

► Grade 2 Topic 15: Shapes and Their Attributes

Big Conceptual Idea: [K-6 Progression on Geometry](#) (pp. 2-5, 10-12)

Prior to instruction, view the *Topic 15 Professional Development Video* located in Pearson Realize online. Read the *Teacher's Edition (TE): Cluster Overview/Math Background* (pp. 851A-851E), the *Topic Planner* (pp. 851I-851K), the *Topic Performance Assessment* (pp. 917-918A), and all 8 lessons.

<p>Mathematical Background: Read Cluster Overview (TE, pp. 851A-851E)</p>	<p>Topic Essential Question: How can shapes be described, compared, and broken into parts?</p> <p><i>Reference Answering the Topic Essential Question (TE, pp. 913-914) for key elements of answers to the Essential Question.</i></p>
--	---

The lesson map for this topic is as follows:

15-1	15-2	15-3	15-4	15-5	15-6	15-7	15-8	Assessment
------	------	------	------	------	------	------	------	------------

4 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 15 focuses on how 2-D and 3-D shapes can be described, classified and analyzed by their attributes. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) standard cluster 2.G.A.

2.G.A Reason with shapes and their attributes.

1. *Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.*
2. *Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.*
3. *Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.*

The NVACS Critical Areas for 2nd Grade, identifies geometry as one of four critical areas for instruction. The National Research Council of the National Academies also articulates this importance:

There is expert consensus that two areas of mathematics are particularly important for young children to learn:

- (1) *Number, which includes whole number, operations, and relations,*
- (2) *Geometry, spatial thinking, and measurement.*

National Research Council of the National Academies. (2009). *Mathematics Learning in Early Childhood*. Washington D.C.: The National Academies Press.

Although labeled as an "Additional Cluster", the content of Topic 15 is necessary to build foundations for 3rd grade fractional concepts and **MUST NOT BE SKIPPED**. Specifically, students work with partitioning shapes in lessons 15-5 to 15-8. Refer to the Conceptual Understanding section of the Math Background pages (TE, p. 851E) for clarification on how Topic 15 concepts will become necessary background for future third grade math learning.

Geometry is often taught with an emphasis on terminology for naming shapes. This approach can limit student access to learning opportunities that build their spatial sense and geometric reasoning. It is often said that some people are "spatial thinkers" while others are not. This is not true. As Pierre van Hiele and Dina van Hiele-Geldof found in their research, we all have the capability of developing spatial reasoning and this geometric thought evolves through a hierarchy of levels. These levels, referred to as "The van Hiele Theory of Geometric Thought" are not age dependent. They are, however, sequential and require geometric *experiences* to advance from one level to the next (Van de Walle, Karp, Lovin, Bay-Williams, 2014).

Topic 15
Shapes and Their Attributes

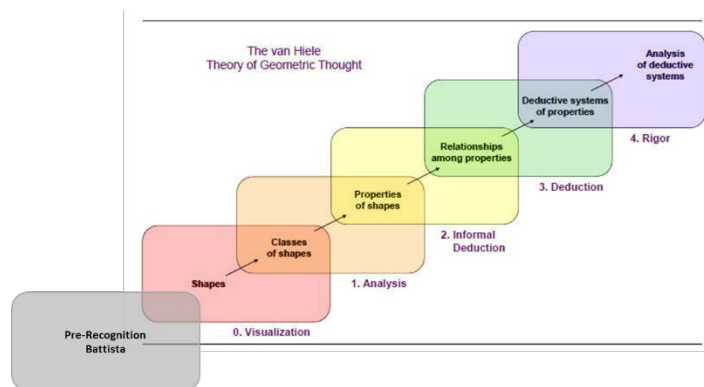
Number of lessons: **8**

A/D/E: **4 days**

NVACS Focus:
G.A

Total Days: ~12

[2nd Grade Curriculum Pacing Framework:](#)
[Balanced Calendar](#)



