Evidence	Look for
12-2	*Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10	Focus CTC around the big idea: • accurate creation of a line plot to display given data. • use of data to solve a problem, use of operations on fractions.
12-2	*Homework and Practice* (student work samples) Items 1 and 2	Focus CTC around the big idea: • accurate creation of a line plot to display given data. • use of data to solve a problem, use of operations on fractions.
12-2	*Quick Check* (digital platform)	• Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 727-730	Use *Scoring Guide* TE pp. 727-730A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 12-1: Analyze Line Plots		
5.MD.B.2 MP.1 MP.2 MP.5 MP.6 MP.7	Access Prior Learning: Students have worked with line plots in previous grades. In 4th grade, students used line plots to display data in fractions of a unit (4.MD.B). Developing the Big Idea: Students build conceptual understanding and procedural skill working with data displayed on a line plot.	Solve and Share: Students use a given line plot to solve a problem. Where does the information used to create a line plot come from? Why is data recorded on a line plot? How do line plots help us to analyze data? Visual Learning: The connection between a data table and a line plot is shown. Why is data organized into a line plot? Can students generalize ideas about when a line plot can be used as a tool for analyzing data? The term outlier is explained. Are students able to make observations about the data as a whole? Fractional units are used throughout the *Guided and Independent Practice* problems. How is reading a line plot similar to working with number lines? Consider items such as 10 and 14 on the *Math Practices and Problem Solving* page to give experience with more in depth questions. Are students able to determine what data is needed to answer the question being asked? Assess and Differentiate: *Another Look!* models using a line plot to find the mode for a set of data. However, it is not necessary to use the term mode in the explanation.
Lesson 12-2: Make Line Plots		
5.MD.B.2 MP.1 MP.2 MP.4 MP.6 MP.7 MP.8	Access Prior Learning: Students have worked with line plots in previous grades. In 4th grade, students used line plots to display data in fractions of a unit (4.MD.B). Developing the Big Idea: Students build procedural skill in making a line plot to display data including fractions of units.	Solve and Share: Students organize a given set of raw data. Look for students creating frequency tables and line plots. Consider facilitating a discussion comparing these two visual representations. How does each help make data easier to analyze? Visual Learning: A similar problem is given and creating a frequency table and a line plot is modeled. Can a frequency tables be used to organize data before creating a line plot? How do these representations of the data help to solve problems? The *Look Back!* asks a question that can be answered using a line plot display. *-continues on next page-*

		Assess and Differentiate: The *Intervention Activity* asks a question and student responses to gather information then use this to make a frequency table and a line plot. This activity can be used whole class or with small groups. *CTC: *Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10 *CTC: *Homework and Practice* (student work samples) Items 1 and 2 *CTC: *Quick Check* (digital platform)
Lesson 12-3: Solve Word Problems Using Measurement Data		
5.MD.B.2 5.NF.A.2 5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.8	Access Prior Learning: Students have worked with line plots in previous grades and lessons. Securing the Big Idea: Students build procedural skill using data from a line plot to solve problems.	Solve and Share: Students are asked to make observations about what is different in the amounts of rainfall using data in a line plot. This open ended question may generate varying solutions. Consider using student observations to facilitate a discussion comparing found differences. Were similar operations used to find the differences? How do we know which operations to apply when analyzing data? Visual Learning: Using multiplication/repeated addition is modeled as a strategy to solve a problem using data found in a line plot and a frequency table. Are students able to draw connections between the methods used in the *Visual Learning Bridge* and their own strategies used during the *Solve and Share*? *Guided and Independent Practice* items focus on modeling a problem with an equation to solve a problem. On item 5 on the *Independent Practice* page, look for students adding up the numbers from the line plot and ignoring the frequency. Why will this method not work for this problem type? Assess and Differentiate: *Another Look!* gives a problem very similar to the *Visual Learning Bridge* with room on the frequency table for students to apply multiplication and calculate values to solve a problem. The *Reteach* page reviews using operations with fractions before asking students to apply and solve problems using a frequency table.
Lesson 12-4: Math Practices and Problem Solving- Critique Reasoning		
5.MD.B.2 5.NF.A.2 5.NBT.B.6 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students have used the thinking habits of MP.3 in previous grades and lessons. Students have worked with line plots in previous grades and lessons. Securing the Big Idea: Students apply knowledge of line plots and use habits of MP.3 to solve a real world problem.	Solve and Share: Students critique the reasoning of given statements using a line plot. Consider facilitating a discussion focused on student using mathematical reasoning to justify their explanations. How are they modeling the thinking habits of MP.3? Visual Learning: The habits of MP.3 are modeled to solve a problem using data contained in a line plot. Can students connect the thinking modeled in the *Visual Learning Bridge* to their own thinking and explanations shared during the *Solve and Share*? Students may see the thinking they used explained in written form with precise mathematical vocabulary. Assess and Differentiate: *Another Look!* Provides a problem very similar to the *Visual Learning Bridge.* Have students practice the habits modeled on the items on the *Homework and Practice* page.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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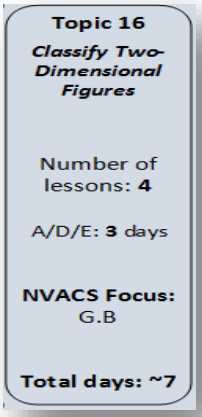
Grade 5 Topic 16: Geometric Measurement: Classify Two-Dimensional Figures



