2nd Grade

WCSD Curriculum Guides Elementary Mathematics



Version 3: 2019/2020

Curriculum is one component of a larger mathematics instructional program in Washoe County School District (WCSD) for Kindergarten through 5th grade students. The purpose of curriculum guides are to bridge the district's K-5 Philosophy of Mathematics Education with the Nevada Academic Content Standards (NVACS) through a connection of the Curriculum Pacing Frameworks, instructional materials (*Bridges in Mathematics* or *enVisionmath2.0*), research based instructional practices and clarification of the standards when necessary. The following describes a course of study for the specified grade for one year. *ALL* students must receive quality instruction in *ALL* grade level standards in one instructional year.

This guide is designed to be **used with the instructional materials** during planning. *This guide is not meant to supplant any portion of the instructional materials*. Teachers will continue to read through Units/Topics during instructional planning.

Guide language:

Throughout the guide the following language is used to describe the level of understanding expected at the lesson level. This language is found in the lesson-by-lesson section in the column labeled "Big Idea Mathematical Development".

Beginning: Indicates students initial explorations with the mathematical idea(s) explored in the lesson. *Instruction continues* to the next lesson.

Developing: Students have worked with the mathematical ideas in previous grades or previously during the year. The focus of the lesson is to connect and build student understanding. Teachers provide intensified support to students who may exhibit misconceptions, partial understanding, no or limited understanding. **Instruction continues** to the next lesson.

Secure: Indicates that students have worked previously with these ideas and are expected to be at a level of secure understanding. Students with secure understanding are able to make connections and use the mathematics in a variety of situations; yet may still struggle expanding the understanding to non-routine situations. Students who are secure may still make mistakes at times; yet these students demonstrate that they have mathematical understanding with limited if any misconceptions. Students not secure in the understanding by the end of that Unit/Topic might benefit from small group intensification on these ideas. Teachers may choose to use an A/D/E (Assessment, Differentiation or Extension) day to provide additional instructional opportunity; yet should be cautious to not spend too long exploring these ideas to ensure students have ample opportunity for instruction to ALL of the Nevada Academic Content Standards (NVACS) for mathematics.

	NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
	Lesson 2-1: E	ven and Odd Numbers	
This lesson indicates a level	2.OA.C.3 2.OA.B.2 MP.4 MP.5	Access Prior Learning: In first grade, students had the opportunity to work with the classification of even and odd numbers. Securing the Big Idea:	Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be proposed with doubles facts. Topic Opener: Consider limiting the Topic Opener to discussion of the Topic Essential Question (TE p.77), Review What You Know (TE p. 78-80) and the Topic 2 Vocabulary Words Activity with the words even and odd. Introduce remaining vocabulary words as they appear in the lessons. Post
of secure understanding.	MP.7	In this lesson, students are securing understanding that numbers GRO, be Classified as even or odd by showing numbers as two equal parts.	Wisual Learning: Have students make cube towers to increase understanding and engagement. Although the Visual Learning discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that can be broken into two equal

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Note:

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▶ Grade 2 Topic 1: Fluently Add and Subtract Within 20

Big Conceptual Idea: K-5 Progression Operations and Algebraic Thinking (pp. 18-21)

Prior to instruction, view the Topic 1 Professional Development Video located in Pearson Realize online. This takes less than 3 minutes. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 1A-1E), the Topic Planner (pp.1I-1K), the Topic Performance Assessment (pp. 75-76A) and all 10 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 1A-1F)	Topic Essential Question: What are strategies for finding addition and subtraction facts?
	Reference TE p. 1 and Answering the Topic Essential Questions (TE, pp. 71-72) for key elements of answers to the Essential Questions.

The lesson map for this topic is as follows:

1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	Assessment
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5 A/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on fluently adding and subtracting within 20, and the big idea that the operations of addition and subtraction are related. Focus instruction on Nevada Academic Content Standard (NVACS, 2010) 2.OA.B.2.

Topic 1 Fluently Add and Subtract Within 20 Number of lessons: 10 over 12 days A/D/E: 5 days NVACS Focus: OA.B Total Days: ~17

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

2.OA.B.2 Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

As defined by the NVACS, fluency refers to "skill in carrying out procedures **flexibly**, **accurately**, **efficiently and appropriately**" (2010, p. 6). It does NOT mean memorization. When instruction focuses on memorization, students are less willing to think about numbers and their relationships and to apply and develop their number sense (Boaler, 2009).

High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed testing stand in the way of number sense, giving students the impression that sense making is not important (Boaler, 2015).

Rather, development of fluency occurs in three phases: 1) Constructing meaning and counting strategies (e.g., count on, count back) 2) Reasoning strategies (e.g., making 10, near doubles) and 3) Working toward quick recall. The third phase, quick recall is defined as ~3 seconds, allowing students to use a known fact to quickly derive an unknown fact without resorting to inefficient counting methods (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

In first grade, students used strategies to add and subtract within 20, demonstrating fluency within 10 (NVACS, 2010, 1.OA.C.6). These strategies included counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums. First grade students connect concrete and representational models to abstract equations. Instruction in Topic 1 focuses on developing these strategies and others through relationships and number sense. The part-part-whole relationship- conceptualizing a number as being composed of parts is the most important numerical relationship that can be developed (Van de Walle, et al., 2014). Maintain focus on reasoning and discuss strategies that students invent. Although the lessons focus on a particular strategy, encourage students to use the strategy but do not require them to do so. A requirement such as this removes the reasoning from strategy development. Instead, honor student strategies by emphasizing their ability to determine the appropriateness of a strategy and justify its use.

When assessing fluency, AVOID timed tests. Approximately one-third of students begin to experience math anxiety at the onset of timed testing (Boaler, 2014). Brain research also concludes that stress blocks the working memory, preventing students from accessing math facts they know (Beilock, 2011; Ramirex, et al., 2013). In addition, timed tests do not tell us which strategies a student used or their level of flexibility. It is important that our instruction and assessment focus on numbers and their relationships. Better options for assessment include student interviews, observations, journaling or quizzes based on strategies (Bay-Williams, Kling, 2014). For examples, reference "Assessing Basic Fact Fluency" and My Fluency Progress (Teaching Tool 63).

Math Practice 3: Construct viable arguments and critique the reasoning of others.

Focus on opportunities for students to develop MP.3 behaviors. This is the focus of lesson 1-10. Reference the Teacher's Edition (TE, pp. F25-F25A) and the Nevada Academic Content Standards for Mathematical Practice.

Finally, please note that lessons 1-1 and 1-2 could be 2-day lessons giving you additional time to establish class routines and expectations for:

- Accessing and returning manipulatives
- Classroom discussion norms
- Mathematical Mindset
- Integrating ideas from the Math Practices and Problem Solving Handbook (TE, p. F19-F35)
 - o Problem Solving Guide and Problem Solving Recording Sheet (TE, p. F31-F32)
 - Pay particular attention to "A Caution" (TE, p. F32, last paragraph)

Anchor Chart of Addition and Subtraction Strategies: Throughout the topic, have students construct a class anchor chart of addition and subtraction strategies. It is helpful to include representations of each strategy. These strategies include, but are not limited to: Count On to Add, Doubles, Near Doubles, Make a 10 to Add, Patterns on the Addition Fact Table, Count On to Subtract, Count Back, Think Addition, Make a 10 to Subtract (Add on to Make 10 and Subtract to Make 10).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction and post on math focus wall.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
addends doubles near doubles	equal sign, = equation sum difference		

Additional terminology that students may need support with: add, break apart, compare, connect, contrast, minuend (the whole), part, subtract, subtracted), whole

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students developing conceptual understanding of the relationship between addition and subtractions to help develop fluency within 20?" (every subtraction fact has a related addition fact-inverse relationships)

Lesson	Evidence	Look for
1-6	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		use of reasoning to analyze the relationship between addition and subtraction
		use part/part/whole understanding when subtracting or adding
1-6	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
		(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 71-76
Assessments (summative)	SE pp. 71-76	

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations		
Lesson 1-1: Addition Fact Strategies				
2.OA.B.2	Access Prior Learning:	Possible 2-day lesson		
MP.2 MP.4 MP.7 MP.8	(Coherence: TE p.1C-1D) In first grade, (1.OA.C.6) students used the counting on strategy to add within 20, demonstrating fluency within 10. Students also applied the commutative and associative properties of addition (1.OA.B.3).	Day 1: Topic Opener: Limit use of the <i>Topic Opener</i> to the <i>Topic Essential Question, Review What You Know</i> and <i>Topic 1 Vocabulary Words Activity</i> with the word, <i>sum.</i> Introduce remaining vocabulary words as they appear in the lessons. Consider establishing class discussion norms and activating student schema by asking students to discuss the <i>Topic Essential Questions</i> (TE, p.1). This conversation will inform your instruction while establishing routines and expectations. Post the questions and student strategies on your math focus wall.		
	Securing the Big Idea: In this lesson, students are securing understanding that counting on is a strategy to find sums within 10.	Solve & Share: Introduce routines for tool use and management. Students MUST have access to, and be encouraged to use tools throughout math instruction daily. Use student solutions to begin an anchor chart of addition and subtraction strategies (see sample below). Add to this chart throughout the topic. Although the image is teacher-made, student made resources are encouraged.		
	Developing the Big Idea: In this lesson, students are developing understanding that counting on is a strategy to find sums extending to within 20. They are also developing understanding of the Commutative Property of Addition, that the order of the addends does not change the sum. Associative Property of Addition: (a + b) + c = a + (b + c)	Addition Strategies Count On Count Back Cou		
	Commutative Property of Addition: a + b = b + a	Visual Learning: Have students use connecting cubes to explore 6 + 3 and 3 + 6 to increase conceptual understanding of the commutative property and increase engagement. Add any new strategies to the anchor chart started on Day 1. • Add the Commutative Property and other student-invented strategies to the anchor chart • Reinforce with representations and examples Independent Practice/Math Practices and Problem Solving: Throughout this topic, students do NOT need to do all of the problems in their student edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The Independent Practice page offers problems that support procedural skill and fluency. The Math Practices and Problem Solving page offers problems that support application. The Quick Check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate. For item 14, consider using the Problem Solving Recording Sheet (Teaching Tool 1) to model how this tool can help students solve problems but do not require them to complete the sheet in written form during this lesson. Assess and Differentiate: If time permits, teach students how to play Listen and Learn from lesson 1-8 (TE, p. 51A). All students should have the opportunity to play this game throughout the topic. Based upon childwatching for the Commutative Property of Addition and counting on, identify students who need additional support and pull them in a small group to do the Intervention Activity (TE, p.9A)		

Lesson 1-2: Doubles and Near Doubles

2.OA.B.2 Access Prior Learning:

MP.4 MP.6 MP.7 MP.8 Doubles facts are often referred to as "equivalent but easier or known sums". In first grade (1.OA.C.6) students created equivalent but easier or known sums within 20, demonstrating fluency within 10. For example, when adding 6 + 7, they used a doubles plus one equivalent: 6 + 6 + 1 = 12 + 1 = 13.

Securing the Big Idea:

In this lesson, students are securing understanding that doubles facts can be used to find basic addition facts that are near doubles within 10.

Developing the Big Idea:

In this lesson, students are developing understanding that doubles facts can be used to find basic addition facts that are near doubles extending to within 20.

Possible 2-day lesson

Day 1:

To assess readiness for near doubles, consider having students create a word web (Teaching Tool 60; also see TE pp. 437-438 for examples) for doubles using pictures or equations. The word web can be used again in the future for explicit vocabulary work.

Solve & Share:

Continue to build routines for tool use and management. Students should be encouraged to model with math (MP.4) using connecting cubes. As the near doubles strategy emerges from student solutions and the class discussion, add it to the anchor chart of addition strategies.

- Add Doubles and other student-invented strategies to the anchor chart
- Reinforce with representations and examples

Day 2:

Visual Learning:

Have students use connecting cubes to explore 7+7, 7 + 8, and 7 + 9 to increase understanding of the near doubles strategy and increase engagement.

- Add Near-Doubles (Doubles +1, Doubles +2, Doubles -1, Doubles -2) and other student-invented strategies to the anchor chart
- Reinforce with representations and examples

Independent Practice/Math Practices and Problem Solving:

Students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Assess and Differentiate:

If time permits, teach or revisit-*Listen and Learn* center game from lesson 1-8 (TE, p. 51A). All students should have the opportunity to play this game throughout the topic. Child-watch to identify students who need additional support with doubles/near doubles. Pull these students in a small group to do the *Intervention Activity* (TE, p.15A)

Lesson 1-3: Make A 10 to Add

2.OA.B.2

MP.1 MP.2

MP.3

MP.5 MP.7

Access Prior Learning:

In kindergarten, (K.OA.A.4) students worked on sums to 10. In first grade, (1.OA.C.6) students used the making ten strategy to add within 20, demonstrating fluency within 10.

Developing the Big Idea:

In this lesson, students are developing understanding that some addition facts with an addend near 10 can be found by changing to an equivalent fact with 10. (9 + 3 = 9 + 1 + 2 = 10 + 2 = 12) Students will relate this strategy to the Making 10 to Subtract strategy in lesson 1-8.

The Make a 10 strategy may be the most important strategy for children to know (Van de Walle, et al., 2014, p. 160). It helps students develop flexibility in their ability to add and subtract. The use of tools such as ten-frames and math racks* help students to visualize number relationships. Although students begin with the use of concrete manipulatives, they will eventually be able to apply the strategy mentally. As articulated in the instructional note at the start of this document, fluency development occurs in three phases. Pushing students to quick recall too early is detrimental to their mathematical mindset and ability to apply strategies flexibly. Understanding of the Make A 10 to Add strategy will later extend to their application of the Make a 10 to Subtract strategy and to their work with adding and subtracting multiples of ten within 100.

*In first grade students worked with number racks. Number racks are not provided with the instructional materials.

Example of Number Racks



Solve & Share:

Students had extensive experience with ten-frames and number racks in kindergarten and first grade. These tools reinforces students' understanding of the structure of 10 in our number system. Look for students who make use of the structure provided by placing 9 counters on one ten frame, decomposing the 3 into 2 and 1, then combine the 9 and 1 to make 10 before placing the remaining two counters on the second ten frame. These students are inherently using the associative property by decomposing the 3 into a 1 and a 2, then associating the 9 and 1 before adding 10 and 2. Also, look for students who count all or count on without attending to the ten structure. Use the *Intervention Activity, Stacking and Making 10* (TE, p. 21A) with these students.

- Add Make a 10 and other student-invented strategies to the anchor chart
- Reinforce with representations and examples

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Assess and Differentiate:

Consider replacing the *On-Level* and *Advanced* Activity Center with *Listen and Leam* center game from lesson 1-8 (TE, p. 51A). All students should have the opportunity to play this game. As noted above, child-watch during the *Solve & Share*. Identify students who did not use the structure of ten to solve 9+3. Engage these students in the *Intervention Activity: Stacking and Making 10* (TE, p. 21A).

Lesson 1-4: Addition Fact Patterns

2.OA.B.2

MP.8

Access Prior Learning:

Earlier in this topic, students
learned strategies for developing
fluency with addition facts within 20
(reference the anchor chart you've built thus far).

Beginning the Big Idea:

In this lesson, students are *beginning* find and recognize patterns in the addition facts table. They begin to see these patterns as useful for adding numbers and developing mental math strategies.

A note of CAUTION: We do not want students to rely on the Addition Facts Table to find answers to basic facts. Watch for students who want to use the Addition Facts Table to find sums and missing addends. **Emphasize that this table is a tool to help them see patterns** in addition facts and a relationship between the parts of a problem (addends and sum).

Solve & Share:

Consider extending the guiding questions in the *During* phase (TE, p.23) by asking students to also reason about the addends with questions such as: "How are the first addends changing? How are the second addends changing?" The goal of these questions is for students to make a connection between changes in the sums and changes in the addends. These conversations will link to the *Visual Learning* and offer another entry point for students into the content.

If your students do not identify the patterns displayed in *Analyze Student Work* (Manny's Work and Gordon's Work), consider displaying the samples provided (TE, p. 23, and available online under the *Solve & Share* as "Teacher Resources").