Students who are at Van Hiele's *Level 0: Visualization* refer to shapes by what they “look like” (e.g., it looks pointy, or it looks like a house). These students are able to notice that shapes are alike or different, but need experiences to help them build an understanding that shapes have properties; and that properties can be used to classify shapes (Van de Walle et al., 2014, p. 302). Reference the list below for Van de Walle's suggestions for supporting student movement from Level 0 to Level 1:

- Challenge children to test ideas about shapes using a variety of examples from a particular category. Say to them, “Let’s see if that is true for other rectangles,” or “Can you draw a triangle that does not have a right angle?” In general, encourage children to see whether observations made about a particular shape apply to other shapes of a similar kind.
- Provide ample opportunities for children to draw, build, make, put together (compose), and take apart (decompose) shapes in both two and three dimensions. These activities should be built around understanding and using specific characteristics or properties. (2014, p. 305)
- Emphasize the properties of figures rather than simple identification. As new geometric concepts are learned, the number of properties that figures have can be expanded.

Students who are at Van Hiele's *Level 1: Analysis*, are learning to look at classes of shapes, rather than just individual shapes. According to Van de Walle, et al.,

...in describing a shape, level 1 thinkers are likely to list as many properties of a shape as they know. They do not see relationships between these properties and so cannot determine which properties are sufficient in describing a shape. They are able to consider all shapes within a class rather than just the single shape on their desk. Instead of talking about this rectangle, they can talk about all rectangles. By focusing on a class of shapes, children are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides of the same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background and children begin to appreciate that a collection of shapes goes together because of properties. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class (2014, pp. 302-303).

Now that we've explored two of the levels, let's consider the implications for instruction. As van Hiele found, students must have geometric experiences to advance in the levels of geometric thought. Therefore, we should strive to incorporate four features of effective geometry instruction for young children as identified in *Teaching Student-Centered Mathematics* (2014):

- Show and compare diverse examples and nonexamples.
- Facilitate discussions about shapes and their attributes.
- Examine a wider variety of shape classes.
- Challenge children with a wide range of geometric tasks.

As you navigate through Topic 15, look for opportunities to emphasize these features in your geometry instruction. Examples are included in the lesson level instructional notes below.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop MP.2 behaviors. This is the focus of the Math Practices and Problem Solving lesson 14-6. Reference the Teacher's Edition (TE, pp. F24-F24A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary		
Use these words consistently during instruction.		
New Academic Vocabulary: (First time explicitly taught)		Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
vertices, vertex	right angle	<i>polygon</i>
quadrilaterals	equal shares	<i>cube</i>
pentagons	halves	<i>face</i>
hexagons	thirds	<i>edge</i>
angle	fourths	

Additional terminology that students may need support with: *alike, attributes, different, fraction, partition, plane shape, properties, quarter, solid figure*

***Collaborative Team Conversations (CTC)**

Consider using **one** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:


Guiding question: "Are students able demonstrate their understanding by identifying and drawing plane shapes that have specified attributes?"
"Are students able to analyze the attributes of different shapes to find similarities and differences?"

Lesson	Evidence	Look for
15-2	Solve & Share (student work samples)	Focus CTC around the big idea: <ul style="list-style-type: none"> student strategies and models identifying attributes that define a polygon
15-5	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". <ul style="list-style-type: none"> understand rows and columns counting or adding unit squares to find the total square units

Learning Cycle Assessments (summative)	Topic Assessments SE pp. 913-918	Use <i>Scoring Guide</i> TE pp. 913-918
--	--	---