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: K-6 Geometry, (pp. 17-18)

Prior to instruction, view the Topic 16 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 845A-845F), the Topic Planner (pp.845I-845J), all 4 lessons, and the Topic Assessments (pp. 881-882A).

Mathematical Background: Read Topics 15 Cluster Overview/Math Background (TE, pp. 845A-845F)	Topic Essential Question: How can triangles and quadrilaterals be described, classified, and named? *Reference Answering the Topic Essential Questions (TE, pp. 879-880) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

16-1	16-2	16-3	16-4	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

Instruction for Topic 16 focuses on Nevada Academic Content Standards (NVACS) cluster 5.G.B, "Classify two-dimensional figures into categories based on their properties" (2010). The cluster contains two standards:

- 5.G.B.3- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

- 5.G.B.4- Classify two-dimensional figures in a hierarchy based on properties.

The WCSD Pacing Frameworks moves Topic 16 between Topics 9 and Topic 10. The 2017-2018 WCSD pacing committee, composed of Washoe County teachers, made the decision to move this topic. The reasoning is stated on page 2 of the pacing framework as "because two-dimensional figures lead into the understanding of three-dimensional figures and volume. Students will need to have a strong understanding of the attributes of polygons to decompose solid figures when volume is additive."

Instructional use of mathematical language and vocabulary is important to consider during Topic 16. Students will learn to identify, draw and classify two-dimensional shapes based on their attributes. All students benefit from an increased focus on language in mathematics, especially when the language support is connected to the mathematics (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The use of mathematical language plays a crucial role as students build understandings necessary to analyze and relate categories of triangles and quadrilaterals based on their attributes.

The Frayer Model, a vocabulary graphic organizer, is provided in the enVisionmath2.0 instructional materials as Teaching Tool 24. It is beneficial for English Language Learners to support the connection of ideas and relationships. For the development of conceptual understanding, knowing which properties that an object does not include is just as important as knowing which properties are included (Dunston and Tyminski, 2013).

The Progression Documents provide an example of how a Venn diagram can be used to model quadrilaterals existing in several classes at once (Common Core Standards Writing Team (CCSWT), 2013, p. 18). Use of the Venn diagram supports student observation of how properties defining a shape in one category extend to all subclasses. For example, students placing a square into the diagram will find that a square possesses all the properties of a rectangle in addition to the special attributes that define it as a square. Visuals can enhance students' abilities to use multiple attributes to classify a single shape. Students are learning that shapes exist in a hierarchy and the properties of a given category extend to its subsets as well. Students may need clarification about the meaning of hierarchy. Referencing real world contexts that use a hierarchy may help students to build understanding of how hierarchies are used for classification.

