## - Grade 2 Topic 7: More Solving Problems Involving Addition and Subtraction

Big Conceptual Idea: $K-5$ Progression on Operations and Algebraic Thinking (pp. 6-7; 18-21) Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 389A-389E), the Topic Planner (pp.3891-389J), the Topic Performance Assessments (pp. 433-434A), and all 6 lessons.

| Mathematical Background: | Topic Essential Question: <br> Read Cluster Overview <br> (TE, pp. 389A-389E) |
| :--- | :--- |
| How can you solve word problems that use adding and |  |
| subtracting? |  |
|  | Reference Answering the Topic Essential Questions (TE, pp. 431-432) for <br> key elements of answers to the Essential Questions. |

The lesson map for this topic is as follows:

| $6-6$ | $6-7$ | $6-8$ | $6-9$ | $7-1$ | $7-2$ | $7-3$ | $7-4$ | $7-5$ | $7-6$ | Assessment |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

4 A/D/E days used strategically throughout the topic.

## Instructional note:

The big idea of Topic 7 focuses on the meaning of operations through word problems that illustrate multiple interpretations of addition and subtraction. Focus instruction on Nevada Academic Content Standard (NVACS, 2010) 2.OA.A.

## 2.OA.A Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

The standard references one- and two-step word problems involving various situations. These situations or problem types (inserted below for reference) help students develop meanings for addition and subtraction. It is important to be mindful that second grade students are expected to demonstrate security with all problem types, including unknowns in each place, by the end of the year. This expectation is supported by work done in kindergarten and first grade. Kindergarten students work with the problem types outined with a thick, solid border as the actions of composition/decomposition are more easily modeled. First grade students interact with all problem types, but the problem types outlined with a thick, dashed border are the most challenging and are not expected to be secure until the end of second grade (CCSWT, 2011, p.20-21).

In general, students find Add To and Take From problem types easier because they include explicit action. Put Together and Take Apart problem types are generally more challenging, as they do not include explicit action and involve combinations of different kinds of objects into one collection as noted in the table below (Van de Walle, et al., 2014, p. 129). Finally, Compare problems tend to be the most challenging problem types, because one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011).

As stated in the "Progressions for the Common Core State Standards in Mathematics", two-step problems should not involve the most difficult subtypes (2011, p.20-21). Carpenter, Fennema, Loef-Franke, Levi and Empson developed a framework for students' strategies for solving these problems. The framework indicates that students often start direct modeling, then move to counting strategies and derived fact strategies (2015). When students use direct modeling strategies, they make an "Explicit physical representation of each quantity in a problem and the action or relationship involving those quantities before counting the resulting set." (Carpenter, et al., 2015, p.29). These models can include concrete objects, tally marks or pictures.

Counting strategies refer to when "... a child recognizes that it is not necessary to physically construct and count the two sets described in a problem." (Carpenter, et al., 2015, p.24). These students may keep track of counts using their fingers, counters or tallies, but most give no evidence of a physical action when counting. Finally, derived fact strategies "can be represented with equations or other notation using written numerals" (Carpenter, et al., 2015, p.32), and where "the notation becomes a tool for reflection." (Carpenter, et al., 2015, p.33). Although students will progress to more efficient strategies over time (e.g., from direct
modeling to counting strategies to derived facts), it is common for students to move among these strategy types based upon the numbers, context and the difficulty of the problem type.

As students interact with the problem types, help them draw connections to the meanings of the operations (addition and subtraction) as well as the relationship between operations. Bar diagrams help students make sense of word problems and see the relationship between quantities supporting part-part-whole reasoning. In addition, making bar diagrams helps students write an equation to solve the problem. The use of a question mark (?) to represent the unknown quantity helps students develop algebraic thinking. In order to offer support or challenge, consider factors that affect problem difficulty. As previously discussed, the structure of a word problem affects the level of difficulty, as do the numbers and wording of the problem. If a student is struggling with put together, take apart or compare problems, consider modifying the problem to include an explicit action (add to or take from), returning to the original problem type after support is offered. In addition, adjusting the numbers in a problem allows teachers to respond to learners. For students who are struggling, help them focus on the context of the problem type by decreasing the numbers and then return to the original level of difficulty. For students who are ready for extension, a challenge might be offered by changing the numbers to elicit strategies based on decomposing or recomposing.


Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

## Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop MP. 2 behaviors. This is the focus of the Math Practices and Problem Solving lesson 76. Reference Teacher's Edition (pp. F24-F24A) and the Nevada Academic Content Standards for Mathematical Practice.