Develop: Problem-Based Learning



Math Practices & Problem Solving: Construct Arguments: Solve & Share

Assign Info T Teacher resources

- Add patterns to the anchor chart
- Reinforce with representations and examples

Independent Practice/Math Practices and Problem Solving:

Consider adding item 10 to guided practice. Provide students with time to write as many addition facts with a sum of 12 as they can. Look for students who organize these facts in a way that promotes the use of patterns. Facilitate a whole class discussion around how patterns can help us solve the problem.

Lesson 1-5: Count On And Count Back To Subtract

2.OA.B.2

MP.1

MP.2 MP.4 MP.5

Access Prior Learning:

In first grade, (1.OA.C.6) students used counting on and the relationship between addition and subtraction to add and subtract within 20, demonstrating fluency within 10. The number line was used for both addition and subtraction.

Securing the Big Idea:

In this lesson, students are securing understanding of the count on and count back strategies to subtract on a number line within 10.

Developing the Big Idea:

In this lesson, students are *developing* understanding of the count on and count back strategies to subtract on a number line within 20.

The use of the number line helps students connect counting to adding and subtracting. When students count on to subtract, help them to understand that they are adding to subtract and that subtraction is an unknown-addend problem. This will be a helpful connection to lesson 1-6: *Think Addition to Subtract*.

If students have trouble keeping track of how many spaces they are counting, or if they are counting the tick marks instead of spaces, refer to the Error Intervention: Item 2 suggestion (TE, p.30) for an appropriate scaffold. Students generally find counting back to be a more difficult strategy, but certain contexts lend themselves to this strategy.

Consider extending student understanding throughout the lesson by asking students to show another way to solve the problem. This might elicit a greater balance between the count on and count back strategies should students choose to use the number line again rather than a picture, equation or concrete manipulative such as counters. To extend even further, encourage students to make a generalization by considering when *count on* is a more appropriate strategy (e.g., the minuend and subtrahend are close together such as in 14 - 11 = ?) and when *count back* is a more appropriate strategy based upon the numbers (e.g., the minuend and subtrahend are further apart such as in 12 - 4 = ?).

- Add Count On, Count Back and other student-invented strategies to the anchor chart
- Reinforce with representations and examples

Lesson 1-6: Think Addition To Subtract

2.OA.B.2

MP.2 MP.7 MP.8

Access Prior Learning:

In first grade, (1.OA.C.6) students used the relationship between addition and subtraction to add and subtract within 20, demonstrating fluency within 10. Students also understood subtraction as an unknown-addend problem (1.OA.B.4).

Securing the Big Idea:

In this lesson, students are securing understanding of the inverse relationship between addition and subtraction and that this relationship can help them find subtraction facts within 10.

Developing the Big Idea:

In this lesson, students are developing understanding of the inverse relationship between addition and subtraction and that this relationship can help them find subtraction facts within 20. Students are developing understanding that every subtraction fact has a related addition fact.

Students continue to develop fluency by using previously learned addition facts to solve subtraction facts. This work develops their understanding of the inverse relationship of addition and subtraction. It also develops student understanding of part-part-whole relationships. The terms *part* and *whole* are not explicitly taught, but are essential to students developing understanding of the relationship of numbers. As Van de Walle, et al., (2014) states, "To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers. For example, 7 can be thought of as a set of 3 and a set of 4 or a set of 2 and a set of 5" (p. 108).

Solve & Share:

- Add Think-Addition and other student-invented strategies to the anchor chart
- Reinforce with representations and examples

Visual Learning:

In the *Do You Understand? Show Me!*, allow students to provide examples of addition facts that help them solve subtraction facts. Use these examples to facilitate a class discussion on how mathematicians can explain their thinking using words and sentences.

As a class, craft a written response to model Math Practice 3: Construct Viable Arguments and Critique the Reasoning of Others, and second grade expectations including:

- Provides complete and clear explanations of one's thinking.
- Uses examples and counterexamples when appropriate.
- See the Math Practices and Problem Solving Handbook for ideas on how to develop, connect and assess the Math Practices (TE, p.F25A)

This crafted response can stand as a model in future lessons, reflecting expectations for student written work.

*CTC: Solve & Share (student work samples)
*CTC: Quick Check (digital platform)

Lesson 1-7: Make A 10 To Subtract

2.OA.B.2

MP.3 MP.4 MP.5 MP.7

MP.8

Access Prior Learning:

In lesson 1-3, students learned to add by making 10.

Developing the Big Idea:

In this lesson, students are developing understanding that some subtraction facts can be simplified by making use of the numbers' relationships to 10. They will learn that there are two different ways to Make A 10 to Subtract: add on to make a 10, and subtract to make a 10.

Students use ten-frames as tools to represent the "Make A 10 strategy", which helps build fluency. This also helps students understand that a subtraction fact can be changed into a fact with a 10 without changing the difference. For students who struggle with this strategy, consider offering another entry point through the use of the make a 10 strategy using a number line. Help these students connect the different representations (number lines and ten-frames) of the same strategy.

Solve & Share:

As students problem solve, child-watch for students who add on to make a 10 (similar to Laura's Work, TE, p. 41) and students who subtract to make a 10 (similar to Amari's Work, TE, p. 41). Consider selecting students to share so that the strategies build in efficiency or accuracy. For example, you may have a child who does not use make a 10 share first, and then follow with a child who added on to make a 10 and finish with a child who subtracted to make a 10. If these strategies were not used by your students, use the work samples provided on TE, p. 41. Focus the conversation so that students make connections between the various strategies and evaluate which are the quickest and most accurate.

Visual Learning:

If students struggle to make a 10 in the *Solve & Share*, consider having them solve 13 – 7 (the problem posed in the animation) using their strategy of choice prior to viewing and discussing the *Visual Learning Animation*. Provide students with counters and ten-frames so they can represent the problem and connect to their experience in the *Solve & Share*.

Guided Practice:

Consider using ten frames (Teaching Tool 8) and counters to support students' understanding of the Make A 10 to Subtract strategies. Item 3 from Independent Practice may also be used during Guided Practice.

Lesson 1-8: Practice Addition and Subtraction Facts

2.OA.B.2

MP.2 MP.3 MP.4 MP.8

Access Prior Learning:

In first grade, (1.OA.C.6) students selected strategies to add and subtract within 20, demonstrating fluency within 10.

Securing the Big Idea:

In this lesson, students are securing understanding that addends determine efficient strategies and the use of efficient strategies builds fluency within 10.

Developing the Big Idea:

In this lesson, students are developing understanding that addends determine efficient strategies and the use of efficient strategies builds fluency within 20.

The strategies for fluently adding and subtracting in lessons 1-1 to 1-7 are interconnected. In this lesson, students apply these strategies to continue to build fluency.

Before the Solve and Share, consider using the English Language Learners example (TE, p.47A) to support vocabulary and context.

Solve & Share:

Modify the problem to remove "...as quickly as you can. Hold up your hand when you are done". As discussed in the instructional note at the beginning of this document, an emphasis on speed causes anxiety and negatively affects students' mathematical mindsets (Boaler, 2016). In the discussion, help students to determine that many math problems have more than one correct answer. In this case, students could accurately write four related facts for 9, 7, and 2 or 9, 7, and 16.

Visual Learning:

It is easy to fall into old habits. Although the lesson refers to "recall", our emphasis should be on students' flexible, accurate and efficient use of strategies, not on memorizing. When students struggle with a fact or group of facts, ask them the *Essential Question* (TE, p.48): How do you decide which strategy to use to add and subtract quickly* and accurately? Encourage students to revisit the anchor chart of addition and subtraction strategies.

*Quickly is defined as approximately 3 seconds, as indicated in the instructional note at the beginning of this document.

Lesson 1-9: Solve Addition and Subtraction Word Problems

2.OA.A.1 2.OA.B.2

MP.1 MP.2 MP.6

Access Prior Learning:

In kindergarten, (K.CC.C.6) students compared the number of objects in groups. In kindergarten, (K.OA.A.1, K.OA.A.2) students also understood addition as putting together and adding to, and subtraction as taking apart and taking from.

In first grade, (1.OA.A.1) students represented and solved addition and subtraction word problems within 20. These problem types included add to, take from, put together, take apart and compare with an unknown in all positions. First grade students had an opportunity to solve and discuss start unknown and compare problem types but were not expected to demonstrate security.

Developing the Big Idea:

In this lesson, students are developing understanding that objects, diagrams, and equations can help solve different types of addition and subtraction word problems including add to, take from, put together, take apart and compare with an unknown in all positions.

Students will solve and discuss various problem types including add to, take from, put together, take apart and compare. For clarification on these problem types, and their progression, reference page 6-7, and 18-21, of the K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking.

Although the lesson specifies using counters also provide access to connecting cubes, as it is beneficial to allow students to choose an appropriate tool when problem solving (MP. 5). Their ability to physically connect or group these cubes supports place value understanding.

Solve & Share:

This word problem is a compare difference unknown problem type. Although students have experience with compare problems from prior grades, the language and context can be challenging for some students. As stated in the Progression Documents (linked above), "in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation..." (p. 12) For this reason, the use of labels (e.g., D or Diego and L or Leslie) and matching with objects and drawings can be helpful. Monitor students' solving the problem, ask questions that will encourage students to utilize these tools to conceptualize the context of the problem. "How can you use counters/connecting cubes to help you model and solve the problem?"

Visual Learning:

In the discussions that occur during *Visual Learning*, ask questions to help students connect their solutions in the *Solve & Share* to the comparison bar diagram. In doing so, students will gain conceptual understanding of the comparison bar diagram by connecting to their work with concrete and representational drawings or models. This will help students develop understanding of part-part-whole relationships. The comparison bar diagram is different than the bar diagram because the part sections are proportionally sized, rather than the same size.

Example of comparison bar diagram

8 <u>?</u>

-continues on next page-

Independent Practice/Math Practices and Problem Solving:

Encourage students to begin by solving the quick check items (indicated with pink check marks) using concrete objects and/or drawings before connecting to the abstract equation. If a student is struggling with these word problems using objects and drawings, consider asking them to solve a word problem of a different type. For example, if a student struggles with item 5 (a put together, addend unknown problem), consider modifying the problem as shown below:

-continues on next page-

Original item 5 (Put Together, Addend Unknown):

Juan reads 5 books. Susan reads some books. They read 11 books in all. How many books did Susan read?

Possible modification (Add to, Change Unknown):

Juan held 5 books. Susan handed him some more. Now Juan is holding 11 books. How many books did Susan hand him?

The inclusion of an explicit action (handing books) in the modified problem can offer an easier entry point for the student, if needed.

Possible modification (Add to, Result Unknown):

Juan held 5 books. Susan handed him 6 more. How many books is Juan holding now? The inclusion of an explicit action (handing books) AND changing from a "change unknown" to a "result unknown" offers another scaffold for students, if needed.

Lesson 1-10: Math Practices And Problem Solving: Construct Arguments

2.OA.A.1 2.OA.B.2

Access Prior Learning:

In first grade, students engaged in Math Practice 3.

MP.1 MP.2 **MP.3** MP.4

Earlier in this topic, students used the make a 10 strategy to solve addition and subtraction problems.

Developing the Big Idea:

In this lesson, students are developing understanding of Math Practice 3: Construct viable arguments and critique the reasoning of others. Students will use a combination of words, symbols, pictures and numbers to construct a clear and concise explanation of their thinking.

This lesson provides an opportunity to focus on the Thinking Habits associated with Math Practice 3. Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F25-F25A, F31) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the student edition (SE, p.F25).

Solve & Share:

In addition to having your students' share their work, consider displaying the samples provided in *Analyze Student Work* (TE, p. 59, and available online under the *Solve & Share* as "Teacher Resources").

Develop: Problem-Based Learning



Math Practices & Problem Solving: Construct Arguments: Solve & Share

Assign 🖺 Info **T** Teacher resources

Facilitate a discussion comparing Michaela's Work to Robin's Work. Ask questions such as, "How did Michaela/Robin show their work? Is their work accurate? What makes their work clear and easy to understand?" Display student work and label the pictures, numbers, symbols and words that illustrate their thinking.

Visual Learning:

After the *Visual Learning Animation*, work on *Do You Understand? Show Me!* (p. 60), by facilitating a class discussion on how mathematicians can explain their thinking, focusing on words and sentences.

As a class, craft a written response to model Math Practice 3: Construct Viable Arguments and Critique the Reasoning of Others, and second grade expectations including:

- Provides complete and clear explanations of one's thinking.
- Uses examples and counterexamples when appropriate.
- See the Math Practices and Problem Solving Handbook for ideas on how to develop, connect and assess the Math Practices (TE, p.F25A)

This crafted response can stand as a model in future lessons, reflecting expectations for student written work. In future topics, students will craft these responses individually or with partners.

WCSD K-5 Mathematics Curriculum Guide

References

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▶ Grade 2 Topic 2: Work with Equal Groups

Big Conceptual Idea: Equivalence

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teachers Edition (TE): Cluster Overview/Math Background (pp. 77A-77E), the Topic Planner (pp. 77I-77J), the Topic Performance Assessments (pp. 117-118A), and all 5 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 77A-77F)	Topic Essential Questions: How can you show even and odd numbers? How do arrays relate to repeated addition?
	Reference Answering the Topic Essential Questions (TE, p. 115-116) for key elements of answers to the Essential Questions.

Assessment

The lesson map for this topic is as follows:

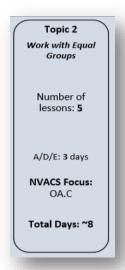
2-4

2-3 3 A/D/E days used strategically throughout the topic.

Instructional note:

2-2

The big idea of Topic 2 is equivalence. This topic focuses on a) securing understanding of the categorization of numbers as even or odd, and b) developing understanding for finding the total objects in situations involving equal groups. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.OA.C which supports the Topic 1 cluster 2.OA.B.



2nd Grade Curriculum Pacing Framework: **Balanced Calendar**

2.OA.C Work with equal groups of objects to gain foundations for multiplication.

- 3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Focus instruction on students' construction of a definition of even numbers as numbers that can be broken into two equal sets with no leftovers, reinforcing the big idea of equivalence. Likewise, investigate and discuss how odd numbers cannot be split into two equal sets. Do not define even and odd numbers by the patterns in the ones digits (e.g., 0, 2, 4, 6, 8 for even); 1, 3, 5, 7, 9 for odd). These patterns describe attributes of even and odd numbers but do not support the big idea of equivalence. Building on the work from Topic 1, students will apply their understanding of doubles to even numbers and their understanding of near doubles to odd numbers. It is important that students explore with concrete objects before moving to representations including drawings, arrays, bar diagrams and equations.

Work with equal groups requires students to apply the big idea of unitizing (Fosnot, Dolk, 2001). "Unitizing requires that children use number to count not only objects but also groups- and to count them both simultaneously" (Fosnot, 2007, p. 7). As students begin to work with arrays, they will connect their understanding of even and odd numbers. Even numbers can be represented using arrays with two equal groups (rows or columns), while odd numbers cannot be represented in arrays with two equal groups.

Students will also apply their ability to unitize by grouping objects in arrays by rows or columns, and develop the understanding that the total items in an array can be found through repeated addition of these units. Students will write an equation reflecting the sum of equal addends as equivalent to the total items in the array. Finally, students will apply these understandings to problem situations that involve equal groups. Look for opportunities to connect equal groups to students' real-world experiences. Work with arrays supports skip counting by 5s, 10s, and 100s to 1,000 in Topic 9, and the partitioning of rectangles into equal rows and columns of squares in Topic 15. Ultimately, work around the big idea of equivalence lays the foundation for algebraic reasoning and multiplication and division in grade 3.

Math Practice 4: Model with mathematics

Focus on opportunities for students to develop MP.4 behaviors. This is the focus of the Math Practices and Problem Solving lesson 2-5. Reference the Teacher's Edition (TE, pp. F26-F26A) and the Nevada Academic Content Standards for Mathematical Practice.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
even	equal		
odd	part		
equal groups*	whole		
* Add to vocabulary explicitly taught			
in lesson 2-3			
array			
rows			
columns			
bar diagram			

Additional terminology that students may need support with: addends, equation, model, pairs, sum

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding questions:

- "Are students developing conceptual understanding and moving their thinking counting all objects to repeated addition of arrays?"
- "Are students developing conceptual understanding of organizing models to represent math equations?"