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 15-1: 2-Dimensional Shapes		
2.G.A.1 MP.3 MP.4 MP.6	<p>Access Prior Learning: In first grade (1.G.A.1), students distinguished between defining attributes and non-defining attributes, and built and drew shapes with those defining attributes. First grade students (1.G.A.2), also composed 2-dimensional shapes including rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles).</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding that the number of sides and vertices can be used to classify and sort shapes based on their attributes. Students will work with polygons including triangles, quadrilaterals, pentagons and hexagons.</p>	<p>Provide families with the <i>Home-School Connection</i> Letter for Topic 15 (<i>Teacher's Resource Masters, Volume 2</i>)</p> <p>Prior to the <i>Topic Opener</i>, engage students in shape sorts. Provide small groups of students with cut outs of various shapes including regular polygons, irregular polygons, and non-examples of polygons. It is helpful to pull shapes from the Lesson 15-1 Visual Learning, and Lesson 15-2 Visual Learning and Independent Practice. Then follow the suggestions from <i>Teaching Student-Centered Mathematics</i>:</p> <ul style="list-style-type: none"> Each student selects a shape and tells something they find interesting about it. Each student selects two shapes and finds something that is alike and something that is different about their two shapes. The group selects a target shape and makes a rule. Then, they find all other shapes that fit their rule. For example, the rule may be, "This shape has a straight side and a curved side". Repeat using the same target shape, but with a different rule. Do a "secret sort". One student selects 3-5 shapes that fit a rule. The other students choose from the shapes left in the pile. They try to find shapes that fit in the set and guess the secret rule. (Van de Walle, et al., 2014, p. 309) <p>In this lesson, students draw shapes from a given set of attributes while engaging in MP.1 behaviors (Van de Walle et al., 2014, p. 320). This process helps students develop understanding of the defining attributes of shapes, and how they can be used to describe, classify and analyze 2-dimensional geometric figures.</p> <p style="text-align: right;"><i>-continues on next page-</i></p>

		<p>Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 851), <i>Review What You Know</i> (TE, p. 852), <i>Vocabulary Review Activity</i> (TE, p. 852) for the term <i>plane shape</i>, and the <i>Topic 15 Vocabulary Words Activity</i> (TE, pp. 853-854) for the word <i>vertices</i> (<i>vertex</i>) only. Introduce remaining vocabulary words as they appear in instruction. Post the essential question and student strategies on your math focus wall.</p> <p>Solve & Share: During the share, look for opportunities to ask students to identify examples and non-examples in each other's work. If you have difficulty finding non-examples in student work, consider displaying Shawn's Work in <i>Analyze Student Work</i> (TE, p.859 and available online under the <i>Solve & Share</i> as "Teacher Resources"). By comparing examples and non-examples, students are able to engage in a conversation grounded in MP.3 behaviors and the defining attributes of shapes.</p> <div data-bbox="646 506 1474 625" style="border: 1px solid #ccc; padding: 5px;"> <p>Develop: Problem-Based Learning</p> <hr style="border: 1px solid #92d050;"/> <div style="display: flex; align-items: center;">  <p>Math Practices & Problem Solving: Construct Arguments: Solve & Share</p> </div> <div style="display: flex; align-items: center; margin-top: 5px;"> Assign Info Teacher resources </div> </div> <p>Return to the Topic Essential Question posted on the math focus wall to add new student thinking.</p> <p>Visual Learning: During the animation, encourage students to write and draw responses on individual whiteboards to increase engagement and understanding.</p> <p>Independent Practice/Math Practices and Problem Solving: As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The <i>Independent Practice</i> page offers problems that support procedural skill and fluency. The <i>Math Practices and Problem Solving</i> page offers problems that support application. The <i>Quick Check</i> items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.</p> <p>After students solve item 12, engage the class in a conversation about their thinking. This problem provides an opportunity for students to consider the classification of geometric figures.</p>
--	--	--