Students should explore these hierarchies using the two definitions for trapezoids. See figures below:

• Note that in the U.S., that the term "trapezoid" may have two different meanings. In their study *The Classification of Quadrilaterals* (Information Age Publishing, 2008), Usiskin et al. call these the exclusive and inclusive definitions:

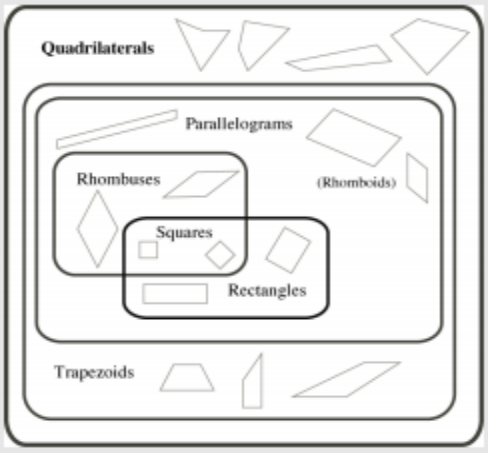
T(E): a trapezoid is a quadrilateral with exactly one pair of parallel sides

T(I): a trapezoid is a quadrilateral with at least one pair of parallel sides.

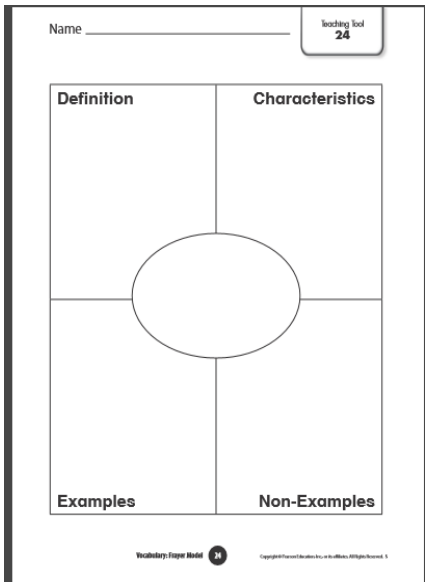
These different meanings result in different classifications at the analytic level. According to T(E), a parallelogram is not a trapezoid; according to T(I), a parallelogram is a trapezoid.

Both definitions are legitimate. However, Usiskin et al. conclude, "The preponderance of advantages to the inclusive definition of trapezoid has caused all the articles we could find on the subject, and most college-bound geometry books, to favor the inclusive definition."

Venn diagram showing classification of quadrilaterals



Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref "T(E)"]).



Math Practice 3: Construct viable arguments and critique the reasoning of others
Focus on opportunities for students to develop *Mathematical Practice 3* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 16-4. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p. 6).

This topic has 4 lessons with 2 additional days for assessment, differentiation and enrichment (A/D/E). Consider using the provided enrichment time to focus on geometric vocabulary used during Topic 16. Teaching Tool 27 (word map) and vocabulary cards (TE, pp. 847-850) are provided to support vocabulary acquisition.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	equilateral triangle *isosceles triangle* *scalene triangle* *right triangle* *acute triangle* *obtuse triangle* *trapezoid* *parallelogram* *rectangle* *rhombus* *square*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to name and classify two-dimensional shapes as a hierarchy?"
"Do students recognize that a shape retains all of the attributes of subcategories in a hierarchy?"