Topic 6 has been decomposed with lessons either omitted or distributed to Topics 5 and 7. Lessons 6-6, 6-7, 6-8, and 6-9 are paced before lesson 7-1 and are included in this document.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

| New Academic Vocabulary: <br> (First time explicity taught) | Review Academic Vocabulary: <br> (Vocabulay explicitly taught in prior grades or topics) |  |
| :--- | :--- | :--- |
|  | break apart |  |
|  | equation | mental math |
|  | difference | compensation |
|  | bar diagram | subtract |
|  | tens |  |
|  | ones |  |
|  | open number line |  |

Additional terminology that students may need support with: algorithm, backward, column, forward, minuend (whole), model, row, separate, subtrahend (part subtracting)

## *Collaborative Team Conversations (CTC)

Consider using one of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding question: "Are students developing conceptual understanding and connecting real world context with numbers, strategies, and/or equations?"

| Lesson | Evidence | Look for |
| :---: | :--- | :--- |
| $7-3$ | Solve \& Share (student work samples) | Focus CTC around the big idea: <br> $\bullet \quad$ student strategies and models <br> $\bullet$ sense-making strategies |
| $7-5$ | Quick Check (digital platform) <br> Items 1, 3, and 4 | Focus CTC around data analysis and collection of student workspace <br> (scratch paper). Printable version available under "Teacher Resources". |


| Learning Cycle <br> Assessments (summative) | Topic Assessments <br> SE pp.431-434 | Use Scoring Guide TE pp. 431-434 |
| :---: | :--- | :--- |

Standards listed in bold indicate a focus of the lesson.

| NVACS <br> (Content and Practices) | Mathematical Development of the Big Idea | Instructional Clarifications \& Considerations |
| :---: | :---: | :---: |
| Lesson 6-6: Use Addition to Check Subtraction |  |  |
| 2.NBT.B. 5 <br> 2.NBT.B. 9 <br> MP. 1 <br> MP. 2 <br> MP. 3 <br> MP. 4 | Access Prior Learning: <br> In first grade, (1.OA.B.4) students understood subtraction as an unknown-addend problem. Students also (1.OA.C.6) added and subtracted within 20 using strategies including the relationship between addition and subtraction. <br> In lesson 1-6, second grade students used think-addition to subtract. In prior topics, students represented the relationship between numbers and operations through bar diagrams. <br> Developing the Big Idea: In this lesson, students are developing understanding of the inverse relationship between addition and subtraction by using addition to check subtraction. | In first grade, students worked with part-part-whole relationships. Continue to use the language of part-part-whole. However, you can help students make a connection to this relationship and prior learning by also referencing fact families. <br> Encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper for solving problems. A blank workspace reflects the value we place on students' selection of appropriate strategies. <br> Topic Opener: <br> In planning, consider the four lessons from Topic 6 as part of Topic 7. Use the Topic 7 Opener. Consider limiting the Topic Opener to discussion of the Topic 7 Essential Question (TE, p. 389), Review What You Know (TE, p.390), and Vocabulary Review Activity (TE, p. 390) with the word equation or bar diagram. Post the essential question and student strategies on your math focus wall. <br> Solve \& Share: <br> Providing students with a blank workspace to solve the problem honors a variety of student strategies. Encourage students to draw a bar diagram as a tool for sense making that reinforces the part-part-whole relationship, as the bar diagram will be a major representation utilized throughout this Topic. In addition, have students write an explanation to answer the question, "How can you use addition to check your answer?" <br> -continues on next page- |


