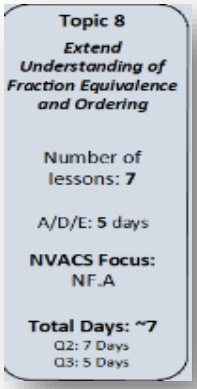
Grade 4 Topic 8: Extend Understanding of Fraction Equivalence and Ordering



[4th grade Curriculum Pacing Framework: Balanced Calendar]

Big Conceptual Idea: [Number and Operations- Fractions] (pp. 121-125)

Prior to instruction, view the Topic 8 Professional Development Animation located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 407A-407F), the Topic Planner (pp. 407I-407K), all 7 lessons, and the Topic Assessments (pp. 459-460A).

Mathematical Background: Read Cluster Overview- (TE, pp. 407A-407F)	Topic Essential Questions: What are some ways to name the same part of a whole? How can you compare fractions with unlike denominators? *Reference TE p. 407 and Answering the Topic Essential Questions (TE, pp. 457-458) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	Assessment

5 A/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on finding equivalent fractions and comparing fractions. Focus instruction on Nevada Academic Content Standards (NVACS) 4.NF.A.1. and 4.NF.A.2. Emphasis for standard 4.NF.A, is to "extend understanding of fraction equivalence and ordering" (2010). Students will find equivalent fractions "by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size" (NVACS, 2010, 4.NF.A). In Grade 3, students worked with denominators of 2, 3, 4, 6 and 8. In Grade 4, students will add in denominators of 5, 10, 12 and 100.

"A key idea about fractions that students must come to understand is that a fraction does not say anything about the size of the whole or the size of the parts. A fraction tells us only the *relationship between* the part and the whole" (Van de Walle, Karp, Bay-Williams, 2010, p. 288). When students compare fractions, they need to consider that both fractions are part of the same whole. A common misconception students have when comparing fractions is that "a/b > c/d if a > c and b > d; in other words, they think that a fraction with both a greater numerator and a greater denominator than another fraction has to be greater than that second fraction" (Small, 2104, p. 44). Using various tools, models, benchmark fractions and strategies to compare fractions will help students in developing their fractional knowledge.

Models are important when students are beginning to understand fractional concepts. The different models give students various opportunities to learn fractions. These different models include; region or area models, length models or number line models and set models. For example, "an area model helps students visualize parts of the whole. A linear model shows that there is always another fraction to be found between any two fractions-an important concept that is underemphasized in the teaching of fractions" (Van de Walle, et al., 2010, p. 288). Van de Walle, et al., continue to emphasize the use of models, "It is important to remember that students must be able to explore fractions across models. If they never see fractions represented by length, they will struggle to solve any problem or context that is linear. As a teacher you will not know if they really understand the meaning of fractions unless they can model a fraction using different context or models" (2010, pp. 290-291).

"In a problem-based classroom, students can develop an understanding of equivalent fractions and also develop from that understanding a conceptually based algorithm" (Van de Walle, et al., 2010, p. 302). Students need to understand equivalence, because "it is a critical but often poorly misunderstood concept" (Van de Walle, et al., 2010, p. 301). Equivalent fractions name two different names for the same point in a number line and same sized parts of the whole.

Students need a conceptual understanding of equivalent fractions before a procedural understanding. A conceptual understanding of equivalent fractions is, "two fractions are equivalent if they are representations for the same amount or quantity-if they are the same number" (Van de Walle, et al., 2010, p. 302). A procedural understanding of equivalent fractions is "to get an equivalent fraction, multiply or divide to top and bottom numbers by the same nonzero number" (Van de Walle, et al., 2010, p. 302). Giving students multiple opportunities with models or representations will help students build a conceptual understanding for finding equivalent fractions. While some of these lessons teach procedurally to find equivalent fractions, continue teaching the conceptual understanding of why fractions are equivalent.