Lesson	Evidence	Look for
16-3	*Solve and Share* (student work samples)	Focus CTC around the big idea: • are students understanding that a shape can belong to more than one category? • are students able to identify the two-dimensional shapes?
16-3	*Convince Me!* (student work samples)	Focus CTC around the big idea: • can students give specific examples of how a two-dimensional? • shape can belong to more than one category in a hierarchy?
16-3	*Quick Check* (digital platform)	• Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 879-882	Use *Scoring Guide* TE pp. 879-882A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 16-1: Classify Triangles		
5.G.B.3 5.G.B.4 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: In 4th grade, students learned about geometric measurement of angles and classified shapes based on properties (4.MD.C) (4.G.A). Developing the Big Idea: Students use properties to analyze and classify triangles to build conceptual understanding.	Solve and Share: Students work to draw and classify several triangles based on their properties. Consider asking students to share their ideas and justify their solutions (MP.3). What properties are used to classify triangles? How are these attributes described using mathematical language? Consider discussing the image of the musical instrument triangle, why would this shape not be classified as a triangle? Visual Learning: Mathematical terms for the properties of triangles are given. Can any descriptions shared during the *Solve and Share* be connected to mathematical terminology? Students are more likely to remember vocabulary if they can make connections. Consider exploring concepts and labeling rather than front loading vocabulary. Use the *Convince Me!* to assist students in thinking about what combinations of attributes are possible for a single triangle. For example, is it possible for a triangle to be isosceles and have a right angle? Why or why not? Assess and Differentiate: A breakdown of attributes similar to what was shown in the *Visual Learning Bridge* is given on the *Reteach* page.
Lesson 16-2: Classify Quadrilaterals		
5.G.B.3 5.G.B.4 MP.1 MP.2 MP.3 MP.6 MP.8	Access Prior Learning: In 4th grade, students learned about geometric measurement of angles and classified shapes based on properties (4.MD.C) (4.G.A). Developing the Big Idea: Students use properties to analyze and classify quadrilaterals to build conceptual understanding.	Solve and Share: Students may need to review the meaning of parallel lines to begin this task. Are students able to use geometric vocabulary to classify their created shapes? Consider using student ideas to facilitate a discussion gathering students' ideas about attributes of quadrilaterals. Visual Learning: We use geometric vocabulary to classify quadrilaterals. Can students connect these terms to descriptions shared during the *Solve and Share*? The *Convince Me!* asks students to use knowledge of attributes to distinguish between a parallelogram and a rhombus. The *Guided Practice* reiterates that shapes retain the properties of subclasses. Multiple attributes are listed for single shapes. During the *Independent Practice*, support students that stop after naming a single attribute of the shapes (SE, p. 859). Prompt students to analyze the shapes for additional attributes. *-continues on next page-*

		Assess and Differentiate: The *Homework and Practice* page contains a classification hierarchy beginning with a trapezoid and ending with a square. Do students understand that each shape after the trapezoid has all of the attributes of the classes before it, as well as the attributes that make it unique?
Lesson 16-3: Continue to Classify Quadrilaterals		
5.G.B.3 5.G.B.4 MP.2 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: In 4th grade, students learned about geometric measurement of angles and classified shapes based on properties (4.MD.C) (4.G.A). Students analyzed properties to classify quadrilaterals in the previous lesson. Developing the Big Idea: Students continue to build conceptual understanding of quadrilaterals through analyzing properties to determine that the properties of a two-dimensional shape belong to all subcategories of that shape.	Solve and Share: Students use knowledge of the properties of quadrilaterals to classify. Do students understand that quadrilaterals can have many attributes? How are the quadrilaterals related? What properties define some quadrilaterals as unique? Visual Learning: Notice the visual hierarchy for classifying quadrilaterals. This diagram might remind students of a family tree. How does this organizer help to classify quadrilaterals? Students must decide if statements about quadrilaterals are true or false on the *Guided Practice* and *Independent Practice* pages. Can students use geometric vocabulary to justify their thinking? Assess and Differentiate: See the classification hierarchy on the *Reteach* page and the *Homework and Practice* page. *CTC: ***Solve and Share*** (student work samples) *CTC: ***Convince Me!*** (student work samples) *CTC: ***Quick Check*** (digital platform)
Lesson 16-4: Math Practices and Problem Solving- Construct Arguments		
5.G.B.3 5.G.B.4 MP.1 MP.2 MP.3 MP.6 MP.7	Access Prior Learning: Students analyzed properties to classify triangles and quadrilaterals in previous lessons. Students worked with MP.3 in previous lessons. Securing the Big Idea: Students apply knowledge of properties of shapes to solve a problem and work to model the thinking habits of MP.3.	Solve and Share: Students work to construct an argument to justify a solution. Consider providing concrete tools such as paper and scissors that will allow students to test their ideas. A class discussion focused on the thinking habits of MP.3 can be used to share strategies and explain thinking. Visual Learning: The thinking used to solve a similar problem is modeled. How do the thinking habits shown connect to student ideas shared during the *Solve and Share*? The *Convince Me!* offers an opportunity for students to form a counter example. How can counter examples be helpful for classifying and constructing arguments? Assess and Differentiate: Students use the thinking habits of MP.3 to construct an argument to justify a statement about the measure of angles in a triangle using the task given on *Another Look!*.