Lesson	Evidence	Look for
2-4	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		use of repeated addition
		understanding arrays as equal rows and columns
2-4	Do You Understand: Show Me!	Focus CTC around data analysis and collection of student workspace
	(digital platform) *Optional in SE	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessment	Use Scoring Guide TE pp. 115-118
Assessments (summative)	SE pp. 115-118	·

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 2-1: E	ven and Odd Numbers	
2.OA.C.3 2.OA.B.2 MP.4 MP.5 MP.6 MP.7	Access Prior Learning: In first grade, students had the opportunity to work with the classification of even and odd numbers. Securing the Big Idea: In this lesson, students are securing understanding that numbers can be classified as even or odd by showing numbers as two equal parts.	Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be represented with doubles facts. Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p.77), <i>Review What You Know</i> (TE, pp. 78-80) and the <i>Topic 2 Vocabulary Words Activity</i> with the words <i>even</i> and <i>odd</i> . Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. Visual Learning: Have students make cube towers to increase understanding and engagement. Although the <i>Visual Learning</i> discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that can be broken into two equal parts with no leftovers and odd numbers as numbers that cannot. The patterns of ones digits should be understood as an attribute of even/odd numbers, not as their definition. (See instructional note at beginning of this document.) Encourage children to revise and add to their word webs for both concepts: even and odd.
		-continues on next page-

Independent Practice/Math Practices and Problem Solving:

Students do NOT need to do all of the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. For item 12, consider using the *Problem Solving Recording Sheet* (Teaching Tool 1) to help students make sense of the problem. Allow students to work on this problem collaboratively in pairs with manipulatives before incorporating into whole group guided practice. Watch for students who use concrete objects, drawings, equations or tables to organize their thinking. Focus the discussion on the mathematical generalization that can be drawn from this work: that adding two whole odd numbers will always have an even sum. Explore why this works.

Lesson 2-2: Continue Even and Odd Numbers

2.OA.C.3 2.OA.B.2

Access Prior Learning: In first grade, students had the opportunity to skip count by 2s and identify patterns in skip counting.

Have students revisit and add to the word webs for *even* and *odd* to assess understanding and inform instructional decisions. Look for students who demonstrate understanding that even numbers can be broken into two equal parts, and that odd numbers cannot.

MP.2 MP.4 MP.6 MP.7

MP.8

In the prior lesson, second grade students broke apart cube towers to classify numbers as even or odd.

Solve & Share:

Before problem solving, ask students to make sense of the problem. Clarify vocabulary such as addends and sum as needed. During problem solving, child-watch for students who are able to use cubes to represent equations with two equal addends and demonstrate understanding that the addends in the equations represent the number of squares in each row. This idea will be reinforced in the *Visual Learning*. Strategically select two students to share their solutions building in sophistication or accuracy. In the discussion, focus your guiding questions on facilitating student connections between strategies and connections to the big mathematical idea of equivalence with questions such as, "What connections can you make to addition strategies we used in Topic 1?" [doubles facts]. "How can skip counting help us to answer the question?" [skip count by 2s]

Securing the Big Idea:

In this lesson, students are securing understanding that numbers can be classified as even or odd by analyzing skip-counting patterns and writing even numbers as a sum of equal addends.

Visual Learning:

Engage students in discussion throughout the *Visual Learning Animation*, intentionally connecting back to the *Solve & Share*. Ask questions to help students connect the cube towers they built in the *Solve & Share* to the representational drawings, arrays and equations seen in the animation. This will strengthen their conceptual understanding.

Lesson 2-3: Use Arrays to Find Totals

2.OA.C.4 2.OA.B.2

MP.1 MP.3 **MP.4**

MP.7

Access Prior Learning:
In the prior lesson, second

In the prior lesson, second grade students wrote equations to represent even numbers, connecting rows of objects to the addends in the equation.

Beginning the Big Idea:

In this lesson, students are beginning understanding that arrays show equal groups, and that equations using repeated addition can be used to find the total objects in an array.

Work with arrays and repeated addition develop students' understanding of equivalence and lay the foundation for multiplicative thinking in grade 3. Students work with equal groups in rows and columns, as well as equivalence in repeated addition equations to represent the total objects in an array (e.g., For a 4 x 3 array, 4 + 4 + 4 = 3 + 3 + 3 + 3 = 12.) After the *Solve & Share*, introduce the vocabulary word "array" using the *Graphic Organizer 5: Frayer Model* (Teaching Tool 62). This organizer includes the definition, characteristics, examples and non-examples.

Solve & Share:

In the teacher notes, omit step 2. *Build Understanding* (TE p. 93) as it provides too much up front and removes opportunities from students for problem solving. **Allow students to work on the** *Solve & Share without prior instruction*. Child-watch for students who are able to unitize (work with equal rows or columns) and identify equal groups of 5 and equal groups of 3. If students count by 1s, support students in unitizing, by clarifying the terms: rows and columns and ask "How can you use equal groups to help you find how many circles in all more efficiently?" The use of two-sided counters (red/yellow) can help students to visualize rows or columns as equal groups by alternating colors (see below). Students may also make connections to equal groups on ten-frames.



	Trylsjon matter at the state of						
	Make Arrays to Find Totals						
2.OA.C.4 2.OA.B.2 MP.2 MP.4 MP.5 MP.8	Access Prior Learning: In the prior lesson, second grade students used repeated addition to find the total objects in an array. Beginning the Big Idea: In this lesson, students are beginning understanding that making arrays, and using repeated addition can be used to solve addition problems.	Solve & Share: During problem-solving, child-watch for students who confuse columns and rows. During the share, ask two children to share their solutions, beginning with the student who built an array with 4 rows and 3 columns, followed by a student who built an array with 3 rows and 4 columns, as was asked. Engage children in a discussion regarding equivalence (both arrays have the same total). Then use this share as an opportunity to highlight MP. 6: Attend to Precision (TE, pp. F28-28A) by reviewing the meaning of columns and rows. Visual Learning: Use the Do You Understand? Show Me! to formatively assess students' understanding of arrays and equal groups. Independent Practice/Math Practices and Problem Solving: In preparation for item 9, look for opportunities for your students to interact with arrays in the real world. They can draw upon this experience to write a story problem using repeated addition. For example, have students walk in 2 equal lines when they come in from recess, put supplies away in an array formation, etc.					
		*CTC: Solve & Share (student work samples) *CTC: Do You Understand: Show Me! (digital platform)					
Lesson 2-5: N	Math Practices and Problem Solvi	ing: Model with Math					
2.OA.A.1 2.OA.C.4 MP.1 MP.3	Access Prior Learning: In first grade, students engaged in Math Practice 4, modeling with ten frames, number lines and open number lines.	MP.4: Model with Mathematics, encourages students to solve problems using the mathematics they know. Support students in determining whether their solutions make sense based on the context of the problem (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p.3). Also, focus on the MP.4 Model with Mathematics thinking habits included in the <i>Math Practices and Problem Solving Handbook</i> (TE, pp. 26-26A). Add these habits to the math focus wall for reference throughout the year.					
MP.4 MP.5 MP.6 MP.7 MP.8	Developing the Big Idea: In this lesson, students are developing understanding that mathematicians use math they know to show and solve problems. They model problems with drawings, arrays, bar diagrams and equations.	Solve & Share: During problem solving, look for students who use their prior knowledge to solve the problem. Strategically select students who used different models to share. In the share, ask questions such as, "How does [student A's] model show the problem?" and "How does this model show that you used what you know to solve the problem?" After discussion, encourage students to write an explanation of their use of MP.4. Visual Learning: If students are having trouble understanding the bar diagram, reference Error Intervention: Item 1 note (TE, p.106).					

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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Fosnot, C.T. (2007). The t-shirt factory: Place value, addition, and subtraction. Heinemann: Portsmouth, N.H.

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▶ Grade 2 Topic 3: Add Within 100 Using Strategies

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 3 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 119A-119E), the Topic Planner (pp. 119I-119K), the Topic Performance Assessments (pp. 187-188A) and all 9 lessons.

Mathematical Background:

Read Cluster Overview (TE, pp. 119A-119E)

Topic Essential Question:

What are strategies for adding numbers to 100?

Reference Answering the Topic Essential Questions (TE, pp. 183-184) for key elements of answers to the Essential Questions.

The lesson map for this topic is as follows:

3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9	Assessment

3 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic is adding using strategies.

...there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p.176).

Topic 3

Add Within 100
Using Strategies

Number of lessons:
9 over 11 days

A/D/E: 3 days

NVACS Focus:
NBT.B

Total Days: ~14

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).

Topics 3-6 compose a major cluster focused on the big idea of the base-10 numeration system. Focus instruction on Nevada Academic Content Standards (NVACS) cluster 2.NBT.B. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above, **place-value instruction does not need to occur in isolation** (Van de Walle, et al., 2014, p. 176). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand 10 as both 10 ones and 1 ten. In second grade, students extend these place value understandings to three-digit numbers, understanding 100 as a bundle of 10 tens and as a "hundred". To foster this development, the use of groupable models, models that children can group into tens (connecting cubes, beans in cups, bundles of straws, etc.) are essential. **Groupable models allow children to move from operating with ones only, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. Telling students that a pre-grouped model, such as a tens rod, is worth ten is ineffective. When considering language, help students connect standard language, "thirty-five", to base-ten language, "3 tens and 5 ones; 3 groups of ten and 5 ones, etc". To best support EL Learners, it is recommended that you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).**

2.NBT.B Use place value understanding and properties of operations to add and subtract.

- 5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

The Properties of Operations: Addition and Subtraction

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a+0=0+a=a

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Students work on adding within 100 using strategies such as adding tens and ones on a hundred chart and open number line, breaking apart numbers into tens and ones, and using compensation (reference lesson-level instructional notes below for additional content related to each strategy). These models reflect the three common types of invented strategy models (a) split strategies, which involve decomposition, (b) jump strategies similar to counting on and counting back, and (c) shortcut strategies such as compensation, which involve the adjustment of numbers (Van de Walle, et al., 2014, p. 210).

In order for students to develop computational fluency, it is important that they be able to use a variety of strategies with understanding and flexibility, adapting to the numbers and context. Van de Walle, et al., stated, "the issue is no longer a matter of 'knows how to subtract three-digit numbers'; rather it is the development over time of an assortment of flexible skills that will best serve children in the real world" (2014, p. 204). Although the lessons focus on a particular strategy, encourage students to use the strategy but do not require them to do so. A requirement such as this removes the reasoning from strategy development. Instead, honor student strategies by emphasizing their ability to determine the appropriateness of a strategy and justify its use. As identified in 2.NBT.B.9, second grade students are expected to, "Explain why addition and subtraction strategies work, using place value and the properties of operations." It also notes that explanations may be supported by drawings or objects. The flexible application of strategies using decomposing and composing numbers also builds students' number sense. It remains important to ensure that all students engage in the *doing* of mathematics through the eight mathematical practices. In particular, all students should engage in MP.5 Use Appropriate Tools Strategically on a daily basis. Students should be encouraged to select and use tools throughout math instruction, with teachers being cognizant of the effect their actions and tool storage systems have on these developing habits of mind.

Math Practice 5: Use appropriate tools strategically

Focus on opportunities for students to develop MP.5 behaviors. This is the focus of the Math Practices and Problem Solving lesson 3-9. Reference the Teacher's Edition (pp. F27-F27A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.				
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)			
break apart mental math compensation	tens ones open number line			

Additional terminology that students may need support with: landmarks

*Collaborative Team Conversations (CTC)

Consider using one of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding question: "Are students developing conceptual understanding and moving their thinking adding on by one's to a deeper understanding of place value?"

Lesson	Evidence	Look for
3-7	Solve & Share (student work samples)	Focus CTC around the big idea: student strategies and models using different strategies to add two-digit numbers (number lines, hundreds chart, compensation, break apart, etc.)
3-4	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 183-188
Assessments (summative)	SE pp. 183-188	

Standards listed in bo	Standards listed in bold indicate a focus of the lesson.							
NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations						
Lesson 3-1: A 2.NBT.B.5 2.NBT.B.9	Access Prior Learning: In first grade, (1.NBT.C.4) students	The use of a hundred chart reinforces students' understanding of the sequence of numbers to 100. It is also a helpful tool for analyzing the structure of our number system through patterns						
2.NBT.B.5		The use of a hundred chart reinforces students' understanding of the sequence of numbers to						
		[See a note of caution above about rote use of the hundred chart.] Instead, add directions for both children to use whiteboards and markers to solve the problem with a second strategy and compare answers with that found on the hundred chart.						

Lesson 3-2: Add Tens on an Open Number Line

2.NBT.B.5 2.NBT.B.9

MP.1 MP.3 MP.4 **MP.5**

MP.6

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students used regular number lines and open number lines as models.

In the prior lesson, second grade students broke apart 2-digit numbers into tens and ones to add using a hundred chart. The hundred chart can be less efficient, so students will move into the use of an open number line in this lesson

-continues on next page-

Developing the Big Idea:

In this lesson, students are developing understanding that 2-digit numbers can be added by breaking apart the tens and ones. They will represent this thinking with hops or jumps on an open number line.

The open number line is an effective tool to support students in explaining their reasoning when using a jump strategy. The open number line offers more flexibility than a regular number line as it allows students to work with any numbers, reduces confusion between hash marks and spaces, and results in fewer computational errors (Van de Walle, et al., 2014, p. 211). In addition, the open number line is a versatile tool that reinforces the inverse relationship between addition and subtraction, supports the development of place value understanding, number sense and computational fluency. It is unnecessary to label the jumps with the operation (+/-). Labeling the jump with the number only, reinforces the inverse relationship between addition and subtraction.

Solve & Share:

Child-watch for students who demonstrate varying levels of place value understanding. If a student uses jumps of ones allow them to finish, then ask, "Can you also solve this problem using jumps of ten? Which jumps were more efficient?" For students who make jumps of ten with understanding, ask them, "What patterns do you notice as you count by tens on the number line? OR "Can you think of an even more efficient way to jump?" Finally, if you see students making jumps of multiples of ten, such as 40 either refer to the *Extension for Early Finishers* (TE, p. 129) or ask, "Does it matter which addend you start with? Is one way more efficient than the other?"

Students' explanation may also be supported using the suggestions in the English Language Learners example on TE p. 129A. If the opportunity does not arise from your students' work, consider displaying Nico and Sheri's Work (TE p.129) to facilitate a conversation around the Commutative Property.

Visual Learning:

It is encouraged that teachers stop and discuss at any point in the animation they deem necessary for their students. At the second stopping point in the animation, students are asked "How many tens are in 30?" Use this as an opportunity to formatively assess place value understanding by having students use concrete manipulatives (connecting cubes or place value blocks) to model their answer. If students indicate that 30 is made of 30 tens, they may still be developing understanding of tens as a unit.

Lesson 3-3: Add Tens and Ones on an Open Number Line

2.NBT.B.5 2.NBT.B.9

MP.3 MP.4 **MP.5**

MP.6

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students used regular number lines and open number lines as models.

In the prior lesson, second grade students added tens to a 2-digit number on the open number line

Developing the Big Idea:

In this lesson, students are *developing* understanding that open number lines can be used to show how they broke apart a number into tens and ones to add two 2-digit numbers.

See instructional note in lesson 3-2 for information regarding the open number line. In this lesson, students break apart numbers to add two 2-digit numbers, deepening their place value understanding and computational fluency. Encourage discussion that compares students' different ways of jumping on the number line to solve a problem, reinforcing students' understanding of equivalence and their strategic selection of jumps.

Visual Learning:

The *Error Intervention: Item 2 note* (TE, p. 136) offers a suggestion that supports students' use of compatible numbers.

Independent Practice/Math Practices and Problem Solving:

Item 10 offers students an opportunity to demonstrate understanding of the Commutative Property of Addition (a+b=b+a). In addition to the three quick check items (marked with pink check marks), ask students to complete item 8 (Part-part-whole problem type) or item 9 (Compare problem type) to offer continued practice with word problems as specified in 2.OA.A.1.