Focus Math Practice 3: Construct arguments

Focus on opportunities for students to develop *Mathematical Practice 3* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 8-7. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
common factor benchmark fraction	*equivalent fraction* *fraction* *numerator* *denominator* *Identity Property of Multiplication*

Additional terminology that students may need support with: unit fraction, equality, inequality

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding questions: "Are students able to name the same part of a whole? Are students able to compare fractions using various strategies?"

Lesson	Evidence	Look for
8-1	*Convince Me!* (student work samples)	Focus CTC around the big idea: • students recognize equivalent fractions based on the same size whole.
8-6	*Quick Check* (digital platform) Items 3 and 5	Focus CTC around the big idea: • students' understanding around the comparison of fractions. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 457-460	Use *Scoring Guide* TE pp. 457-460A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 8-1: Equivalent Fractions- Area Model		
4.NF.A.1 MP.1 MP.2 MP.5	Access Prior Learning: In third grade, students used an area model to find simple equivalent fractions and explain why they are equivalent. Developing the Big Idea: In this lesson, students will find equivalent fractions using an area model and determine why they are equivalent.	Note: For fractions to be equivalent, the equivalent fractions must name the same part of the same whole. Convince Me: Consider changing the *Solve & Share* to the *Convince Me!* problem, "Mia ate ¼ of a pizza. Matt ate 2/8 of another pizza. Did Mia and Matt eat the same amount of pizza? Explain." This will elicit more strategies from students. Consider having concrete tools, like fraction strips available for students to solve the problem. Read *Convince Me!* note in the Teacher's Edition for an explanation about the *Convince Me!* (p. 412) problem. Child-watch for students who may have misconceptions when it comes to equivalent fractions. Solve & Share: After students answer the *Convince Me!* and strategies are shared, consider having students work together on the *Solve & Share* to find equivalent fractions. Consider including the *Look Back!* question, "How do you know your fraction is equivalent to ¼?" as the discussion around the *Solve & Share* is taking place. Visual Learning: Students are asked to use an area model to explain equivalency between 5/6 & 10/12. Consider having students use fraction strips along with the area model to support the conceptual understanding of equivalent fractions. Assess and Differentiate/Intervention Activity: Consider facilitating a discussion around the *Intervention Activity* with all students. The activity has students compare fraction strips. You may want to consider having students make their own construction paper fraction strips kit. *CTC: *Convince Me!* (student work samples)