References

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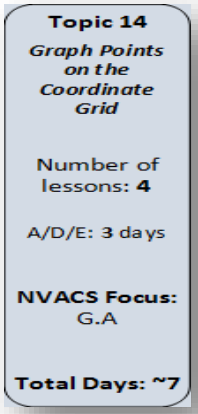
Grade 5 Topic 14: Graph Points on the Coordinate Plane



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: K-6 Geometry, (pp. 17-18)

Prior to instruction, view the Topic 14 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 773A-773F), the Topic Planner (pp. 773I-773J), all 4 lessons, and the Topic Assessments (pp. 807-808A).

Mathematical Background: Read Topics 14 Cluster Overview/Math Background (TE, pp. 773A-773F)	Topic Essential Question: How are points plotted? How are relationships shown on a graph? *Reference Answering the Topic Essential Questions (TE, pp. 805-806) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

14-1	14-2	14-3	14-4	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses instruction on Nevada Academic Content Standards (NVACS) cluster 5.G.A; "Graph points on the coordinate plane to solve real-world and mathematical problems" (2010). Two standards make up this cluster:

- 5.G.A.1; Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

- 5.G.A.2; Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Fifth grade is the first time students will have formal instruction on graphing points in the coordinate plane. Many students have prior experiences with this content because it is commonly integrated into other content areas such as science or used to play popular games. These experiences may have familiarized students with the basics of using coordinate grids. However, students might still lack an understanding of conventions such as beginning at the origin, knowing that an ordered pair always contains an X-coordinate and a Y-coordinate, and that an ordered pair indicates a location on a two-dimensional plane.

A common misconception for fifth graders using the coordinate plane is confusing the x and y values. Students mistakenly try to move a point up on the y-axis before moving over on the x-axis. An analogy to help students remember the convention correctly could be to think about trying to reach a window on the second story of a house. The ladder always starts from where it is stored (the origin). Then the ladder must be moved underneath the window (along the x axis) before it is climbed (moving up the y axis). Notice that this analogy connects formal mathematical language to a real world situation.

Students might wonder about the purpose of the coordinate plane and the conventions needed for its use. Sharing real world examples of using coordinates to describe locations and represent data can help students to understand its uses and its importance as a mathematical tool. Students may be surprised to learn that computer graphics are created using the same ideas they will be studying during Topic 14! Students will use this knowledge extensively in middle school mathematics and beyond (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

In fifth grade, students use only the first quadrant which contains positive numbers on both the x and y axis. In sixth grade, students will use all four quadrants and work with negative numbers.

Although Topic 14 will not ask students to use fractional ordered pairs, the idea may be raised by students thinking about how to represent real world contexts. The Progression Documents for the Common Core Math Standards (2013) also mention fractions and state, "students extend their knowledge of the coordinate plane, understanding the continuous nature of two-dimensional space and the role of fractions in specifying locations in that space" (p. 17). In lesson 14-2, the growth of a plant is shown to always occur in whole numbers. Yet, in real life, measurements often contain fractional pieces of units. How would this data be represented on a coordinate plane? Fifth graders have had lots of practice with fractions and using number lines. Have students extend this understanding to observe that the x and y axis are essentially two number lines placed perpendicularly and used to describe a location in two-dimensional space. From this understanding, have students decide how a fractional ordered pair can be placed.

Students will discover that real world relationships can be represented as a table of ordered pairs and as points on the coordinate grid. These representations are used to analyze information and relationships in order to solve problems. Students will practice the 3 ways to use these representations to solve problems. Students can:

- continue to generate ordered pairs and create a ratio table,
- find the relationship between the x and y coordinates for y given any value of x, or
- extend the line on the coordinate plane to find a solution for any given value of x or y.

Do students realize that when they create ordered pairs and use them to graph a line, every single ordered pair on that line may represent a solution? These strategies are different ways of representing and analyzing information which can be used to quantify and solve real world problems.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop *Mathematical Practice 2* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 14-4. Reference the Teacher's Edition (TE, pp. F22-F22A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
coordinate grid ordered pair x-axis y-axis origin x-coordinate y-coordinate	

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to accurately place ordered pairs on a coordinate grid?"
"Are students using the information represented in a graph to solve real world problems?"