Assess and Differentiate:

The *Intervention Activity,* "The Numbers Under the Line" (TE, p. 139A) offers students an additional entry point for understanding the open number line as a tool for place value addition strategies through the use of connecting cubes. Use this activity with students who demonstrate inaccuracies with the open number line, or who do not demonstrate understanding of breaking apart numbers into tens and ones with corresponding jumps.

Lesson 3-4: Break Apart Numbers to Add

2.NBT.B.5 2.NBT.B.9

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value.

Mental math strategies refer to strategies used without writing down steps, and often involve the break apart strategy. This promotes flexibility and helps build fluency.

Some students may be ready to do computations mentally, others may still be in the direct modeling stage, or need to write down intermediate steps to keep track as they think through the problem. You may be concerned about the time and effort some students use, however,

-continues on next page-

MP.4

MP.5 MP.7 MP.8 Students added tens and tens, and ones and ones, and sometimes composed a ten when needed.

In the prior lesson, second grade students broke apart 2-digit numbers to add using tens and ones on the open number line.

Developing the Big Idea:

In this lesson, students are developing understanding that 2-digit numbers can be broken apart using tens and ones and added in different ways. They break apart both addends and consider how breaking apart numbers can help them solve problems mentally.

time spent cultivating these early stages in a meaningful way will yield long lasting understanding and ultimately reduce the need for re-teaching. As students become more proficient with flexible methods, encourage them to do appropriate computations mentally (Van de Walle, et al., 2014, p.208). Oftentimes, students will find that using mental strategies based on place value are quicker than using other written strategies, including standard algorithms. An example of such a problem is: 26 + 48. It is quicker to add 50 and 26 to get 76, and then subtract 2 to get 74, than it would be to use the U.S. Traditional Algorithm.

Visual Learning:

The guided practice items offer students support with breaking apart numbers by including the structure of number frames (boxes for them to write the value of the tens and value of the ones). Students should progress to breaking apart numbers without the frames to support. Alternatively, if students need additional support, encourage them to build addends with connecting cubes, and then physically break the numbers apart into tens and ones. The focus should be helping students construct meaning by connecting the concrete model and breaking action to the abstract numbers (e.g., 17 is composed of 1 ten and 7 ones).

Independent Practice/Math Practices and Problem Solving:

If time allows, consider using item 13 as an extension in guided practice using the *Problem Solving Recording Sheet* (Teaching Tool 1). This word problem is a Compare Bigger Unknown problem and one of the more challenging problem types.

*CTC: Quick Check (digital platform)

Lesson 3-5: Continue to Break Apart Numbers to Add

2.NBT.B.5 2.NBT.B.9

> MP.1 MP.4 MP.7

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students added tens and tens, and ones and ones, and sometimes composed a ten when needed.

In the prior lesson, second grade students broke apart both addends into tens and ones when adding two 2-digit numbers.

Developing the Big Idea:

In this lesson, students are developing understanding that 2-digit numbers can be broken apart using tens and ones and added in different ways. They break apart just the second addend and continue to consider how breaking apart numbers can help them solve problems mentally.

See the instructional note in Lesson 3-4 regarding the break apart strategy.

Solve & Share:

Encourage students to solve the problem using two different strategies to promote flexibility with addition strategies. Strategically select students to share based upon their method. First, select a student whose work reflects the understanding of the majority of students. This allows most students an entry point into the discussion through a strategy they understand. Then, have a child who is using a more sophisticated strategy such as break apart or mental math share. Focus the conversation so students can connect the strategies and make meaning of the more sophisticated method.

Visual Learning:

When using the *Error Intervention*: Item 2 note in the guided practice, have students use connecting cubes and/or drawings to build the second addend, and then "break" the number apart into tens and ones.

Lesson 3-6: Add Using Compensation

2.NBT.B.5

MP.2 MP.3

MP.8

Access Prior Learning:

In first grade, students broke apart numbers using the 5 and 10 structure.

In Topic 1, second grade students used the making ten strategy. Earlier in this topic, students worked with a variety of addition strategies for adding two 2-digit numbers.

Developing the Big Idea:

Possible 2-day lesson

Compensation for addition makes the problem easier to solve mentally. The same amount is added to one addend, and subtracted from the other addend. For example, students solving 38 + 23 may add 2 to 38, and subtract 2 from 23, resulting in the problem 40 + 21. This strategy shows students' flexibility with numbers, increases their understanding of the inverse relationship between addition and subtraction, and builds fluency.

Day 1:

Solve & Share:

During the discussion around student strategies, encourage students to extend their thinking beyond *what* steps they did to compensate to *why* compensation works for addition. Ask them questions about equivalence, such as "Are 35 + 8 and 40 + 3 equivalent? How do you know?"

-continues on next page-

In this lesson, students are *developing* understanding of adding 2-digit numbers using the compensation strategy.

Visual Learning:

During the guided practice, watch for a misconception that the sum changes when the addends change. Have students use concrete manipulatives to justify their thinking by building both addends for each equation.

Assess and Differentiate:

The *Intervention Activity* "Using Compensation to Make a 10" (TE, p. 157A) offers another entry point for students, while supporting the Make a 10 strategy.

Day 2:

Consider selecting a problem from the *Independent Practice* and *Problem-Solving* pages (TE, pp. 155-156) and making it a *Solve & Share* OR facilitate a mini-lesson using a string of numbers intentionally structured to promote student use of the compensation strategy. These strings are intended to develop students' use of mental math, but do not require students to only solve the problems *in* their heads. Instead, focus on their ability to examine the numbers and select a clever and efficient way to solve the problem. As students verbally explain their thinking, make a written record so that students can "see" the strategy using an open number line. This becomes a picture for the class to discuss. Relying only on verbal explanations will limit access for children to understand (Fosnot, 2007, p.7). Although children may begin by using a variety of strategies, through discussion they will notice patterns in the string of problems and in the answers. These patterns will encourage students to examine the numbers *before* selecting a strategy.

58 + 22

60 + 20

30 + 50

28 + 52

32 + 48

33 + 47

98 + 42

97 + 34

Child-watch for students who identify that the first six problems are equivalent expressions. If this is unnoticed, point out that the first six problems have the same answer and ask, "Why is this happening? Which problem is the easiest?" As students demonstrate understanding of the compensation strategy, encourage them to use it to make the last two problems into equivalent but easier expressions. For example, change 98 + 42 to 100 + 40.

Child-watch for students who have difficulty deciding how to adjust the addends. Support these students by encouraging the use of tools, such as number lines and ten frames, to identify landmarks of ten close to the addends.

As part of their habits of mind, strategic thinkers look at the context and numbers in a problem

to determine the best strategy for solving it. Posting students' strategies on a math focus wall

Lesson 3-7: Practice Adding Using Strategies

2.NBT.B.5 2.NBT.B.6 2.NBT.B.9

Access Prior Learning:

Earlier in this topic, second grade students developed strategies for adding 2-digit numbers.

throughout the year can help students select appropriate strategies. Solve & Share:

MP.2 MP.4

MP.5

Developing the Big Idea:

In this lesson, students are *developing* understanding that there are different ways to add 2-digit numbers and that a strategy may be better for one problem than others.

During problem solving, child-watch for students who select a strategy based upon the numbers. Some students may break apart, and others may use compensation For example, in 39 + 43, students may reason that 39 is close to 40. Watch for students who use this landmark of ten to adjust and compensate, resulting in a new equation of 40 + 42 or 40 + 43 - 1. Also, watch for students who use their knowledge of doubles to solve the problem by adding 40 + 40 + 2. Use questioning to guide the class discussion to focus on strategy selection and evaluating the "better" strategy for the given problem.

Visual Learning:

Consider omitting the *Visual Learning Animation*. Instead, extend time spent in the *Solve & Share*, focusing on a variety of student selected strategies (look for and strategically select students who use the open number line, break apart, and compensation to facilitate discussion around the appropriateness of each strategy for the given problem.)

*CTC: Solve & Share (student work samples)

Lesson 3-8: Solve One-Step and Two-Step Problems

2.OA.A.1

MP.1 MP.2

MP.4 MP.6

Access Prior Learning:

In first grade, (1.OA.A.1) students used addition and subtraction within 20 to solve word problems of varying types, with unknowns in all positions.

In lesson 1-9, second grade students solved word problems of varying types, with unknowns in all positions, involving addition and subtraction within 20.

Developing the Big Idea:

In this lesson, students are *developing* understanding that some word problems can be solved in one-step.

Beginning the Big Idea:

In this lesson, students are beginning understanding that some word problems can be solved in two-steps, requiring a sub-problem or hidden question to be answered first in order to solve original question.

Possible 2-day lesson

The NVACS (2.OA.A.1), indicate that second grade students will solve one-step and two-step word problems involving addition and subtraction within 100. These word problems include add to, take from, put together, take apart and compare problem types with unknowns in all positions. Reference the NVACS, Table 1. Common addition and subtraction situations included on the last page of this document, for examples of these problem types (CCSSO, 2010, p. 88). Also reference page 6-7, and 18-21, of the K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking .

Students used bar diagrams and equations to solve word problems in lesson 1-9, and will continue to do so in this lesson. The use of bar diagrams reinforces understanding of the relationship between addition and subtraction, and helps students understand the relationship between the numbers in the problem.

Day 1:

Solve & Share:

A note of CAUTION: Compare Bigger Unknown problems are a more challenging problem type, as they do not include a specific action that students can more easily model, such as "she found __ more" or "he lost ____". For this reason, anticipate that students may need additional time to problem-solve. If students are having trouble understanding the context of the problem, encourage reasoning with concrete manipulatives and bar diagrams to make sense of the problem. Avoid "helping" students by modeling the problem for them and removing the "problem" from problem solving.

Visual Learning:

Consider asking students to connect the use of comparison bar diagrams in the animation to MP.1 Make Sense of Problems and Persevere in Solving Them by asking questions such as, "How can we use bar diagrams to help us make sense of the problem?" Listen for students who identify the changing placement of the unknown based on the context.

Independent Practice/Math Practices and Problem Solving:

Consider using some of the items from these pages (TE, pp. 167-168; SE, pp. 167-168) for a *Solve & Share* format in the WCSD Lesson 2 to follow.

Day 2:

In general, students find *add to* and *take from* problem types easier because they include explicit action. *Put together* and *take apart* problem types are generally more challenging, as they do not include explicit action. Finally, *compare* problems tend to be the most challenging problem types, as one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011). Keep this in mind as you respond to learners. All students need to solve all problem types, but we can use this information to scaffold and extend.

The Independent Practice and Math Practices and Problem Solving pages from lesson 3-8 (TE, pp. 167-168; SE, pp. 167-168) contain word problems of varying types. If your students struggled with the Compare Bigger Unknown problem from the 3-8 Solve & Share, consider strategically selecting problems from pages 167-168 to facilitate growth towards a compare problem type (see Suggestion A below).

If your students demonstrated understanding of the *Compare Bigger Unknown* problem in the 3-8 *Solve & Share*, strategically select problems from pages 167-168 to foster continued growth with *compare* problems and two-step word problems (see Suggestion B below). In both cases, support students in their sense making of the numbers and context with manipulatives and bar diagrams. Students will continue to use bar diagrams and solve word problems in enVision, spending a full topic on these problem types in Topic 7.

Classification of items (TE, pp. 167-168; SE pp. 167-168) by Problem Type: Reference the NVACS, Table 1. Common addition and subtraction situations for examples of these problem types (CCSSO, 2010, p. 88). A copy can be found on the last page of this Topic 3 document.

- Item 2: Compare Bigger Unknown
- Item 3: Separate Result Unknown/Join Result Unknown
- Item 4: Join Result Unknown
- Item 5: Put Together Total Unknown
- Item 6: Join Result Unknown/Separate Change Unknown
- Item 7: Compare Bigger Unknown
- Item 8: Put Together Total Unknown
- Item 9: Compare Smaller Unknown/Compare Bigger Unknown
- Item 10: Join Result Unknown/Put Together Total Unknown

-continues on next page-

		Suggestions below are highly recommended to promote understanding: Suggestion A: Select and use the following items in a Solve & Share format: Item 4, 10, 7. This sequence moves from a one-step join problem, to a two-step join/put together problem, and finishes with a one-step compare problem. Suggestion B: Select and use the following items in a Solve & Share format: Item 2, 6, 9. This sequence moves from a one-step compare problem, to a two-step join/take apart problem, and finishes with a two-step compare problem.
Lesson 3-9: M	Math Practices and Problem Solvi	ng: Use Appropriate Tools
2.OA.A.1 2.NBT.B.5 MP.1	Access Prior Learning: In first grade, students engaged in the Standards for Mathematical Practice including using appropriate tools strategically.	This lesson provides an opportunity to focus on the Thinking Habits associated with Math Practice 5. Refer to the <i>Math Practices and Problem Solving Handbook</i> (TE, pp. F27-F27A) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the student edition (SE, p. F27). If you have not done so already, add a "Tools" section to your math focus wall for student reference throughout the year.
MP.2 MP.3 MP.5	Developing the Big Idea: In this lesson, students are developing understanding that mathematicians choose tools that are appropriate for the problem, then use them accurately.	Solve & Share: During problem solving, child-watch for students who select appropriate tools and use them correctly. If you see students solve the problem using a count by 1s approach ask, "Is there a more efficient way to use the tool you chose, or is there a more efficient tool that can help you solve this problem?" These prompts aim to get students to make use of place value using the structure of tens and ones.

References

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Fosnot, C. T. (2007). *Ages and timelines: subtraction on the open number line.* Portsmouth, NH: Firsthand/Heinemann. National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

Table 1. Common addition and subtraction situations.6

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together/ Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3+?=5, 5-3=?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, $5 = 5 + 05 = 1 + 4$, $5 = 4 + 15 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare ³	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2+?=5,5-2=?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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▶ Grade 2 Topic 4: Fluently Add Within 100

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 4 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 119A-119E), the Topic Planner (pp.189A-189C), all 8 lessons, and the Topic Performance Assessments (pp. 251-252A).

Mathematical Background: Read Cluster Overview (TE, pp. 119A-119E)	Topic Essential Question: What are strategies for adding numbers to 100?
117A-117L)	Reference Answering the Topic Essential Questions (TE, pp. 247-248) for key elements of answers to the Essential Questions.

The lesson map for this topic is as follows:

4-1	4	-2	4-3	4-4	4-5	4-6	4-7	4-8	Assessment

2 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 4 focuses on fluently adding within 100 using efficient strategies.

...there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, et al, 2014, p.176).

Topic 4
Fluently Add
Within 100

Number of
lessons: 8

A/D/E: 2 days

NVACS Focus:
NBT.B

Total Days: ~10
Q1: 5 Days
Q2: 6 Days

2nd Grade Pacing Curriculum Framework: Balanced Calendar

It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).

Topics 3-6 compose a major cluster focused on the big idea of the base-10 numeration system. Focus instruction on Nevada Academic Content Standards (NVACS) cluster 2.NBT.B. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above, **place-value instruction does not need to occur in isolation** (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 176). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

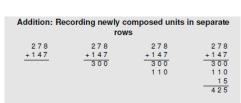
In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding one hundred as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that children can group into tens (connecting cubes, beans in cups, bundles of straws, etc.) are essential. Groupable models allow children to move from operating with ones only, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a tens rod, is worth ten is ineffective. When considering language, help students connect standard language, "thirty-five", to base-ten language, "3 tens and 5 ones; 3 groups of ten and 5 ones, etc". It is also recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

Topic 4, specifically, focuses on the following standards in NVACS cluster 2.NBT.B:

2.NBT.B Use place value understanding and properties of operations to add and subtract.

- 5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
- 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

In Topic 4, students apply the strategies from Topic 3 for adding within 100 to addition algorithms including partial sums and the U.S. Traditional algorithm. (Focus instruction on NVACS cluster 2.NBT.B) The authors of enVisionmath2.0 placed the algorithms at the end of this sequence of strategies with the intent that students connect their understanding of place value strategies to construct meaning of the algorithms. They also intended for students to see the algorithm as one of many strategies for addition, not the pinnacle of addition strategies. Standard 2.NBT.B.5 expects students to "Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" (2010, pg. 19). Looking ahead to the focus of Topics 10-11, standard 2.NBT.B.7 states, "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." The "Progressions for the Common Core State Standards in Mathematics" elaborate on what it means to "relate to a written method", by including the following examples:



The computation shown proceeds from left to right, but could have gone from right to left. Working from left to right has two advantages: Many students prefer it because they read from left to right; working first with the largest units yields a closer approximation earlier.