|  |  | Visual Learning: <br> Omit the Visual Learning animation. Instead, spend more time on the Solve \& Share. Focus on sharing student work that demonstrates why addition can be used to check subtraction. When working with regrouping a ten for 10 ones, ensure that students have support through concrete manipulatives and representational drawings. <br> Independent Practice/Math Practices and Problem Solving: <br> As previously indicated, students do NOT need to do all of the problems in their student edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The Independent Practice page offers problems that support procedural skill and fluency. The Math Practices and Problem Solving page offers problems that support application. The quick check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate. <br> Assess and Differentiate: <br> To support students in understanding the bar diagram, refer to the Intervention Activity, "Check It with Cubes" (TE, p.357A). |
| :---: | :---: | :---: |
| Lesson 6-7: Practice Subtracting |  |  |
| 2.NBT.B. 5 <br> 2.NBT.B. 9 <br> MP. 1 <br> MP. 2 <br> MP. 6 <br> MP. 7 | Access Prior Learning: <br> In first grade, (1.NBT.B.2) students understood that the two digits of a 2-digit number represent amounts of tens and ones. <br> In Topic 5, second grade students learned subtraction strategies. <br> Securing the Big Idea: <br> In this lesson, students are securing understanding of subtraction strategies and that at times it is necessary to regroup 1 ten as 10 ones. | If students have consistent opportunities to play the Regrouping Game (Reference the last page of this document), they should secure the understanding of regrouping 1 ten as 10 ones. <br> Solve \& Share: <br> Ask students to solve the problem a second time using place-value blocks. Child-watch for students who appropriately trade in, or regroup, 1 ten for 10 ones in order to subtract. Highlight this student's strategy in the share to develop the essential understanding of the lesson, without relying on the standard subtraction algorithm. <br> Visual Learning: <br> The Visual Learning animation includes the break apart strategy and the U.S. traditional standard subtraction algorithm. Students should use place value blocks to support their understanding of the strategies presented. Students may view and discuss standard algorithms to expose the idea that it is one more strategy, but do not focus on it. Additional learning opportunities around written methods and standard algorithms have been included in Topic 11. <br> Independent Practice/Math Practices and Problem Solving: <br> Consider using item 12 ( $34-8=35-$ ?) to engage students in a discussion around the meaning of the equal sign (=). Avoid defining the meaning of this symbol. Instead, use questioning and examples to formatively assess students' understanding. If students believe that the equal sign means, "the answer is" continue to move them toward an understanding of equivalence as "the same as". Presenting equations with the answer first $(7=2+5)$ causes students to examine why it is the same as when the answer is last $(2+5=7)$. In addition, asking students to find equivalent expressions, rather than just the answer, helps students to focus on the meaning of this symbol. For example, given $19+23$, students may come up with $19+23=20+22$. (Van de Walle, et al., 2014, p. 230). |
| Lesson 6-8: Solve One-Step And Two-Step Problems |  |  |
| 2.OA.A. 1 <br> MP. 1 <br> MP. 2 <br> MP. 4 <br> MP. 7 | Access Prior Learning: <br> In lesson 3-8 and 4-7 students solved one-step and two-step word problems. <br> In lesson 5-8, second grade students solved word problems using the relationship between addition and subtraction. <br> Developing the Big Idea: <br> In this lesson, students are developing understanding that models and equations can be used to solve one-step and two-step word problems. They are also developing understanding that solving two-step word problems requires them to solve the first step before solving the second step. | Solve \& Share: <br> Provide a blank workspace for students to solve the problem. This may simply be placing a sticky note on the page. Child-watch for students who demonstrate understanding. Offer an extension by asking them to represent the two-steps of the problem using bar diagrams. Continue to encourage all students to use two strategies to check for accuracy, and evaluate those strategies for efficiency. <br> Visual Learning: <br> Focus the discussion on the Essential Question: Why is it helpful to complete a bar diagram and write an equation to solve word problems? (TE, p.366). |

## Lesson 6-9: Math Practices And Problem Solving: Reasoning

| 2.OA.A.1 | Access Prior Learning: <br> 2.NBT.B.9 |
| :---: | :--- |
| In first grade, students engaged in <br> Math Practice 2. |  |
| MP.1 | Developing the Big Idea: |
| MP.2 | In this lesson, students are |
| MP.4 | developing understanding of Math |
| MP.5 | Practice 2: Reasoning Abstractly <br> and Quantitatively by thinking <br> MP. 6 |
|  | about words and numbers to solve |
| problems. |  | online. Engage students in discussions around MP. 2 behaviors (TE, pp.F24 - F24A). The focus of this lesson should be on these habits of mind, rather than on computation. Students use reasoning to contextualize addition and subtraction problems, then decontextualize by writing and solving an equation.

## Solve \& Share:

Provide a blank workspace for students to solve the problem. This may simply be placing a sticky note on the page.

## Visual Learning:

Before the Visual Learning animation, give students time to make sense of and solve the problem presented in the animation. In Guided Practice item 2, if students are confused about the comparison bar diagram, reference Error Intervention Note: Item 2 (TE, p.372).
Representing the quantities of 46 and 18 with cubes will help them understand the proportion of the boxes in the diagram, and why 46 is placed in the larger "part" box.

## Topic 6 Performance Assessment:

It is not necessary to give the Topic 6 Performance Assessment at this time. Teachers may choose to select items to add to the Topic 7 Performance Assessment, skip the assessment, or use a few items to formatively assess and inform instruction.

## Lesson 7-1: Represent Addition and Subtraction Problems

### 2.0A.A.1 Access Prior Learning:

In first grade, (1.OA.A.1) students
MP. 2
MP. 4
MP. 5
MP. 8 solved addition and subtraction word problems within 20 , with unknowns in all positions.