Lesson	Evidence	Look for
14-2	*Math Practices and Problem Solving* (student work samples) Item 19	Focus CTC around the big idea: • explanation of conventions for graphing a point on a coordinate plane. • inclusion of "origin" as a starting point for placing a point on a coordinate plane.
14-2	*Quick Check* (digital platform) Items 1, 3 and 4	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
14-3	*Math Practices and Problem Solving* (student work samples) Items 7 and 8	Focus CTC around the big idea: • correct placement of points on the coordinate plane. • understanding that coordinates represent information for a given situation which can be used to solve problems.
14-3	*Quick Check* (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 805-808	Use *Scoring Guide* TE pp. 805-808A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 14-1: The Coordinate System		
5.G.A.1 MP.2 MP.3 MP.4 MP.5 MP.6	Access Prior Learning: Students have experience finding points on number lines and representing data using line plots from previous grades and lessons. Developing the Big Idea: Students learn procedures for graphing in the first quadrant of a Cartesian coordinate plane.	Solve and Share: Students are asked to plot points on a coordinate grid and then describe where the points are located to a partner. Look for partner pairs having difficulty describing locations successfully. How can we use the coordinate grid more accurately? Students are discovering the need for conventions such as x-coordinates and y-coordinates, starting from the origin, and moving over on the x-axis before moving up the y-axis. The *Look Back*! can be used to facilitate a discussion about graphing conventions. Visual Learning: A coordinate grid is modeled for finding locations. What other real world contexts could a coordinate grid be used for? *Math Practices and Problem Solving*: Items 20-24 ask students to find missing coordinates needed to complete a picture of a house. As a challenge, consider asking students to change the coordinates by a consistent amount and re-graph the pairs. For example, add 3 to all x values or to both the x and y values. What happens to the picture of the house? Why? Teaching Tool 20 (coordinate grids) could help students complete this task.
Lesson 14-2: Graph Data Using Ordered Pairs		
5.G.A.1 MP.1 MP.2 MP.3 MP.5 MP.6 MP.8	Access Prior Learning: Students graphed points on the coordinate grid in the previous lesson. Developing the Big Idea: Students practice graphing on the coordinate plane to build procedural skill.	Solve and Share: Look for students using the conventions of graphing. What would happen to their results if they did not follow the conventions? Have students describe the shape they graphed using geometric attributes? Challenge: What happens to the picture of the square if we change all the coordinates by the same amount. Students could begin by multiplying all the coordinates by 2. What happens to the picture of the square? Why? Visual Learning: Students see how data can be recorded in a table and then used to create a graph. Why is the visual representation of data on a graph helpful? Can students use the graph to make predictions about future growth of the plant? Assess and Differentiate: *Another Look!* ask students graph a parallelogram. Consider using this shape for the challenge task noted in the Solve and Share. *CTC: *Math Practices and Problem Solving* (student work samples) Item 19 *CTC: *Quick Check* (digital platform) Items 1, 3 and 4
Lesson 14-3: Solve Problems Using Ordered Pairs		
5.G.A.2 5.G.A.1 MP.4 MP.6 MP.7	Access Prior Learning: Students graphed points on the coordinate plane in the previous two lessons. Developing the Big Idea: Students increase procedural skill and apply knowledge of graphing on the coordinate plane to solve problems.	Solve and Share: Students graph the points in the data table and use the line formed to answer a question. Students often notice that it is possible to use the table to answer the question. How does graphing data as ordered pairs help us to look for patterns and solve problems? Consider asking students where the line will intersect the x-axis. Will they use the data table or the coordinate graph to find this point? Visual Learning: The earnings of Ann and Bill are shown as ordered pairs in a data table and graphed. Can students determine what the coordinates represent using the context of the problem? What relationship between the amounts can be found using the graph? *Guided and Independent Practice*: The problems use ordered pairs in the hundreds and thousands. These items can be used to facilitate a discussion about matching scale of the x-axis and y-axis to the context of problems. *CTC: *Math Practices and Problem Solving* (student work samples) Items 7 and 8 *CTC: *Quick Check* (digital platform)