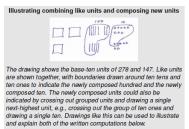
		,	U
Addition: Reco	rding newly con	nposed units in	the same row
278 +147	278 + 147 5	278 + 147 25	278 +147 +25
	Add the ones, 8 + 7, and record these 15 ones with 1 on the line in the tens column and 5 below in the ones place.	record these 12 tens with 1 on the line in the	Add the hundreds, 2+1+1, and record these 4 hundreds below in the hundreds column.
addends, on the Each two-digit p close to each oit top down," usual the easy-to-add holding an altere changed by add numbers (the ad	line. This placen artial sum (e.g., " her, suggesting th lly sums of larger "1" is added to the ad digit in memory ing numbers to the Idends and the to	sed units are place nent has several a 15" is written wit heir origin. In "ad digits are compu at sum, freeing s y. The original nu he first addend; th tal) can be seen heir usual order (e	advantages. In the digits ding from the ted first, and tudents from mbers are not ree multi-digit clearly. It is

5) rather than "write the 5 and carry the 1" (write 5, then 1).

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The first written method, Partial Sums, records intermediate steps and is helpful in building toward the second written method, the U.S. Traditional algorithm. The progression documents articulate that drawings such as the one pictured below, can be used by students in explaining the written methods above. Knowing that the trajectory is building toward the expectation that students will relate strategies to a written method, we can view the lessons in this topic as offering an entry point into algorithms. However, in regards to transitioning from the first written method to the second written method, the progression document also states, "Some students might make this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency requires a more compact method."

Based on this, we can offer opportunities for our students to construct meaning of the algorithms, but we should not expect all students to transition to use of the standard algorithm in second grade. Just as in Topic 3, although the lessons focus on a particular strategy, encourage students to use the strategy but do not require them to do so. A requirement such as this removes the reasoning from strategy development (Van de Walle, et al., 2014). Instead, honor student strategies by emphasizing their ability to use strategies based on place value understanding, properties of operations, and the relationship between addition and



subtraction. Continue to encourage the use of manipulatives throughout math instruction.

Math Practice 4: Model with mathematics

Focus on opportunities for students to develop MP.4 behaviors. This is the focus of the Math Practices and Problem Solving lesson 4-8. Reference the Teacher's Edition (pp. F26-F26A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary		
Use these words consistently during instruction.		
New Academic Vocabulary:	Review Academic Vocabulary:	
(First time explicitly taught)	(Vocabulary explicitly taught in prior grades or topics)	
partial sum	tens	
regroup	ones	
compatible numbers	open number line)	

Additional terminology that students may need support with: algorithm, model

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students developing conceptual understanding of part/part/whole and how it relates to an equation?" "Were students able to effectively communicate their strategies?"

Lesson	Evidence	Look for	
4-7	Show Me! (student work samples)	Focus CTC around the big idea:	
		mental math strategies	
		ability to communicate thinking through writing	
4-6	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace	
	Items 2, 3, and 4	(scratch paper). Printable version available under "Teacher Resources".	

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 247-252
Assessments (summative)	SE pp. 247-252	

Standards listed in **bold** indicate a focus of the lesson.

Statitual us listed III DC	tandards listed in bold indicate a focus of the lesson.		
NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations	
Lesson 4-1: A	Add With Partial Sums		
2.NBT.B.5 2.NBT.B.9 MP.1 MP.4 MP.5 MP.6 MP.7	Access Prior Learning: In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens	As indicated in the instructional note at the beginning of this document, encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Although the text offers tens-and-ones charts as an intended support for students, it might send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper to solve problems on. A blank workspace reflects the value we place on students' selection of appropriate strategies. Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 189) and <i>Review What You Know</i> (TE, pp. 190-192) and the <i>Topic 4 Vocabulary Words Activ</i> ity with the words <i>compatible numbers</i> . Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall.	
	and tens, and ones and ones. In Topic 3, second grade students used addition strategies to add 2-digit numbers within 100. Beginning the Big Idea: In this lesson, students are beginning understanding of the partial sums algorithm when adding within 100.	Solve & Share: As indicated under 2. Build Understanding (TE, p. 193), show students how to draw place value blocks efficiently, using sticks for tens and circles/dots for ones. To extend early finishers, ask them to solve the problem using a second model that demonstrates place value understanding and compare the efficiency of each (e.g., jumps of tens and ones on the open number line). Visual Learning: To facilitate student connections between concrete and abstract models, have students use place value blocks to solve 57 + 28 before showing the animation. During the animation, consider having students model the steps of partial sums using manipulatives or drawings. Focus the discussion around making sense of the algorithm, avoiding a rote, procedural approach. -continues on next page-	

Independent Practice/Math Practices and Problem Solving:

Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that problem. **Instead, offer students a blank piece of paper to solve problems on.** A blank workspace reflects the value we place on students' selection of appropriate strategies.

Lesson 4-2: Continue To Add With Partial Sums

2.NBT.B.5 2.NBT.B.9

MP.1 MP.2 MP.3 **MP.4**

MP.6

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones.

In the prior lesson, second grade students used partial sums to add 2-digit numbers within 100.

Beginning the Big Idea:

In this lesson, students are beginning understanding of the partial sums algorithm when adding within 100.

As indicated in the instructional note at the beginning of this document, encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that problem. Instead, offer students a blank piece of paper to solve problems on. A blank workspace reflects the value we place on students' selection of appropriate strategies.

Visual Learning:

To facilitate student connections between concrete and abstract models, have students use place value blocks to solve 38 + 59 before showing the animation. During the animation, have students model the steps of partial sums using their place value blocks. Focus the discussion around making sense of the algorithm, avoiding a rote, procedural approach.

Assess and Differentiate:

Modify the *Intervention Activity Sum of Sums!* (TE, p.203A). Students should build the addends using manipulatives, and later combine them while the teacher records the corresponding equations. This allows students to focus on constructing the concept of partial sums, without following a rote procedure for recording the steps.

Replace the *On-Level* and *Advanced Activity Centers* (TE, p.203A) with the lesson 4-6 *On-Level* and *Advanced Activity Centers*, "Play a Game" (TE, p. 227A).

Lesson 4-3: Models to Add 2-Digit Numbers

2.NBT.B.5 2.NBT.B.9

MP.2 MP.3 MP.4 MP.5 MP.6

Access Prior Learning:

In first grade, (1.NBT.C.4) students added within 100, adding a two-digit number and a one-digit number and a multiple of 10 using strategies based on place value. Students understood that sometimes it was necessary to compose a ten when adding tens and tens, and ones and ones.

Earlier in this topic, second grade students used partial sums to add 2-digit numbers within 100.

Beginning the Big Idea:

In this lesson, students are beginning understanding of the standard addition algorithm, using place value blocks to model the math, when adding within 100.

As indicated in the instructional note at the beginning of this document, students are expected to relate addition strategies to a written method by the end of second grade (2.NBT.B.7). These written methods include expanded algorithms (e.g., Partial Sums) and standard algorithms (e.g., U.S. Traditional Algorithm). However, the Progression Documents clarify that although some second grade students will transition from an expanded algorithm such as partial sums, to the standard algorithm, some won't make this transition until third grade. Therefore, do not expect all students to understand and use a standard algorithm at this point in the year. Focus instruction on student strategies based on place value, properties of operations and the relationship between addition and subtraction.

As Zachary Champagne, one of the enVision authors states, "there is power in the blank page". Throughout Topic 4 you will find tens-and-ones charts and 2-digit addition guides. These structures, intended to support students, may be perceived as limiting students' approach to algorithm work only. Instead, offer students a blank space to work in, allowing students to self-select strategies with understanding. Blank paper, post-its, math journals, and whiteboards are a few examples of such workspaces.

Solve & Share:

Students are introduced to the vocabulary word, *regroup* in *Visual Learning*. During the share, highlight and discuss the use of regrouping in students' solutions as an entry point for students to connect to the concept of regrouping as presented in the *Visual Learning Animation*. Regrouping is not limited to "carrying the 1" in the U.S. Traditional and other standard algorithms. This is better illustrated in students' work with manipulatives in *Analyze Student Work*: Conor's Work (TE, p.205).

Visual Learning:

Have students model using concrete manipulatives to begin to develop conceptual understanding of the steps in the standard algorithm.

Independent Practice/Math Practices and Problem Solving:

Offer students a blank workspace conducive to a variety of strategies.

Lesson 4-4: Add 2-Digit Numbers Access Prior Learning: 2.NBT.B.5 A note of CAUTION: The Coherence section for lesson 4-4 (TE, p.211A) states, "Now that In first grade, (1.NBT.C.4) students students understand the concepts behind the standard addition algorithm, students can use 2.NBT.B.9 symbols alone to perform addition at the abstract level. Drawings of place-value blocks are added within 100, adding a twoused during instruction to reinforce conceptual understanding." Do NOT push students to the digit number and a one-digit MP.1 abstract level only. Continue to give students opportunities to build conceptual number and adding a two-digit understanding through the use of manipulatives and drawings. MP.2 number and a multiple of 10 using strategies based on place value. MP.3 Independent Practice/Math Practices and Problem Solving: Students understood that MP.4 Continue to offer students a blank workspace and encourage them to use place value sometimes it was necessary to strategies. If you choose to have students try the standard algorithm, allow them to use it as a MP.6 compose a ten when adding tens second strategy and draw connections between both approaches. Students should perceive the and tens, and ones and ones. standard algorithm as another strategy they can choose when solving addition problems. Earlier in this topic, second grade Assess and Differentiate: students used partial sums and The Intervention Activity: Missing Parts (TE, p. 215A) focuses only on the abstract algorithm. place value blocks to add 2-digit Instead, have students play "Play a Game" (Lesson 4-6, TE, p. 227A). numbers within 100. Beginning the Big Idea: In this lesson, students are beginning understanding of the standard addition algorithm, using place value blocks to model the math, when adding within 100. Lesson 4-5: Add More Than Two 2-Digit Numbers 2.NBT.B.6 Access Prior Learning: Standard 2.NBT.B.6 expects students to "Add up to four two-digit numbers using strategies based on place value and properties of operations." Look for evidence of place value In first grade, (1.OA.A.2) students 2.NBT.B.9 understanding as students work with more than two addends. For example, students may break solved word problems that involved apart tens and ones, make jumps of tens and ones, use the Make 10 strategy, etc. Look for addition of three whole numbers MP.2 students' understanding and use of the properties of operations. with a sum within 20, using objects, MP.3 drawings, and equations with a Associative Property of Addition: symbol for the unknown. MP.4 (a + b) + c = a + (b + c)MP.6 In Topics 1 and 3, second grade Commutative Property of Addition: MP.8 students used strategies for a + b = b + aaddition. Earlier in this topic, Solve & Share: students were introduced to As students problem solve, ask them to explain how they used place value to add three algorithms. numbers. After students complete the directions given, encourage students to solve the problem a second time, adding the numbers in a different order to facilitate understanding of the Developing the Big Idea: Associative Property of Addition. Ask, "Did adding the numbers in a different order change the In this lesson, students are sum? Why or why not?" This work will offer students an entry point into the content of the Visual developing understanding that Learning animation. strategies and algorithms can be used to add more than two 2-digit Visual Learning: numbers. They are also developing Have students model the addition problem using manipulatives or drawings to support understanding that numbers can conceptual understanding. In Guided Practice, circling the digits added first is an opportunity for be added in any order using the students to show their reasoning and use of prior learned strategies such as Make 10 and other Commutative (order) and compatible numbers. Associative (grouping) Properties of Addition. Lesson 4-6: Practice Adding Access Prior Learning: 2.NBT.B.5 Review the vocabulary words, compatible numbers. Use students' experiences from the prior In first grade, (1.OA.A.2) students 2.NBT.6 lesson to draw examples of compatible numbers that are easy to add or subtract with mental math. For example, in the lesson 4-5 Solve & Share, students used the compatible numbers: 6 solved word problems that involved 2.NBT.9 and 4 to make a 10 when adding 24 + 16 + 14 + 15 = ? Add compatible numbers to the math addition of three whole numbers focus wall and include student-generated examples. with a sum within 20, using objects, MP.2 drawings, and equations with a MP.3 symbol for the unknown.

-continues on next page-

MP.4

MP.6

In Topics 1 and 3, second grade

students used strategies for

MP.7

addition. Earlier in this topic, students were introduced to algorithms.

Developing the Big Idea:

In this lesson, students are developing understanding that strategies and algorithms can be used to add more than two 2-digit numbers. They are also developing understanding that numbers can be added in any order using the Commutative (order) and Associative (grouping) Properties of Addition.

Continue to look for students' understanding and use of the properties of operations.

Associative Property of Addition:

(a + b) + c = a + (b + c)

Commutative Property of Addition:

a + b = b + a

Solve & Share:

During the share, highlight student solutions that make use of compatible numbers, regrouping, or adding the numbers in a different order to create a "bridge" for students into the content of the *Visual Learning*.

Visual Learning:

During the guided practice, if students use regrouping with a standard algorithm, ensure that they are able to explain their thinking accurately using place value understanding. This means they can convey that 10 ones have been regrouped into 1 ten. In doing so, we are distinguishing between students who *use* standard algorithms versus students who *understand* standard algorithms. If students cannot demonstrate understanding, redirect them to use other place value strategies such as break apart or partial sums.

*CTC: Quick Check (digital platform)

Lesson 4-7: Solve One-Step and Two-Step Problems

2.OA.A.1

MP.1 MP.4 MP.5

MP.8

Access Prior Learning:

In first grade, (1.OA.A.1) students used addition and subtraction within 20 to solve word problems using objects, drawings, and equations with a symbol for the unknown number.

In lesson 3-8, second grade students solved one-step and twostep word problems using bar diagrams and equations.

Developing the Big Idea:

In this lesson, students are developing understanding of onestep and two-step word problems using bar diagrams, equations and tens-and-ones addition charts.

Visual Learning:

Consider using the *Problem Solving Recording Sheet* (Teaching Tool 1) to make sense of the problem (MP.1) presented in the *Visual Learning* animation. Avoid key word strategies as they send a message to students that sense-making is not important, they are often misleading, and cannot be used to solve multi-step problems (Van de Walle, et al., 2014, p. 148). For example, if students have been taught that "join" means to add, students may approach this problem incorrectly (36 + 53 = ?) instead of 36 + ?) = 53.

Independent Practice/Math Practices and Problem Solving:

Students may notice that the items include two different bar diagrams. Refer to the Item 3 note on TE, pp. 231-232 for clarification between the bar diagram and the comparison bar diagram. Reference Teaching Tools 15 and 23 for blackline masters of both bar diagrams.

Assess and Differentiate:

In the *Intervention Activity* "Let's Solve and Check!", have students build the addends on ten frames to support understanding.

*CTC: Show Me! (student work samples)

Lesson 4-8: Math Practices and Problem Solving: Model with Math

2.OA.A.1 2.NBT.B.5

MP.1 MP.2 MP.3

MP.4

Access Prior Learning:

In first grade, students engaged in the Standards for Mathematical Practice including MP. 4 Model with Math.

In Topic 2, second grade students focused on MP.4 Model with Math.

Developing the Big Idea:

In this lesson, students are developing understanding that they can make models to help them solve problems.

Students focused on MP4. Behaviors in Topics 2 and 4. Consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.

References

- Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards Instructional Support/Nevada Academic Standards/Math __Documents/mathstandards.pdf.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

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▶ Grade 2 Topic 5: Subtract Within 100 Using Strategies

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 5 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 119A-119E), the Topic Planner (pp.253A-253C), all 8 lessons, and the Topic Performance Assessments (pp. 319-320A).

Mathematical Background: Read Cluster Overview (TE, pp. 119A-119E) Topic Essential Question:

What are strategies for subtracting numbers to 100?

Reference Answering the Topic Essential Questions (TE, pp. 315-316) for key elements of answers to the Essential Questions.

The lesson map for this topic is as follows:

5-1 5-2 5-3 5-4 5-5 5-6 5-7 5-8 5-9 6-1 Assessment

4 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 4 is to subtract using different strategies.