In previous lessons, second grade
students represented and solved problems using models, pictures and equations.

## Developing the Big Idea:

In this lesson, students are developing understanding that equations can be written to model word problems using a question mark (?) to represent the unknown. Students will model problems with the unknown in any position.

Solve \& Share:
During problem solving, child-watch for students who are able to explain their thinking in a way that reflects understanding of the context of the Put Together Addend Unknown problem. Encourage students to explain how their drawing connects to the abstract equation given. Sequence the share to include strategies seen in the Visual Learning such as bar diagrams, adding on, and the relationship between addition and subtraction.

## Visual Learning:

Give students an opportunity to solve the Add To Change Unknown word problem presented before continuing with the animation. During discussion, strategically ask questions that help students connect student strategies from the Solve \& Share to the strategies presented in Visual Learning. Also, encourage students to think of other models for solving the problem, including open number lines with a jump of 30 and a jump of 1.

In the Guided Practice, use of the Problem Solving Recording Sheet (Teaching Tool 1) may help students make sense of the problem. Use of this tool models questions students should ask themselves when making sense of a problem and devising a plan. Item 1 is a Take From Start Unknown problem. Having the unknown in the start position increases the level of difficulty of the problem. Item 2 is an easier problem type, Add To Result Unknown.

## Assess and Differentiate:

In the Intervention Activity, "Step-by-Step!" (TE, p.395A) further support students by having them represent quantities on the bar diagram with manipulatives or drawings, as a way to give meaning to the numerals in the problem.

## Lesson 7-2: Mixed Practice: Solve Addition and Subtraction Problems

### 2.0A.A.1 Access Prior Learning:

 In lesson 7-1, second grade students wrote equations with a question mark (?) for the unknown, to model and solve word problems.
## Developing the Big Idea:

In this lesson, students are developing understanding of Compare Bigger Unknown problems.

## Securing the Big Idea:

In this lesson, students are securing understanding that drawings, bar diagrams and equations can be used to make

This lesson focuses on Compare Bigger Unknown word problems involving addition and subtraction. As indicated in the instructional note at the beginning of this document, compare problems are more challenging because one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011). Rather than involving an action such as Add To or Take From, these problems involve the relationship between quantities. These quantities are labeled as follows: referent set, compared set, and difference (Carpenter, et al., 2015, p.10)

| Mark has 8 mice. | $\leftarrow$ Referent set |
| :--- | :---: |
| Joy has 12 mice. | $\leftarrow$ Compared set |
| Joy has 4 more mice than Mark. | $\leftarrow$ Difference |

$\left.\left.\begin{array}{|c|l|l|}\hline & \begin{array}{l}\text { sense of word problems and } \\ \text { strategies can be used to solve } \\ \text { them. }\end{array} & \begin{array}{l}\text { Throughout instruction, ensure that students are encouraged to model their thinking with } \\ \text { concrete manipulatives and drawings. As noted in the Error Intervention Note: Item 1 (TE } \\ \text { p.398), labeling the bar diagram helps students make sense of the quantities and relationships } \\ \text { in a problem. Reference Visual Learning for an example. Also, when drawing comparison bar } \\ \text { diagrams, the sizes of the bottom boxes should correspond with the proportion of the quantities } \\ \text { in the problem. } \\ \text { Solve \& Share: }\end{array} \\ \text { If students have difficulty making sense of the problem, engage them in a conversation around } \\ \text { the word, fewer using real-world examples. As indicated by Van de Walle et al., students often } \\ \text { have more experiences with the term more than, so they may need additional experiences with } \\ \text { the terms less than or fewer than (2014, p.131). } \\ \text { Visual Learning: }\end{array}\right\} \begin{array}{l|l|l|}\hline \text { The Do You Understand? Show Me! (TE, p.398) further supports students as they develop } \\ \text { understanding of the more and less than relationships. Consider having students work in } \\ \text { partners to model the statements. }\end{array}\right\}$


## References

Carpenter, T., Fennema, E., Loef Franke, M., Levi, L., Empson, S.B. (2015). Children's mathematics: Cognitively guided instruction (2nd ed.). Portsmouth, NH: Heinemann.

Common Core Standards Writing Team. (2011, May 29). Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards Instructional Support/Nevada Academic Standards/Math Documents/mathstandards.pdf.

Van de Walle, J., Karp, K., Lovin, L., \& Bay-Williams, J. (2014). Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2 (2nd ed.). Boston, MA: Pearson.

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