Lesson 14-4: Math Practices and Problem Solving- Reasoning		
5.G.A.2 5.G.A.1 MP.1 MP.2 MP.4 MP.5 MP.7	Access Prior Learning: Students have used the thinking habits of MP.2 in previous lessons. Students have analyzed and represented data to answer questions in previous grades and lessons. Securing the Big Idea: Students practice using data represented in a table on a coordinate graph to solve a problem.	Solve and Share: Students graph ordered pairs to analyze the data and solve a problem. What is represented by the x-coordinates and y-coordinates? How does the coordinate graph help to solve this problem? Visual Learning: Using a coordinate graph to solve a problem is modeled. Watch for students who create a data table using the given patterns. Orchestrate a whole class discussion around "How does each coordinate represent the context of the problem? If that a point in the data chanced what might that mean? *Independent Practice* items 4-6 require students to extend the pattern beyond what can be displayed on the coordinate grid provided. What generalizations can be made about extending patterns and the graphed line on the coordinate grid? Consider assigning a small sample of items from the *Guided Practice, Independent Practice*, and *Math Practices and Problem Solving* page and asking students to justify their solutions with mathematical reasoning MP.2.

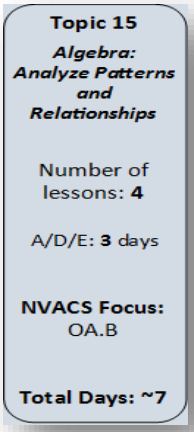
References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Common Core State Standards Writing Team. (2013). *Progressions for the Common Core State Standards in Mathematics (draft). Geometry, K-6.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8*. (2nd ed.). New York, NY: Pearson.

Grade 5 Topic 15: Algebra: Analyze Patterns and Relationships



5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: K-5 Operations and Algebraic Thinking, (pp. 32-35)

Prior to instruction, view the Topic 15 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 809A-809F), the Topic Planner (pp. 583I-583J), all 4 lessons, and the Topic Assessments (pp. 843-844A).

Mathematical Background:	Topic Essential Question:
Read Topics 15 Cluster Overview/Math Background (TE, pp. 809A-809F)	How can number patterns be analyzed and graphed? How can number patterns and graphs be used to solve problems? *Reference Answering the Topic Essential Questions (TE, pp. 841-842) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

15-1	15-2	15-3	15-4	Assessment

3 A/D/E days used strategically throughout the topic

Instructional Note:

This topic focuses on the Nevada Academic Content Standards (NVACS) cluster 5.OA.B: Analyze patterns and relationships and includes one standard:

- 5.OA.B.3- Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane" (2010).

In 4th grade, students used a rule to generate and continue number patterns. In Topic 15, they will extend this knowledge to using rules to generate two separate number patterns. In addition, they will graph lines representing the patterns. The graph will help students identify relationships between terms, make comparisons, and analyze number patterns. Through this process students learn that identifying and representing the relationships between corresponding terms can be used to solve mathematical problems. "This work prepares students for studying proportional relationships and functions in middle school" (Common Core Standards Writing Team (CCSWT), 2011, p. 32).

As students analyze terms, they will discover that relationships can be additive or multiplicative. Students may also observe that some patterns have consistent rates of change starting from different values. Other patterns may start with the same value and change at different rates. Both of these scenarios will be examined through the use of tables and graphing on the coordinate plane.

Although lessons *15-1* and *15-2* will not ask students to graph the ordered pairs found in tables, consider adding this instructional component to help students visually represent the patterns being analyzed. Graphing is an important part of standard 5.OA.B.3 that can aid students' ability to see relationships in data. "The graph allows one to observe at a glance the relationship between (terms). The context gives meaning to the graph, and the graph adds understanding to the context," (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 254). The visual representation allows observations such as "the lines will never intersect" (excluding the point of origin) or "that line is going up faster" (informal observations of slope). These observations can be difficult to make using only the frequency table.

Math Practice 1: Make sense of problems and persevere in solving them

Focus on opportunities for students to develop *Mathematical Practice 1* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 15-4. Reference the Teacher's Edition (TE, pp. F21-F21A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
corresponding terms number sequence	*variable*

Additional terminology that students may need support with:

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to generate a numerical rule from a given mathematical situation?"
"Are students able to analyze the information in a graph to discover patterns and solve a real world problem?"