...there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, Karp, Lovin, & Bay Williams, 2014, p. 176).

It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p. 83).

Topics 3-6 compose a major cluster focused on the big idea of the base-10 numeration system. Focus instruction on Nevada Academic Content Standards (NVACS) cluster 2.NBT.B. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above, **place-value instruction does not need to occur in isolation** (Van de Walle, et al., 2014, p. 176). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding one hundred as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that children can group into tens (connecting cubes, beans in cups, bundles of straws, etc.) are essential. Groupable models allow children to move from operating with ones only, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a tens rod, is worth ten is ineffective. When considering language, help students connect standard language, "thirty-five", to base-ten language, "3 tens and 5 ones; 3 groups of ten and 5 ones, etc". It is also recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

2.NBT.B Use place value understanding and properties of operations to add and subtract.

- 5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

Topic 5
Subtract Within 100
Using
Strategies

Number of lessons:
10 over 15 days
*After 5-9 add lesson 6-1
A/D/E: 4 days

NVACS Focus:
NBT.B

Total Days: ~19

2nd Grade Pacing
Curriculum Framework:
Balanced Calendar

The Properties of Operations: Addition and Subtraction

Associative property of addition	(a+b)+c=a+(b+c)	
Commutative property of addition	a+b=b+a	Council of Chief State School Officers. (2010). The Nevada Academic Content Standards.
Additive identity property of 0	a + 0 = 0 + a = a	Retrieved from

http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Topic 5 focuses on strategies for subtraction within 100. The strategies in this topic parallel those presented in Topic 3 for addition. They include the hundred chart, count back to subtract and add up to subtract on an open number line, break apart numbers, and compensation (reference lesson-level instructional notes below for content related to each strategy). These reflect the three common types of invented strategy models: 1) split strategies, which involve decomposition such as break apart, 2) jump strategies similar to counting back and add up to subtract, and 3) shortcut strategies such as compensation which involve adjustment of numbers (Van de Walle, et al., 2014, p. 210). In order for students to develop computational fluency, it is important that they be able to use a variety of strategies with understanding and flexibility, adapting to the numbers and context. Van de Walle, et al. stated, "...the issue is no longer a matter of 'knows how to subtract three-digit numbers'; rather it is the development over time of an assortment of flexible skills that will best serve children in the real world" (2014, p. 204).

Although the lessons focus on a particular strategy, encourage students to use the strategy but do not require them to do so. A requirement such as this removes the reasoning from strategy development. Instead, honor student strategies by emphasizing their ability to determine the appropriateness of a strategy and justify its use. As identified in 2.NBT.B.9, second grade students are expected to, "Explain why addition and subtraction strategies work, using place value and the properties of operations." The flexible application of strategies using decomposing and composing numbers also builds students' number sense. It remains important to ensure that all students engage in the doing of mathematics through the eight mathematical practices. In particular, all students should engage in MP.5 Use Appropriate Tools Strategically on a daily basis. Students should be encouraged to select and use tools throughout math instruction, with teachers being cognizant of the effect their actions and tool storage systems have on these developing habits of mind.

Math Practice 3: Construct Viable Arguments and Critique the Reasoning of Others

Focus opportunities for students to develop MP.3 behaviors. This is the focus of the Math Practices and Problem Solving lesson 5-9. Reference the Teacher's Edition (pp.F25-F25A) and the Nevada Academic Content Standards for Mathematical Practice.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Topic 5 and 6 Essential Academic Vocabulary Use these words consistently during instruction.					
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)				
	equation difference bar diagram tens ones open number line	break apart mental math compensation subtract			

Additional terminology that students may need support with: algorithm, backward, column, forward, minuend (whole), model, row, separate, subtrahend (part subtracting)

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students able to employ strategies such as break-apart/compensation to mentally solve the Number String?"

Lesson	Evidence	Look for	
5-7	Number String (audio/video	Focus CTC around the big idea:	
	recording)	student strategies and models	
		communicate thinking orally	
		use previous expression to solve future expressions	
5-8	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace	
		(scratch paper). Printable version available under "Teacher Resources".	

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 315-320
Assessments (summative)	SE pp. 315-320	·

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 5-1: S	Subtract Tens And Ones On A Hu	ndred Chart
2.NBT.B.5 2.NBT.B.9 MP.1 MP.3 MP.5 MP.6 MP.7	Access Prior Learning: In first grade, (1.NBT.C.6) students subtracted multiples of 10 in the range of 10-90 from multiples of 10 in the range of 10-90 using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. They related the strategy to a written method to explain their reasoning. In lesson 3-1, second grade students used place value and a hundred chart to add 2-digit numbers within 100. Developing the Big Idea: In this lesson, students are developing understanding that patterns on a hundred chart can be used to subtract 2-digit numbers.	As indicated in the instructional note at the beginning of this document, encourage students to try the strategy indicated in the instructional materials, but do not require them to use it. Look for evidence of place value understanding and flexible use of strategies. The use of a hundred chart reinforces students' understanding of the sequence of numbers to 100. It is also a helpful tool for analyzing the structure of our number system through patterns and can be used to support skip-counting, particularly by 2,s, 5s, and 10s. Consider giving students access to a chart that extends to 200, or even to 1,000 (Van de Walle, et al., 2014, p. 119). A note of CAUTION: Watch for students who use the hundred chart rotely, with limited understanding of, or connection to the structure of the number system. When subtracting 34, these students move up 3 boxes because "that's what you do with the first number" and move left 4 boxes because "that's what you do with the second number". By emphasizing opportunities for students to find and explain patterns, we can facilitate conceptual understanding that connects to the procedural use of this tool. The goal being that students understand that jumping down or up represents adding or subtracting by ten, respectively, while right or left movement represents additive or subtractive jumps of one. Topic Opener: Consider limiting the Topic Opener to discussion of the Topic Essential Question (TE, p. 253), Review What You Know (TE, p. 254), and Vocabulary Review Activity (TE, p. 254) only. Post the essential question and student strategies on your math focus wall. Solve & Share: Child-watch for students who use the structure of the hundred chart to count by tens and ones. When students count on from the subtraction is an unknown-addend problem. This will further develop their understanding of the relationship between addition and subtraction. If students count by ones only, support place value understanding by asking "How can counting by tens help you solve the problem more ef

Lesson 5-2: Count Back To Subtract On An Open Number Line

2.NBT.B.5 2.NBT.B.9

MP.2 MP.3 MP.5 MP.8

Access Prior Learning:

In first grade, (1.NBT.C.6) students subtracted multiples of 10 in the range of 10-90 from multiples of 10 in the range of 10-90 using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. They related the strategy to a written method to explain their reasoning.

In lesson 3-2 and 3-3, students used an open number line to add tens and ones. In the prior lesson, students subtracted on a hundred chart. The hundred chart can be less efficient, so students will move into the use of an open number line in this lesson.

Beginning the Big Idea:

In this lesson, students are *developing* understanding that the open number line can be used to model subtracting tens from a 2-digit number.

The open number line is an effective tool to support students in explaining their reasoning when using a jump strategy. The open number line offers more flexibility than a regular number line as it allows students to work with any numbers, reduces confusion between hash marks and spaces, and results in fewer computational errors (Van de Walle, et al., 2014, p. 211). In addition, the open number line is a versatile tool that reinforces the inverse relationship between addition and subtraction, supports the development of place value understanding, number sense and computational fluency.

Solve & Share:

Refer to *Analyze Student Work* (TE, p. 261) for examples of possible student solutions. Also child-watch for students who make repeated jumps of 10 and students who make a single jump of 20 or 30. Engage students in a discussion of which jumps are more efficient. We want students to develop the understanding that jumps of multiple groups of ten are more efficient than single jumps of ten (e.g., In solving 50 - 30 = ?, starting at 20, a jump forward of 30 is more efficient than three forward jumps of 10). Students do not need to indicate an operation when labeling jumps on the number line (+10 or -10, instead label with just 10) this helps to reinforce the relationship between addition and subtraction

Visual Learning:

Making jumps of ten in the mid-decades (e.g., 56, 46, 36) may be challenging for some students. Have students use concrete manipulatives such as place value blocks to model the jumps as they are made on the open number line. Also engage students in a discussion of the patterns they notice in the tens digit and ones digit when subtracting tens (e.g., the tens digit decreases by one when subtracting ten; the ones digit remains the same when subtracting ten).

Assess and Differentiate:

In the *Intervention Activity*, "Counting Back Tens" (TE, p. 265A) ask students to look for patterns in the tens digit and ones digit as they count back.

Lesson 5-3: Continue To Count Back To Subtract On An Open Number Line

2.NBT.B.5 2.NBT.B.9

MP.1 MP.4 MP.5

Access Prior Learning:

In first grade, (1.NBT.C.6) students subtracted multiples of 10 in the range of 10-90 from multiples of 10 in the range of 10-90 using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. They related the strategy to a written method to explain their reasoning.

In the prior lesson, second grade students used an open number line to subtract tens from a 2-digit number.

Developing the Big Idea:

In this lesson, students are developing understanding that numbers can be broken into tens and ones when subtracting on an open number line. This lesson focuses on the count back strategy.

Student reasoning around addition and subtraction is classified into levels of sophistication (Battista, 2012, p. 9-10). Level 1 reasoning describes students who add or subtract numbers as collections of ones (e.g., count all, count on or down by ones, etc). Level 2 reasoning refers to students who use skip-counting by place value parts. Level 3 reasoning includes students who combine or separate place value parts. Therefore, a student at level 2 who makes single jumps of tens and ones (two jumps of 10 and four jumps of 1) is showing less sophisticated reasoning than a student who makes a single jump of a multiple of ten and a group of ones (one jump of 20 and one jump of 4). As you child-watch, look for students' use of these varying levels of sophistication, supporting the development from one level to the next through strategic questioning. For example, "Can you solve the problem in fewer jumps?"

Solve & Share:

Child-watch for students who count back using two jumps of 10 and four jumps of 1 (Level 2). Also, look for students who count back using one jump of 20 and one jump of 4 (Level 3). Highlighting these different strategies, comparing their answers, and considering the efficiency of each will prepare students for the *Visual Learning* while reinforcing place value understanding.

Visual Learning:

During the discussion, connect back to the *Solve & Share*. Ask students to evaluate the efficiency of each example presented in the animation.

Assess & Differentiate:

In the *Intervention Activity*, "Subtraction Drawings and Equations" (TE, p.271A), some students may need additional support through the use of place value blocks (concrete or drawings) connected to the open number line and equation.

Lesson 5-4: Add Up To Subtract Using An Open Number Line

2.NBT.B.5 2.NBT.B.9

MP.1 MP.2 MP.5

MP.6

Access Prior Learning: In first grade, (1.OA.B.4) students understood subtraction as an

In lessons 3-2 and 3-3, second grade students used an open number line to add tens and ones.

Developing the Big Idea:

unknown-addend problem.

In this lesson, students are *developing* understanding that numbers can be broken into tens and ones when subtracting on an open number line. This lesson focuses on the add up to subtract strategy.

Think-addition strategies such as add up to subtract are powerful ways to solve subtraction problems. The add up to subtract strategy lends itself to problems such as 45 - 19, in which students consider how much they need to add to 19 to get 45. For example, 19 + 1 = 20, then 20 + 25 = 45. Therefore, 1 + 25 = 26, so 45 - 19 = 26. This strategy also supports students' use of place value with tens. However, for problems such as 45 - 6 this strategy is not efficient (Van de Walle, et al, 2014, p.215).

Solve & Share:

During problem solving, child-watch for students who use count back and add up strategies to subtract. Sequence the share to finish with student work using the add up strategy, as that is focus of the *Visual Learning* animation. During discussion, ask students why they chose certain jumps, facilitating conversation around the use of landmark or easier numbers.

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their student edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The quick check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Lesson 5-5: Break Apart Numbers To Subtract

2.NBT.B.5 2.NBT.B.9

MP.3 MP.6 MP.7 Access Prior Learning:

In lesson 1-7, second grade students made a ten to subtract. In lessons 3-4 and 3-5, students used break-apart strategies to help them add mentally.

Developing the Big Idea:

In this lesson, students are developing understanding that when subtracting a 1-digit number from a 2-digit number, 1-digit numbers can be broken apart to make mental subtraction easier.

(Possible 2-day lesson: Choose to extend *either* lesson 5-5 OR 5-6 over two days based upon your students' demonstrated understanding of subtracting using the break apart strategy.)

The break-apart strategy extends students' knowledge of the base-ten number system and of basic facts, therefor removing the need for counting. Child-watch for students who count back or add on by ones, paying no attention to the ten. Encourage these children to attend to the tenstructure using ten frames or the hundred chart as a model (Van de Walle, et al., 2014, p.212).

Solve & Share:

Structure the share to highlight students' use of tens to make mental subtraction easier. Similar to Maureen's Work (TE, p.279), watch for students who break apart 7 into 2 and 5, first subtracting 2 to get to 40 (evidence of attending to the ten-structure) before subtracting 5 to get 35. When students break apart 7 in different ways, focus the discussion on evaluating which method is most appropriate to solve the problem, 42 - 7. This will encourage students to think strategically when breaking apart numbers to subtract.

If students struggle to remove 7 from 42, ask them to show you the value of each digit in the minuend (42). Students who are able to show 4 tens or 40, and 2 ones demonstrate place value understanding. Students who show the tens-digit as 4 ones and the ones-digit as 2 ones, do not show place value understanding of tens. Support these students by asking them to build 42 using groupable models such as counters or connecting cubes. Then, ask these students how many groups of ten they can make. If needed, model how to make a group of ten using ten frames or cube towers. Return to the original prompt, asking the student to show the value of each digit in 42. Help the student connect the visual representation of 40 or 4 tens to the tens-digit in 42. Set a goal with the student, asking them to show the value of each digit in the numbers they work with before subtracting. This support may be needed on an ongoing basis to increase place value understanding. However, it is important that students continue to interact with the grade level content while receiving this support.

Visual Learning:

During the *Guided Practice* section of the lesson, support students who have difficulty breaking apart the subtrahend, by referencing the *Error Intervention* note (TE, p.280). This can be used with a hundred chart or ten frames.

Independent Practice/Math Practices and Problem Solving:

For item 15, have students write an explanation of how the break apart strategy could help them solve the problem. Doing so *before* solving the problem supports MP.1.

Lesson 5-6: Continue To Break Apart Numbers To Subtract

2.NBT.B.5 2.NBT.9

> MP.1 MP.3 MP.6

MP.7

Access Prior Learning: In lessons 3-4 and 3-5, second grade students broke apart 2-digit number to add.

In the prior lesson, students broke apart 1-digit numbers when subtracting from 2-digit numbers.

Developing the Big Idea: In this lesson, students are developing understanding that when subtracting a 2-digit number from a 2-digit number, the numbers

Some students may encounter difficulties decomposing a ten when subtracting with place value blocks. Encourage these students to use groupable models, such as connecting cubes, that can be physically broken apart, or ten frames and counters which can be physically removed. In either case, students should construct 53 with tens and ones (e.g., 5 tens and 3 ones, 4 tens and 13 ones) before subtracting to continue to support use of the ten structure. Then have these students return to the place value blocks to see if they can connect their understanding to the pre-grouped models.

Possible 2-day lesson: Choose to extend *either* lesson 5-5 OR 5-6 over two days based upon

your students' demonstrated understanding of subtracting using the break apart strategy.

Assessment & Differentiate:

Solve & Share:

For the *Intervention Activity*, "Break Apart Tens and Ones to Subtract" (TE, p.289A), also incorporate the support from the Solve & Share note above.

Lesson 5-7: Subtract Using Compensation

2.NBT.B.5 2.NBT.9

MP.1 MP.4 MP.7

MP.8

Access Prior Learning:

can be broken apart to make mental subtraction easier.

In lessons 5-5 and 5-6, second grade students manipulated numbers to solve problems using the break-apart strategy.

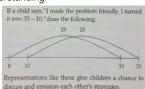
Developing the Big Idea:

In this lesson, students are developing understanding of the compensation strategy for subtraction.