Lesson 8-2: Equivalent Fractions- Number Lines		
4.NF.A.1 MP.1 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: In third grade, students used number lines to find simple equivalent fractions and to explain why they are equivalent. Developing the Big Idea: In this lesson, students continue to find equivalent fractions by using the number line and area model. Look Ahead: Emphasize number lines, as they will be an important concept throughout Topics 9-12.	Note: Students will need multiple opportunities to find equivalent fractions on a number line. Consider teaching this lesson over 2 days. Remember that by teaching this lesson over two days, it will be one of the A/D/E days on the WCSD Pacing Framework. Solve & Share: Consider having multiple tools readily available, like a ruler or Teaching Tool 12. Child-watch for students who may not know how to read a ruler and notice if students connect the ruler to a number line. Look Back: Consider facilitating a discussion around the *Look Back!* by having students find more fractions equivalent to 2/4 by using the ruler or number line. Students may also want to use fraction strips or an area model to find equivalent fractions. Visual Learning: Consider having students "act out" the number line and decide where fractions may fall on the number line to show equivalence. For example, some students may hold the following numbers on a number line; 0, ¼, 2/4, ¾ and 1. Other students will add the eighths onto the already made number line. Continue with twelfths. This reinforces the idea that equivalent fractions must be part of the same-sized whole. Consider using a variety of number line lengths for students to see the number line goes beyond 1. Convince Me: Consider using the *Convince Me!* to facilitate a discussion regarding the question. Have students connect to the *Visual Learning Animation*. Another Example: Consider having a discussion with the whole class about the *Another Example!*. Consider making a number line that is greater than 1 and have students "act out" where equivalent fractions would fall on the already made greater than 1 number line.
Lesson 8-3: Generate Equivalent Fractions- Multiplication		
4.NF.A.1 MP.2 MP.3 MP.4	Access Prior Learning: In third grade, students used the Identity Property of Multiplication for whole numbers. In the previous lessons, students used area models and number lines to find equivalent fractions. Beginning of the Big Idea: In this lesson, students will work with equivalent fractions by applying the Identity Property of Multiplication to develop a procedure for finding equivalency between fractions.	Note: Focus on why fractions are equivalent when multiplying by the same nonzero fraction. The Identity Property of Multiplication is explained in *Essential Question* (TE, p. 424). Solve & Share: Consider giving students opportunities to use tools and models to solve the problem. Students can use two colored counters to solve the set model problem. Look Back: Consider facilitating a discussion around the *Look Back!* to see if students make connections between the numerators and denominators of the equivalent fractions. If students do not make connections, consider using this lesson to continue with a conceptual understanding of equivalent fractions through representations before showing the procedure. Visual Learning: In the *Visual Learning*, students are shown multiplication to find equivalent fractions. Consider reading the questions and *Prevent Misconception* as there is important information and questions to ask students regarding multiplication to find equivalent fractions (TE p. 424). Reiterate the relationship between the numerator and denominator and not the size. Convince Me: Consider facilitating a discussion around the *Convince Me!* as it connects to the *Visual Learning Animation*. Guided Practice: Consider facilitating a discussion around item 1 with the whole class as it compares to the area model with multiplication to find equivalent fractions. Consider giving students an opportunity to use area models, number lines or fraction strips to compare 5/6 and 10/12. Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Assess and Differentiate/Intervention Activity: Consider using area models, number lines or two colored counters for the *Intervention Activity* to support the conceptual understanding of equivalent fractions.

Lesson 8-4: Generate Equivalent Fractions- Division		
4.NF.A.1 MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: In Topic 7, students learned to find factors of whole numbers. In the previous lesson, students learned to find equivalent fractions by multiplying the numerator and denominator by the same whole number greater than 1. Beginning of the Big Idea: In this lesson, students will learn to find equivalent fractions by dividing the numerator and denominator by a number that is a factor of both.	Note: If students are still working conceptually with equivalent fractions, consider using this opportunity to facilitate discussions around using models when comparing equivalent fractions. Solve & Share: Consider giving students the opportunity to use multiple tools or models to find equivalent fractions for 6/10. Visual Learning: The mathematical vocabulary of common factor is discussed in the *Visual Learning Animation*. Students find common factors greater than 1 by which to divide the numerator and denominator by. This idea begins to support simplifying fractions, which is still finding an equivalent fraction. The *Visual Learning Animation* uses denominators beyond those indicated in the standards. Only use if these ideas do not come out during the whole class discussion following the Solve and Share. Ensure student engagement and understanding by stopping the animation and discussing key ideas. Convince Me: Consider facilitating a discussion around the *Convince Me!*, as it connects the division learned in the *Visual Learning Animation* to a number line used to find equivalent fractions in Lesson 8-3. Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems. Assess and Differentiate/Intervention Activity: Consider using the *Intervention Activity* with students, but give students who are having difficulty with equivalent fractions an opportunity to use tools and models.
Lesson 8-5: Use Benchmarks to Compare Fractions		
4.NF.A.2 4.NF.A.1 MP.1 MP.2 MP.3 MP.8	Access Prior Learning: In third grade, students learned how to compare fractions with the same numerator or denominator by reasoning about the size of the fractions. Developing the Big Idea: In this lesson, students will learn about benchmark fractions and how benchmark fractions are used to compare fractions. Look Ahead: Benchmark fractions will be an important part of estimating with unlike denominators in Topic 9. Continue to use estimation language and emphasize accurate when comparing to benchmark fractions throughout Topic 8.	Solve & Share: Consider handing students strips of paper all the same length, and having students shade different portions of each strip. Then have students estimate how much of the strip is shaded, and explain how they estimated. Consider having students critique the reasoning of others or construct an argument of why a student may agree or disagree with another student based on the estimate. Look for students who use benchmark fractions to compare. Look Back: Consider facilitating a discussion around the *Look Back!*, as this will support students thinking in regards to comparing fractions to ½, and the relationship between the numerator and denominator. Visual Learning: In the *Visual Learning Animation*, compare to ½ to determine equality or inequality. Convince Me: In the *Convince Me!*, students use a model or number line to compare fractions. Consider facilitating a discussion around the *Convince Me!*. Another Example: Consider facilitating a discussion around the *Another Example!* as students use one whole as a benchmark to compare fractions. Assess and Differentiate/Intervention Activity: In the *Intervention Activity,* students can solve the activity using fractions strips, Teaching Tool 13 or student-made strips from Lesson 8-1 to continue understanding fractions conceptually.
Lesson 8-6: Compare Fractions		
4.NF.A.2 4.NF.A.1 MP.2 MP.3 MP.5	Access Prior Learning: In third grade, students learned how to compare fractions with the same denominator by comparing numerators. Developing the Big Idea: In this lesson, students will compare fractions by changing them to equivalent	Solve & Share: Consider giving students opportunity to use various strategies, tools or models to compare fractions in the *Solve & Share*. Consider child-watching for misconceptions because they may think larger numbers means a larger fraction. Reiterate the relationship between the numerator, denominator and relative size. *-continues on next page-*