Lesson 15-2: Polygons and Angles

<p>2.G.A.1</p> <p>MP.2 MP.6 MP.7</p>	<p>Access Prior Learning: In first grade (1.G.A.1), students distinguished between defining attributes and non-defining attributes, and built and drew shapes with those defining attributes.</p> <p>In lesson 15-1, second grade students identified plane shapes by the number of sides and vertices.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding that plane shapes can be described by their defining attributes including sides and angles.</p>	<p>This lesson focuses on reasoning at Van Hiele's <i>Level 1: Analysis</i>. Students consider the attributes of polygons through analysis of examples and non-examples. Students are also explicitly taught the words <i>polygon</i>, <i>angle</i> and <i>right angle</i> for the first time. Use the Frayer Model (Teaching Tool 62) to capture students' new understanding of these terms after <i>Visual Learning</i>.</p> <p>Solve and Share: During problem solving, child-watch for students who demonstrate <i>Level 0: Visualization</i> understanding by describing what the shapes "look like" without using attributes. For example, a student may say the shapes look like windows. Refer to the Instructional Note at the beginning of this document for clarification on <i>Level 0</i> and appropriate supports to offer students. Also, child-watch for students who demonstrate <i>Level 1: Analysis</i> understanding. These students will attend to the attributes of the shapes. Select and sequence students to share solutions to progress from less sophisticated to more sophisticated geometric reasoning.</p> <p>Independent Practice/Math Practices and Problem Solving: If students identify Item 5 as having four angles, they may not be counting the concave angle. Engage students in a discussion that clarifies that concave, "dented in", angles are counted as angles. Students will also discover that polygons have the same number of sides, vertices and angles.</p> <p>Extend student thinking on item 5 by asking students to draw another pentagon and give a written explanation that proves that both shapes are pentagons. This can be done on a sticky note or plank piece of paper and used as a formative assessment.</p> <p>*CTC: <i>Solve & Share</i> (student work samples)</p>
---	--	--

Lesson 15-3: Draw 2-Dimensional Shapes		
<p>2.G.A.1</p> <p>MP.1 MP.6 MP.7</p>	<p>Access Prior Learning: In first grade (1.G.A.1), students distinguished between defining attributes and non-defining attributes, and built and drew shapes with those defining attributes. First grade students (1.G.A.2) also composed 2-dimensional shapes including rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles). In the prior lessons, second grade students identified polygons by the number of sides, vertices and angles. Students also learned about right angles.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding of how to define and differentiate shapes based upon attributes. They will also draw 2-dimensional shapes using specific attributes.</p>	<p>NOTE: As suggested in “Pose the Solve-and-Share Problem” (TE, p. 871), have all students construct polygons using toothpicks, crayons or straws before drawing them.</p> <p>Solve & Share: As students problem-solve, remind them of the resources on the math focus wall that may help them compare the shapes they've constructed. These resources include the frayer model graphic organizers from lesson 15-2.</p> <p>Visual Learning: Prior to interacting with the animation, have students draw a polygon with 5 vertices on their whiteboards. At the conclusion of the animation, engage students in a discussion of the Lesson Essential Question: <i>What information should you give to others if you want them to draw a particular polygon?</i> (TE, p. 872) Record students' thoughts on the math focus wall for future reference.</p> <p>Independent Practice/Math Practices and Problem Solving: If students respond to item 7 by drawing a triangle, ask them, “Does your shape have 5 vertices? How does this affect the total number of sides?” to support MP.6 Attend to Precision. Use item 11 as a formative assessment of students' understanding of polygons as closed plane shapes with three or more straight sides. As an extension, consider allowing students to write their own geometry riddles, similar to item 7. These can be recorded on cards for peers to solve.</p>
Lesson 15-4: Cubes		
<p>2.G.A.1</p> <p>MP.2 MP.3 MP.6 MP.7</p>	<p>Access Prior Learning: In first grade (1.G.A.2), students composed 3-dimensional shapes including cubes, right rectangular prisms, right circular cones, and right circular cylinders, to create a composite shape, and compose new shapes from the composite shape. In lesson 15-3, second grade students learned about right angles.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding of how to describe a cube by telling about its faces, edges and vertices. Students also draw a cube using these attributes.</p>	<p>The only 3-dimensional figure included in second grade standards is the cube. Reference NVACS, 2010, 2.G.A.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. In this lesson, students consider how a solid figure (cube) is different from a plane shape (square).</p> <p>Solve & Share: Ensure that all students have access to concrete cubes to explore while problem-solving.</p> <p>Visual Learning: As student understandings emerge, record their thinking around solid figures (3-D) vs. plane shapes (2-D). Add this information to the math focus wall.</p>
Lesson 15-5: Divide Rectangles Into Equal Squares		
<p>2.G.A.2 2.OA.C.4</p> <p>MP.1 MP.3 MP.4 MP.5 MP.7</p>	<p>Access Prior Learning: In first grade (1.G.A.3), students partitioned rectangles into two and four equal shares and described the shares using <i>halves</i>, <i>fourths</i>, <i>quarters</i>, <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Students also described the whole as two of or four of the shares.</p>	<p>The concept of equal sharing comes intuitively for students. Their experiences of sharing with siblings or friends allow fractional concepts to emerge. It is important to capitalize on these meaningful connections to equal shares. As Van de Walle et al. states, “One of the most significant ideas for children to develop about fractions is the sense that fractions are numbers-quantities that have values...Researchers have acknowledged for some time the importance of these two actions [<i>partitioning</i> (splitting equally) and <i>iterating</i> (counting a repeated amount)] to meaningfully working with the numerical nature of fractions” (2014, p. 253).</p> <p>Solve & Share: Child-watch for student understanding of repeated addition, introduced in Topic 2. Select and sequence students to share so that both equations ($5 + 5 + 5 + 5$ and $4 + 4 + 4 + 4 + 4$) are</p>