Possible 3-day lesson

A note of CAUTION: Some students may try to apply the compensation strategy for addition to subtraction. However, the compensation strategy for subtraction works differently. When compensating with subtraction, the same amount can be added to each number OR the same amount can be subtracted from each number to result in an easier problem. For example, with 86 - 29, students can add 1 to each number, resulting in 87 - 30. The *Visual Learn*ing presents two ways to compensate. One way adjusts both numbers before the operation. The second way adjusts the subtrahend, conducts the operation, and then adjusts the final answer.

To support students' conceptual understanding, compensation with subtraction can also be thought of as constant difference. When adjusting the numbers in a subtraction problem, the difference must be kept constant. Representing this idea on an open number line will support students as they construct understanding.



Fosnot, C. T. (2007). Ages and timelines: subtraction on the open number line. Portsmouth, NH: Firsthand/Heinemann.

Day 1: Solve & Share, Visual Learning Visual Learning:

To support understanding of compensation as creating equivalent expressions, have students use connecting cubes to prove that 43-18 and 45-20 have the same difference of 25. Then, lay the 25 cubes along an open number line and mark the endpoints as 18 and 43. Slide the cubes and mark the new placement of each endpoint at 20 and 45. Continue to explore constant difference, by sliding the cubes to new minuends and subtrahends. Relating constant difference to ages on a timeline is also a helpful real world connection for students.

Independent Practice/Math Practices and Problem Solving:

In the Intervention Activity, "Compensate to Subtract!" (TE p.295A), encourage students to use ten frames or the hundred chart to support their work with compensation.

Day 2: Number String, Independent Practice/Math Practices and Problem Solving

Facilitate a lesson using a string of numbers intentionally structured to promote student use of the compensation strategy. These strings are intended to develop students' use of mental math, but do not require students to only solve the problems in their heads. Instead, focus on their ability to examine the numbers and select an appropriate and efficient way to solve the problem. As students verbally explain their thinking, make a written record so that students can "see" the strategy using an open number line. This becomes a picture for the class to discuss. Relying only on verbal explanations will limit access for children to understand (Fosnot, 2007, p.7). Although children may begin by using a variety of strategies, through discussion they will notice patterns in the string of problems and in the answers. These patterns will encourage students to examine the numbers before selecting a strategy.

Child-watch for students who identify that the first four problems are equivalent expressions. If this is unnoticed, point out that the first four problems have the same answer and ask, "Why is this happening? Which problem is the easiest?" Use of the open number line to model student strategies will support students in their understanding of the compensation strategy, or constant difference. Encourage students to apply this understanding to the last four problems in the string.

Day 3: Solve & Share, Assess and Differentiate

Select a problem from the Independent Practice and Math Practices and Problem Solving pages (TE, p. 293-294) and use it as a Solve & Share. Follow with Assess and Differentiate.

Child-watch for students who have difficulty deciding how to make adjustments. Support these students by encouraging the use of tools, such as open number lines and ten frames, to identify landmarks of ten close to the minuend and subtrahend.

*CTC: Number String (audio/video recording)

Lesson 5-8: Solve One-Step and Two-Step Problems

2.OA.A.1

MP.1

MP.2

MP.4

MP.5

Access Prior Learning:

In lesson 3-8, second grade students used comparison bar diagrams and equations to solve one- and two-step word problems.

In lessons 4-7, students continued to solve one- and two-step word problems.

Developing the Big Idea:

In this lesson, students are developing understanding that they can use bar diagrams, equations and the relationship between addition and subtraction to solve one- and two-step word problems.

Possible 2-day lesson

In general, students find *Add To* and *Take From* problem types easier because they include explicit action. *Put Together* and *Take Apart* problem types are generally more challenging, as they do not include explicit action. Finally, *Compare* problems tend to be the most challenging problem types, as one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011). Keep this in mind as you respond to learners. All students need to solve all problem types, but we can use this information to scaffold and extend.

When working with word problems, avoid key word strategies as they send a message to students that sense-making is not important, they are often misleading, and cannot be used to solve multi-step problems (Van de Walle, et al., 2014, p. 148). Instead, honor sense-making through the use of bar diagrams and the math practices. The *Problem Solving Recording Sheet* (Teaching Tool 1) and *Bar Diagrams* (Teaching Tools 15 and 16) support MP.1 behaviors. Students should also be encouraged to draw their own bar diagrams as a tool that is available to them at all times. These diagrams reinforce understanding of the relationship between addition and subtraction, and help students understand the relationship between the numbers in the problem.

Day 1: Solve & Share, Visual Learning Solve & Share:

The *Solve & Share* is a *Add To Start Unknown* problem type. Although *add to* problems are easier than other problem types, having the unknown in the start position makes this one more difficult. In this case, students know the whole and one part, and must solve for the missing part. Students who attempt to use direct modeling to solve this problem will likely use trial and error. In doing so, watch for students who use trial and error systematically. These students will choose a start number, add 16 more and see if they get a sum of 49. Systematic thinking will be reflected in their ability to reason about the answer to determine if their guess was too low or too high and adjust accordingly. This problem parallels the problem in *Visual Learning*, which is also a *Add To Start Unknown* problem.

Visual Learning:

Consider using the *Problem Solving Record Sheet* (Teaching Tool 1) to engage students in MP.1. Give students time to solve the problem presented in the *Visual Learning* **before** proceeding with the animation and discussion. This will allow students to connect their learning in the *Solve & Share* and strengthen their entry point into the content.

Day 2: Independent Practice/Math Practices and Problem Solving, Assess and Differentiate

Independent Practice/Math Practices and Problem Solving:

If your students demonstrated understanding of the *Add To Start Unknown* problems in the 5-8 *Solve & Share* and *Visual Learning*, strategically select problems from pages 299-300 to foster continued growth with other problem types and two-step word problems (see Suggestion A below). Choose one problem to run as a *Solve & Share* and respond accordingly.

If your students struggled with the *Add To Start Unknown* problems, strategically select problems from pages 299-300 to facilitate growth towards more challenging problem types (see Suggestion B below). In both cases, support students in their sense making of the numbers and context with manipulatives and bar diagrams. Students will continue to use bar diagrams and solve word problems in enVision, spending a full topic on these problem types in Topic 7.

Classification of Items (TE, p. 299-300, SE p. 299-300) by Problem Type: Reference the NVACS, Table 1. Common addition and subtraction situations for examples of these problem types (CCSSO, 2010, p. 88).

- Item 2: Add To Start Unknown
- Item 3: Put Together Total Unknown/Take From Result Unknown
- Item 4: Take From Result Unknown/Add To Result Unknown
- Item 5: Add To Change Unknown
- Item 6: Add To Start Unknown
- Item 7: Compare Bigger Unknown/Add To Result Unknown
- Item 8: Put Together Total Unknown/Take From Result Unknown

Suggestion A:

Select and use the following items in a *Solve & Share* format: Item 2, 8, 7. This sequence moves from a one-step *Add To Start Unknown* problem, to a two-step *Put Together Result Unknown/Take From Result Unknown* problem, and finishes with a two-step *Compare Bigger Unknown/Add To Result Unknown* problem.

Suggestion B:

Select and use the following items in a *Solve & Share* format: Item 4, 5, 6. This sequence begins with a two-step *Take From Result Unknown/Add To Result Unknown* problem. This problem was selected because it includes the two easiest problem types for students to direct model due to the explicit action. This problem is followed by a one-step *Add To Change Unknown* problem, and finishes with a one-step *Add To Start Unknown* problem both of which are more challenging problem types due to the placement of the unknown.

Students focused on MP3. Behaviors in Topic 1. Consider using the Math Practice 3 Animation

(Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth

on Pearson Realize Online for an example of MP.3 behaviors. Also, consider having students

self-reflect on their understanding of this math practice using the Self-Assessment Tool

*CTC: Quick Check (digital platform)

Lesson 5-9: Math Practices and Problem Solving: Critique Reasoning

2.NBT.B.5

MP.3

MP.4

MP.7

2.OA.A.1

Access Prior Learning: In first grade, students er

In first grade, students engaged in the Standards for Mathematical Practice including MP. 3 Construct Viable Arguments and Critique the Reasoning of Others.

dents

Include Topic 5 Fluency Practice Activity (TE, p.309).

In Topic 1, second grade students focused on MP.3 behaviors.

Developing the Big Idea:

In this lesson, students are developing understanding that mathematicians construct arguments and critique the reasoning of others.

Topic 5 Performance Assessment:

mindset in mathematics.

NOTE: Give Topic 5 Performance Assessment after lesson 6-1.

Lesson 6-1: Regroup 1 Ten For 10 Ones

2.NBT.B.5 2.NBT.B.9

MP.1

MP.3

MP.5

Access Prior Learning: In first grade, (1.NBT.B.2a) students understood 10 as

students understood 10 as a bundle of ten ones. When adding within 100 (1.NBT.C.4), first grade students also understood that it is sometimes necessary to compose a ten.

In this lesson, students will use number sense and concrete place-value blocks to determine if regrouping is needed when subtracting a 1-digit number from a 2-digit number.

Visual Learning:

During the animation, stop after the regrouping of 1 ten as 10 ones. Ask, "When the ten was regrouped, did the quantity change? Prove it using your place-value blocks." Child-watch for

MP.8	Developing the Big Idea: In this lesson, students are developing understanding that it is sometimes necessary to regroup 1 ten for 10 ones when subtracting.	students who understand that 3 tens and 4 ones is equivalent to 2 tens and 14 ones. Some students may believe that the quantity has changed. Child-watch for students who regroup the 1 ten, but include the 4 ones already in 34, therefore only trading for 6 more cubes. These students will have changed the quantity from 34 to 30 (2 tens and 10 ones). Refer to the <i>Error Intervention</i> Note: Item 3 (TE, p.324) for another common misconception and
		teacher response.

CTC: Collaborative Team Conversations evidence collection

References

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- Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
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- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards Instructional Support/Nevada Academic Standards/Math __Documents/mathstandards.pdf.
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- National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

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▶ Grade 2 Topic 6: Fluently Subtract Within 100

Instructional note:

Topic 6 lessons are omitted or distributed to other topics (Topics 5 and 7) to support students' development.

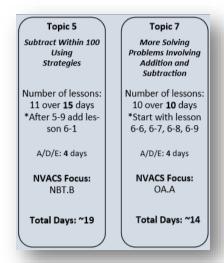
We have paced learning opportunities around strategies, written methods and standard subtraction algorithms in Topic 11, rather than include lessons focused on the U.S. Traditional algorithm at this point in the year. Reference the Topic 11 Curriculum Guide for instructional clarifications and considerations. This adjustment allows for additional opportunities for students to develop subtraction strategies based on place value, properties of operations and the relationship between addition and subtraction in Topics 5 and 7.

Topic 5:

Lesson 6-1 is paced after Lesson 5-9. Refer to the Topic 5 Curriculum Guide for instructional clarifications and considerations.

Topic 7:

Lessons 6-6, 6-7, 6-8 and 6-9 are paced before Lesson 7-1. Refer to the Topic 7 Curriculum Guide for instructional clarifications and considerations.



2nd Grade Curriculum Pacing Framework: Balanced Calendar This page is intentionally left blank

▶ Grade 2 Topic 7: More Solving Problems Involving Addition and Subtraction

Big Conceptual Idea: K-5 Progression on Operations and Algebraic Thinking (pp. 6-7; 18-21)

Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 389A-389E), the Topic Planner (pp.389I-389J), the Topic Performance Assessments (pp. 433-434A), and all 6 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 389A-389E)	Topic Essential Question: How can you solve word problems that use adding and subtracting?
	Reference Answering the Topic Essential Questions (TE, pp. 431-432) for key elements of answers to the Essential Questions.

The lesson map for this topic is as follows:

6-6	6-7	6-8	6-9	7-1	7-2	7-3	7-4	7-5	7-6	Assessment
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⁴ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 7 focuses on the meaning of operations through word problems that illustrate multiple interpretations of addition and subtraction. Focus instruction on Nevada Academic Content Standard (NVACS, 2010) 2.OA.A.

Topic 7

More Solving
Problems Involving
Addition and
Subtraction

Number of lessons:
10 over 10 days
*Start with lesson
6-6, 6-7, 6-8, 6-9

A/D/E: 4 days

NVACS Focus:
OA.A

Total Days: ~14

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

2.OA.A Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

The standard references one- and two-step word problems involving various situations. These situations or problem types (inserted below for reference) help students develop meanings for addition and subtraction. It is important to be mindful that second grade students are expected to demonstrate security with all problem types, including unknowns in each place, by the end of the year. This expectation is supported by work done in kindergarten and first grade. Kindergarten students work with the problem types outlined with a thick, *solid* border as the actions of composition/decomposition are more easily modeled. First grade students interact with all problem types, but the problem types outlined with a thick, *dashed* border are the most challenging and are not expected to be secure until the end of second grade (CCSWT, 2011, p.20-21).

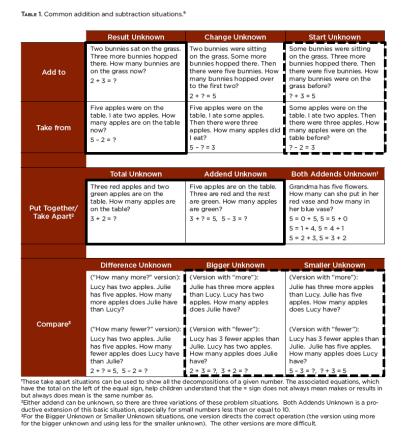
In general, students find *Add To* and *Take From* problem types easier because they include explicit action. *Put Together* and *Take Apart* problem types are generally more challenging, as they do not include explicit action and involve combinations of different kinds of objects into one collection as noted in the table below (Van de Walle, et al., 2014, p. 129). Finally, *Compare* problems tend to be the most challenging problem types, because one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011).

As stated in the "Progressions for the Common Core State Standards in Mathematics", two-step problems should not involve the most difficult subtypes (2011, p.20-21). Carpenter, Fennema, Loef-Franke, Levi and Empson developed a framework for students' strategies for solving these problems. The framework indicates that students often start direct modeling, then move to counting strategies and derived fact strategies (2015). When students use direct modeling strategies, they make an "Explicit physical representation of each quantity in a problem and the action or relationship involving those quantities before counting the resulting set." (Carpenter, et al., 2015, p.29). These models can include concrete objects, tally marks or pictures.

Counting strategies refer to when "...a child recognizes that it is not necessary to physically construct and count the two sets described in a problem." (Carpenter, et al., 2015, p.24). These students may keep track of counts using their fingers, counters or tallies, but most give no evidence of a physical action when counting. Finally, derived fact strategies "can be represented with equations or other notation using written numerals" (Carpenter, et al., 2015, p.32), and where "the notation becomes a tool for reflection." (Carpenter, et al., 2015, p.33). Although students will progress to more efficient strategies over time (e.g., from direct

modeling to counting strategies to derived facts), it is common for students to move among these strategy types based upon the numbers, context and the difficulty of the problem type.

As students interact with the problem types, help them draw connections to the meanings of the operations (addition and subtraction) as well as the relationship between operations. Bar diagrams help students make sense of word problems and see the relationship between quantities supporting part-part-whole reasoning. In addition, making bar diagrams helps students write an equation to solve the problem. The use of a question mark (?) to represent the unknown quantity helps students develop algebraic thinking. In order to offer support or challenge, consider factors that affect problem difficulty. As previously discussed, the structure of a word problem affects the level of difficulty, as do the numbers and wording of the problem. If a student is struggling with *put together, take apart* or *compare* problems, consider modifying the problem to include an explicit action (*add to* or *take from*), returning to the original problem type after support is offered. In addition, adjusting the numbers in a problem allows teachers to respond to learners. For students who are struggling, help them focus on the context of the problem type by decreasing the numbers and then return to the original level of difficulty. For students who are ready for extension, a challenge might be offered by changing the numbers to elicit strategies based on decomposing or recomposing.



Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop MP.2 behaviors. This is the focus of the Math Practices and Problem Solving lesson 7-6. Reference Teacher's Edition (pp. F24-F24A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Topic 6 has been decomposed with lessons either omitted or distributed to Topics 5 and 7. Lessons 6-6, 6-7, 6-8, and 6-9 are paced before lesson 7-1 and are included in this document.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Topic 6 and 7 Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
	equation difference bar diagram tens ones open number line	break apart mental math compensation subtract	

Additional terminology that students may need support with: algorithm, backward, column, forward, minuend (whole), model, row, separate, subtrahend (part subtracting)

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students developing conceptual understanding and connecting real world context with numbers, strategies, and/or equations?"