Lesson	Evidence	Look for
15-4	*Solve and Share* (student work samples)	Focus CTC around the big idea: • explanation of conventions for graphing a point on a coordinate plane. • inclusion of "origin" as a starting point for placing a point on a coordinate plane.
15-4	*Quick Check* (digital platform) Items 1 and 2	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 805-808	Use *Scoring Guide* TE pp. 805-808A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 15-1: Numerical Patterns		
5.OA.B.3 MP.2 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: In 4th grade, students worked with number patterns that followed a given rule (4.OA.C). Developing the Big Idea: Students will build conceptual understanding analyzing number patterns and using discovered relationships to solve a problem.	Solve and Share: Students use the context of the problem to find a rule and complete the tables. While students are likely to focus on the rule of +10, they may also observe patterns between the two tables. Why does Emma always have more money than Jorge? Can this relationship also be expressed as a rule? Consider asking students to graph a line for each table on a single coordinate grid. Can seeing the data as two separate line graphs help students discover relationships? Visual Learning: A similar problem type is modeled using two tables. The vocabulary words *corresponding terms* and *number sequences* are introduced. Consider revisiting the *Topic 15 Essential Question*. How did the tables help to solve these problems? The *Convince Me!* adds the idea that patterns can be extended. Will the Rosemary plant always be taller?
Lesson 15-2: More Numerical Patterns		
5.OA.B.3 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In 4th grade, students worked with number patterns that followed a given rule (4.OA.C). Student used a rule to extend number patterns in the previous lesson. Developing the Big Idea: Students build conceptual understanding analyzing number patterns and using discovered relationships to solve a problem.	Solve and Share: Students can build on observations from the previous lesson to help analyze the two tables. A multiplicative relationship is shown here while the previous lesson used an additive pattern. A different starting point is also given for each pattern. What relationships can be discovered between the two tables? Questioning can help students move from observing differences to noticing relationships between quantities. Visual Learning: The *Visual Learning Bridge* in lesson 15-2 moves from representing two patterns in two separate tables to using the same table to show both patterns. The constant (weeks) is only shown once. Students may need clarification on how to read this table or benefit from seeing the same problem represented in two separate tables.
Lesson 15-3: Analyze and Graph Relationships		
5.OA.B.3 MP.1 MP.3 MP.4 MP.7	Access Prior Learning: Students learned to use a ratio table to represent data and graph points on a coordinate plane during Topic 14. Developing the Big Idea: Students work to build conceptual understanding and procedural skill by extending two patterns, finding relationships, and graphing the points on a coordinate grid.	Solve and Share: Students are asked to graph the ordered pairs after using a rule to complete a table. The table uses muffins for the x- and y-coordinates. Alternatively, students may decide to create two separate graphs using boxes as the independent variable (x-axis) and muffins as the dependent variable (y-axis). Both graph lines could be drawn onto the same coordinate grid. Either graphing strategy will allow students to analyze the relationships in the number sequences. How does the visual representation of the graph help to discover relationships between the terms of two sequences? *-continues on next page-*

		Visual Learning The problem used in this *Visual Learning Bridge* is very similar to the *Solve and Share.* Using mathematical reasoning to analyze a graph is demonstrated. This context can also be modeled using two separate tables and two separate graphs. Consider revisiting the *Topic 15 Essential Question*. How do tables and graphs help to analyze number patterns and solve problems?
Lesson 15-4: Math Practices and Problem Solving- Make Sense and Persevere		
5.OA.B.3 MP.1 MP.2 MP.5 MP.6	Access Prior Learning: Students have practiced the thinking habits of MP.1 in previous grades and topics. Students analyzed patterns and graphed points on the coordinate plane to solve a problem in previous lessons. Securing the Big Idea: Students apply knowledge to solve a problem and practice the thinking habits of MP.1.	Solve and Share: Students create two separate number sequences and graph each to determine the best location for a birthday party. This problem uses a different starting point and a different change for each table. Students will need to make sense of the problem and create a plan. How did the graphs help to solve this problem? Students may observe that the graphed lines intersect. What does this mean for the given context? What happens to the two lines after the intersection? What does this represent? Visual Learning: A similar problem is modeled. Can students connect the thinking and problem solving skills demonstrated to strategies and thinking habits they used to complete the *Solve and Share* problem? Assess and Differentiate: The *Reteach* page focuses on using rules to complete a number sequence and does not ask students to graph. The *Homework and Practice* page has more opportunities for students to practice graphing and analyzing two number sequences. Can students reason why time units are usually placed on the x-axis? *CTC: ***Solve and Share*** (student work samples) *CTC: ***Quick Check*** (digital platform) Items 1 and 2

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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