-continues on next page-

	<p>Securing the Big Idea: In this lesson, students are <i>securing</i> understanding of repeated addition.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding of how to divide rectangles into rows and columns of equal squares.</p>	<p>presented. Ask students to compare both solutions for accuracy and equivalence. Also, ask them which unit each equation utilizes- rows (5 + 5 + 5 + 5) or columns (4 + 4 + 4 + 4 + 4). This conversation will offer an entry point for students into the <i>Visual Learning</i>.</p> <p>Visual Learning: Look for opportunities to help students connect their understanding of measurement concepts, such as no gaps or overlaps, to tiling of squares over a rectangle. Students should use the foam square tiles from their student manipulative kits.</p> <p><i>*CTC: Quick Check (digital platform)</i></p>
--	---	--

Lesson 15-6: Partition Shapes

<p>2.G.A.3</p> <p>MP.2 MP.4 MP.6 MP.8</p>	<p>Access Prior Learning: In first grade (1.G.A.3), students partitioned rectangles into two and four equal shares and described the shares using <i>halves, fourths, quarters, half of, fourth of, and quarter of</i>. Students also described the whole as two of two shares or four of four shares.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding of how to divide circles and rectangles into halves, thirds and fourths; and that you can show halves, thirds and fourths of the same whole in different ways.</p>	<p>“Fraction symbolism represents a fairly complex convention that can be misleading to children. That is why it is important in grades pre-K-2 to use fraction words and postpone introducing fraction symbolism. Let children first focus on making sense of fractions without the complication of also trying to make sense of the symbolism.” (Van de Walle, et al, 2014, p. 256) In other words, do not use fraction symbols. As indicated by the text, expect students to use fraction words instead.</p> <p>As Van de Walle et al. suggests, introduce fraction words, <i>halves, thirds, fourths</i>, during discussion when students are sharing their solution strategies. He goes on to clarify, “Children need to be aware of two aspects of fractional parts: (1) the number of parts and (2) the equality of the parts (in size, not necessarily in shape). Emphasize that the number of equal parts or fair shares that make up a whole determines the name of the fractional parts or shares.” (2014, p. 256)</p> <p>Solve & Share: During problem-solving, child-watch for students who demonstrate understanding of lesson 15-5 by partitioning the rectangle with no gaps or overlaps. Select students who demonstrate a misconception to share first. This will allow other students to help clarify the error and move forward with accurate partitioning. During discussion of accurate solutions, introduce fraction words as appropriate to the work shown.</p> <p>Visual Learning: Prior to interacting with the animation, ask students to draw two circles on their whiteboard. Ask them to split one into equal shares, and the other so that it does NOT show equal shares. Continue to ask students to draw examples and non-examples throughout the animation.</p> <p>Independent Practice/Math Practices and Problem Solving: For item 10, formatively assess student understanding of equal shares by asking for a written explanation of how/why the drawings show fourths. Offering a sticky note or blank piece of paper will encourage responses that are more thoughtful as they provide more room to write.</p>
--	--	--