	fractions with a common denominator or numerator.	Look Back: Consider using the *Look Back!* as an extension for early finishers or use the *Look Back!* to facilitate a discussion around comparing fractions using inequalities. Visual Learning: In the *Visual Learning Animation*, students use their prior knowledge of multiples to find common denominators to compare fractions. Consider facilitating a discussion around why this procedure works in comparing fractions conceptually. Number lines are a model used to determine the equality and inequalities of fractions. Another Example: Consider facilitating a discussion around the *Another Example!* as students apply the knowledge of equivalent fractions to compare numerators. The *Another Example!* gives students another strategy to use when comparing fractions. Independent Practice/Math Practices and Problem Solving: Consider having students do item 6-15. Consider having students explain how they compared the fractions and why it may be correct. Child-watch to see if students apply number sense using tools, strategies or models to solve the items. Consider facilitating a discussion around item 16 with the whole class, as students need to understand when comparing fractions, the fractions need to represent the same whole. Concepts of area could be used to explain why the fractions are not equivalent. Assess and Differentiate/Intervention Activity: Consider facilitating a discussion with all students around the *Intervention Activity* as students compare fractions with like numerators, with like denominators or neither. *CTC: *Quick Check* (digital platform)
Lesson 8-7: Math Practices and Problem Solving- Construct Arguments		
4.NF.A.1 4.NF.A.2 MP.3 MP.1 MP.2 MP.5	Access Prior Learning: In previous topics and lessons, students have constructed arguments. Developing the Big Idea: In this lesson, students will construct arguments about comparing fractional amounts.	Note: This lesson focus is on constructing arguments in math, but the mathematical idea is to compare fractions based on the same-sized whole. Solve & Share, Visual Learning and Convince Me all focus on the mathematical idea when comparing fractions the comparison needs be part of the same-sized whole. Independent Practice/Math Practices and Problem Solving: Consider having students do items 6-9, as students need to understand the values of the fractions and where the fractions fall on a number line. Similar to Van de Walle, et al. (2010) problem "Who is Winning?" (p. 290).

References

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Small, M. (2014). *Uncomplicating fractions to meet common core standards in math, K-7.* New York, NY: Teachers College Press, Nelson Education.

Van de Walle, J.A., Karp, K., Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally*. New York, NY: Pearson.

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