Lesson 15-7: Equal Shares, Different Shapes

<p>2.G.A.3</p> <p>MP.1 MP.2 MP.3 MP.4 MP.7</p>	<p>Access Prior Learning: In first grade (1.G.A.3), students partitioned rectangles into two and four equal shares and described the shares using <i>halves, fourths, quarters, half of, fourth of, and quarter of</i>. Students also described the whole as two of two shares or four of four shares.</p> <p>In lessons 15-5 and 15-6, second grade students partition rectangles and other shapes into equal shares and described the shares.</p> <p>Developing the Big Idea: In this lesson, students are <i>developing</i> understanding that equal shares of the same whole do not have to have the same shape.</p>	<p>Van de Walle et al., suggests, “In addition to helping children use the words <i>halves, thirds, fourths, and quarters</i>, be sure to make regular comparisons of fractional parts to the whole. Make it a point to use the term <i>whole, one whole</i>, or simply <i>one</i> so that children have a language they can use regardless of the model involved” (2014, p. 256).</p> <p>Solve & Share: During problem solving, extend everyone’s thinking by using the “Extension for Early Finishers” on TE p. 895.</p> <p>Visual Learning: Prior to interacting with the animation, have students use their whiteboards to show different ways to divide a square into 3 equal shares.</p> <p>Independent Practice/Math Practices and Problem Solving: For items 7 and 11, child-watch for students who do not use the whole when dividing into equal shares. Ask, “How can you share the <i>whole</i> rectangle equally?” Using a context such as brownies, students will understand that we do not want to waste the extras. Due to similarity in the sound of the terms, some students may confuse <i>shares</i> with <i>squares</i>.</p>
---	--	---

Lesson 15-8: Math Practices And Problem Solving: Repeated Reasoning		
<p>2.G.A.2 2.G.A.3 2.OA.C.4</p> <p>MP.1 MP.2 MP.3 MP.4 MP.7 MP.8</p>	<p>Access Prior Learning: Second grade students focused on Math Practice 8: Repeated Reasoning in Topic 10.</p> <p>Securing the Big Idea: In this lesson, students are <i>securing</i> understanding of Math Practice 8: Reason abstractly and quantitatively behaviors in the context of second grade. Students will use repeated reasoning to find different ways to divide rectangles into rows and columns and then into equal shares. Students will also write equations using repeated addition.</p>	<p>Students focused on Math Practice 8: Repeated Reasoning in Topic 10. Reference the <i>Math Practices and Problem Solving Handbook</i> for suggestions for developing, connecting and assessing MP.8 (TE p.F30-F30A). Also, consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.</p> <p>MP. 8 Behaviors:</p> <ul style="list-style-type: none"> • Notices and describes when certain calculations or steps in a procedure are repeated • Generalizes from examples or repeated observations • Recognizes and understands appropriate short cuts • Evaluates the reasonableness of intermediate results <p>Solve & Share: As recommended in the "Pose the Solve-and-Share Problem" (TE p. 901), provide students with $\frac{3}{4}$ inch squares (Teaching Tool 52) to help them problem solve.</p> <p>Visual Learning: Prior to interacting with the animation, have students use their whiteboards to show different ways Sam could design his quilt square. Reference <i>Error Intervention</i>: Item 1 (TE p. 902) to support students who have difficulty coming up with a second design.</p>

References

Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Geometry, K-6*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). *The Nevada Academic Content Standards*. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Cross, C.T., Woods, T.A., Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood*. Washington D.C.: National Academies Press.

National Research Council of the National Academies. (2009). *Mathematics learning in early childhood*. Washington D.C.: National Academies Press.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

This page is intentionally left blank