Lesson	Evidence	Look for
7-3	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		sense-making strategies
7-5	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	Items 1, 3, and 4	(scratch paper). Printable version available under "Teacher Resources".

La amaina a Consta	Tania Assassments	Has Cooring Civida TE no. 421-424
Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 431-434
Assessments (summative)	SE pp.431-434	

Standards listed in bo	Standards listed in bold indicate a focus of the lesson.				
NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations			
Lesson 6-6: L	Jse Addition to Check Subtraction	n			
2.NBT.B.5 2.NBT.B.9	Access Prior Learning: In first grade, (1.OA.B.4) students understood subtraction as an	In first grade, students worked with part-part-whole relationships. Continue to use the language of part-part-whole. However, you can help students make a connection to this relationship and prior learning by also referencing fact families.			
MP.1	unknown-addend problem. Students also (1.OA.C.6) added	Encourage students to try the strategy indicated in the instructional materials, but do not require			
MP.2	and subtracted within 20 using	them to use it. Although the text offers tens-and-ones charts as an intended support for students, it may send a conflicting message that there is only one acceptable strategy for that			
MP.3	strategies including the relationship	problem. Instead, offer students a blank piece of paper for solving problems. A blank			
MP.4	between addition and subtraction.	workspace reflects the value we place on students' selection of appropriate strategies.			
	In lesson 1-6, second grade students used think-addition to subtract. In prior topics, students represented the relationship between numbers and operations through bar diagrams.	Topic Opener: In planning, consider the four lessons from Topic 6 as part of Topic 7. Use the <i>Topic 7 Opener</i> . Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic 7 Essential Question</i> (TE, p. 389), <i>Review What You Know</i> (TE, p.390), and <i>Vocabulary Review Activity</i> (TE, p. 390) with the word equation or bar diagram. Post the essential question and student strategies on your math focus wall.			
	Developing the Big Idea: In this lesson, students are developing understanding of the inverse relationship between addition and subtraction by using addition to check subtraction.	Solve & Share: Providing students with a blank workspace to solve the problem honors a variety of student strategies. Encourage students to draw a bar diagram as a tool for sense making that reinforces the part-part-whole relationship, as the bar diagram will be a major representation utilized throughout this Topic. In addition, have students write an explanation to answer the question, "How can you use addition to check your answer?" -continues on next page-			

Visual Learning:

Omit the *Visual Learning* animation. Instead, spend more time on the *Solve & Share*. Focus on sharing student work that demonstrates why addition can be used to check subtraction. When working with regrouping a ten for 10 ones, ensure that students have support through concrete manipulatives and representational drawings.

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their student edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The quick check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Assess and Differentiate:

To support students in understanding the bar diagram, refer to the *Intervention Activity*, "Check It with Cubes" (TE, p.357A).

Lesson 6-7: Practice Subtracting

2.NBT.B.5 2.NBT.B.9

2.NBT.B.9

MP.2 MP.6

MP.7

Access Prior Learning:

In first grade, (1.NBT.B.2) students understood that the two digits of a 2-digit number represent amounts of tens and ones.

In Topic 5, second grade students learned subtraction strategies.

Securing the Big Idea:

In this lesson, students are securing understanding of subtraction strategies and that at times it is necessary to regroup 1 ten as 10 ones.

If students have consistent opportunities to play the Regrouping Game (Reference the last page of this document), they should secure the understanding of regrouping 1 ten as 10 ones.

Solve & Share:

Ask students to solve the problem a second time using place-value blocks. Child-watch for students who appropriately trade in, or regroup, 1 ten for 10 ones in order to subtract. Highlight this student's strategy in the share to develop the essential understanding of the lesson, without relying on the standard subtraction algorithm.

Visual Learning:

The *Visual Learning* animation includes the break apart strategy and the U.S. traditional standard subtraction algorithm. Students should use place value blocks to support their understanding of the strategies presented. Students may view and discuss standard algorithms to expose the idea that it is one more strategy, but do not focus on it. Additional learning opportunities around written methods and standard algorithms have been included in Topic 11.

Independent Practice/Math Practices and Problem Solving:

Consider using item 12 (34 - 8 = 35 - ?) to engage students in a discussion around the meaning of the equal sign (=). Avoid defining the meaning of this symbol. Instead, use questioning and examples to formatively assess students' understanding. If students believe that the equal sign means, "the answer is" continue to move them toward an understanding of equivalence as "the same as". Presenting equations with the answer first (7 = 2 + 5) causes students to examine why it is the same as when the answer is last (2 + 5 = 7). In addition, asking students to find equivalent expressions, rather than just the answer, helps students to focus on the meaning of this symbol. For example, given 19 + 23, students may come up with 19 + 23 = 20 + 22. (Van de Walle, et al., 2014, p. 230).

Lesson 6-8: Solve One-Step And Two-Step Problems

2.OA.A.1

MP.1 MP.2

MP.4 MP.7

Access Prior Learning:

In lesson 3-8 and 4-7 students solved one-step and two-step word problems.

In lesson 5-8, second grade students solved word problems using the relationship between addition and subtraction.

Developing the Big Idea:

In this lesson, students are developing understanding that models and equations can be used to solve one-step and two-step word problems. They are also developing understanding that solving two-step word problems requires them to solve the first step before solving the second step.

Solve & Share:

Provide a blank workspace for students to solve the problem. This may simply be placing a sticky note on the page. Child-watch for students who demonstrate understanding. Offer an extension by asking them to represent the two-steps of the problem using bar diagrams. Continue to encourage all students to use two strategies to check for accuracy, and evaluate those strategies for efficiency.

Visual Learning:

Focus the discussion on the Essential Question: Why is it helpful to complete a bar diagram and write an equation to solve word problems? (TE, p.366).

Lesson 6-9: Math Practices And Problem Solving: Reasoning 2.OA.A.1 | Access Prior Learning: | Consider using an

2.NBT.B.9

In first grade, students engaged in Math Practice 2.

MP.1 **MP.2**

MP.4 MP.5

MP.6

Developing the Big Idea: In this lesson, students are developing understanding of Math

Practice 2: Reasoning Abstractly and Quantitatively by thinking about words and numbers to solve problems.

Consider using and discussing the *Math Practice 2 Animation* available on Pearson Realize online. Engage students in discussions around MP.2 behaviors (TE, pp.F24 - F24A). The focus of this lesson should be on these habits of mind, rather than on computation. Students use reasoning to contextualize addition and subtraction problems, then decontextualize by writing and solving an equation.

Solve & Share:

Provide a blank workspace for students to solve the problem. This may simply be placing a sticky note on the page.

Visual Learning:

Before the *Visual Learning* animation, give students time to make sense of and solve the problem presented in the animation. In *Guided Practice* item 2, if students are confused about the comparison bar diagram, reference Error Intervention Note: Item 2 (TE, p.372). Representing the quantities of 46 and 18 with cubes will help them understand the proportion of the boxes in the diagram, and why 46 is placed in the larger "part" box.

Topic 6 Performance Assessment:

It is not necessary to give the *Topic 6 Performance Assessment* at this time. Teachers may choose to select items to add to the *Topic 7 Performance Assessment*, skip the assessment, or use a few items to formatively assess and inform instruction.

Lesson 7-1: Represent Addition and Subtraction Problems

2.OA.A.1

MP.2

MP.4

MP.5

MP.8

Access Prior Learning:

In first grade, (1.OA.A.1) students solved addition and subtraction word problems within 20, with unknowns in all positions.

In previous lessons, second grade students represented and solved problems using models, pictures and equations.

Developing the Big Idea:

In this lesson, students are developing understanding that equations can be written to model word problems using a question mark (?) to represent the unknown. Students will model problems with the unknown in any position.

Solve & Share:

During problem solving, child-watch for students who are able to explain their thinking in a way that reflects understanding of the context of the *Put Together Addend Unknown* problem. Encourage students to explain how their drawing connects to the abstract equation given. Sequence the share to include strategies seen in the Visual Learning such as bar diagrams, adding on, and the relationship between addition and subtraction.

Visual Learning:

Give students an opportunity to solve the *Add To Change Unknown* word problem presented before continuing with the animation. During discussion, strategically ask questions that help students connect student strategies from the Solve & Share to the strategies presented in *Visual Learning*. Also, encourage students to think of other models for solving the problem, including open number lines with a jump of 30 and a jump of 1.

In the *Guided Practice*, use of the *Problem Solving Recording Sheet* (Teaching Tool 1) may help students make sense of the problem. Use of this tool models questions students should ask themselves when making sense of a problem and devising a plan. Item 1 is a *Take From Start Unknown* problem. Having the unknown in the start position increases the level of difficulty of the problem. Item 2 is an easier problem type, *Add To Result Unknown*.

Assess and Differentiate:

In the *Intervention Activity*, "Step-by-Step!" (TE, p.395A) further support students by having them represent quantities on the bar diagram with manipulatives or drawings, as a way to give meaning to the numerals in the problem.

Lesson 7-2: Mixed Practice: Solve Addition and Subtraction Problems

2.OA.A.1

MP.1

MP.2

MP.3

MP.4

Access Prior Learning:

In lesson 7-1, second grade students wrote equations with a question mark (?) for the unknown, to model and solve word problems.

Developing the Big Idea:

In this lesson, students are *developing* understanding of *Compare Bigger Unknown* problems.

Securing the Big Idea:

In this lesson, students are securing understanding that drawings, bar diagrams and equations can be used to make

This lesson focuses on *Compare Bigger Unknown* word problems involving addition and subtraction. As indicated in the instructional note at the beginning of this document, *compare* problems are more challenging because one of the quantities must be conceptualized, as it is not present physically in the problem (CCSWT, 2011). Rather than involving an action such as *Add To or Take From*, these problems involve the relationship between quantities. These quantities are labeled as follows: *referent set, compared set,* and *difference (Carpenter, et al., 2015, p.10)*.

Mark has 8 mice. ← Referent set

Joy has 12 mice. ← Compared set

Joy has 4 more mice than Mark. ← Difference

sense of word problems and strategies can be used to solve them.

Throughout instruction, ensure that students are encouraged to model their thinking with concrete manipulatives and drawings. As noted in the *Error Intervention Note*: Item 1 (TE p.398), labeling the bar diagram helps students make sense of the quantities and relationships in a problem. Reference *Visual Learning* for an example. Also, when drawing comparison bar diagrams, the sizes of the bottom boxes should correspond with the proportion of the quantities in the problem.

Solve & Share:

If students have difficulty making sense of the problem, engage them in a conversation around the word, *fewer* using real-world examples. As indicated by Van de Walle et al., students often have more experiences with the term *more than*, so they may need additional experiences with the terms *less than* or *fewer than* (2014, p.131).

Visual Learning:

The *Do You Understand? Show Me!* (TE, p.398) further supports students as they develop understanding of the *more* and *less than* relationships. Consider having students work in partners to model the statements.

Lesson 7-3: Continue Practice With Addition And Subtraction Problems

2.OA.A.1

MP.1 MP.2

MP.4 MP.8

Access Prior Learning:

In lesson 7-2, second grade students used drawings and equations to make sense of *Compare Bigger Unknown* word problems.

Developing the Big Idea:

In this lesson, students are developing understanding of Compare Difference Unknown and Compare Smaller Unknown problems.

Securing the Big Idea:

In this lesson, students are securing understanding that drawings, bar diagrams and equations can be used to make sense of word problems and strategies can be used to solve them.

Solve & Share:

Students begin this lesson with a *Compare Smaller Unknown* problem. Encourage all students to use drawings, bar diagrams and equations to show their thinking. Sequence the share to highlight a variety of student strategies for making sense of the problem, including a bar diagram with labels. If no students use a bar diagram with labels, plant the idea with a student during problem solving so that you have a student to share this strategy.

Visual Learning:

Allow students to make sense of and solve the problem in the animation so they are better prepared to discuss. As with lesson 7-3, The Do You Understand? Show Me! (TE p.404) further supports students as they develop understanding of the *more* and *less than* relationships. Consider having students work in partners to model the statements.

Independent Practice/Math Practices and Problem Solving:

For item 7, encourage students to notice patterns in how addends change, while the sum remains constant. This provides another opportunity to build the big idea of equivalence. Notice the instructional note at the beginning of this document, item 7 provides opportunities to write equivalent expressions such as 36 + 22 = 37 + 21 (36 + 22 is the same as 37 + 21).

Assess and Differentiate:

In the *Intervention Activity*, "Dare to Compare!" (TE, p.407A), encourage students to use concrete manipulatives and drawings to develop or reinforce their understanding of the quantities and relationships represented in the comparison bar diagrams.

*CTC: Solve & Share (student work samples)

Lesson 7-4: Solve Two-Step Problems

2.OA.A.1

MP.1 MP.2

MP.4 MP.6

Access Prior Learning:

In lesson 6-8, second grade students solved one-step and two-step word problems.

Developing the Big Idea:

In this lesson, students are developing understanding that drawings and equations can be used to make sense of and solve two-step word problems; and that these problems include a hidden question that needs to be answered first.

As stated in the "Progressions for the Common Core State Standards in Mathematics", two-step problems should not involve the most difficult subtypes (CCSWT, 2011, p.20-21). During this lesson, students will focus on identifying the hidden question that needs to be solved first, before solving for the final answer.

Solve & Share:

This two-step problem is an *Add To Result Unknown/Take Away Result Unknown* problem. The use of connecting cubes supports students as they model the actions within the problem. During the share, focus the discussion on how students made sense of the problem, and how they identified the first, hidden question before finding the final answer.

Independent Practice/Math Practices and Problem Solving:

When solving the quick check items (marked with a pink check), students may find item 8 more challenging than items 3 or 9 because it contains a *change unknown* problem type. Continue to encourage students to use concrete manipulatives.

Item 3: Put Together Whole Unknown/Take Away Result Unknown

Watch for students who reason about the numbers in this problem. These students may subtract 8-7 to get 1, and then add 16, resulting in 17 frogs left. This strategy simplifies the computation needed to solve the problem and demonstrates flexibility with number and operations.

Item 8: Add To Result Unknown/Take Away Change Unknown Item 9: Take Away Result Unknown/Add To Result Unknown

Lesson 7-5:	Continue To Solve Two-Step Prob	nlems
2.OA.A.1	Access Prior Learning:	Solve & Share:
MP.1 MP.3 MP.4 MP.6	In lesson 7-4, second grade students solved two-step word problems by first identifying the hidden question needed to find the final answer. Securing the Big Idea: In this lesson, students are securing understanding that drawings and equations can be used to make sense of and solve two-step word problems; and that these problems include a hidden question that needs to be answered first.	The two-step problem presented contains the <i>Take Away Change Unknown</i> and <i>Add To Start Unknown</i> problem types. In addition to the challenge of <i>change unknown</i> problems, some students may find the context of this problem challenging. The use of role-playing can support students in clarifying the meaning of "return some books" and "take out 15 more books". If students have been told to use the key word strategy (see the Instructional Note at the beginning of this document as to why the key word strategy should be AVOIDED), they may perceive "take out" to mean subtraction, when in fact, in this context it refers to addition. If students continue to struggle with the context of this problem, consider changing the context but maintaining the problem types and numbers. Assess and Differentiate: In the <i>Intervention Activity</i> , "Come and Go" (TE, p.419A), students work with two-step word problems including <i>Add To</i> and <i>Take From</i> problem types. Also, provide these students opportunities to work with two-step word problems that include <i>Put Together</i> and <i>Take Apart</i> problem types with unknowns in all positions. Reference Table 1. Common addition and subtraction situations at the beginning of this document for examples.
		*CTC: Quick Check (digital platform)
Lesson 7-6:	Math Practices and Problem Solv	
2.OA.A.1 MP.1	Access Prior Learning: In first grade, students engaged in Math Practice 2: Reason Abstractly and Quantitatively.	Consider using the <i>Math Practice 2 Animation</i> on Pearson Realize Online for an example of MP.2 behaviors. Also, reference the Math Practices and Problem Solving Handbook for suggestions for developing, connecting and assessing MP.2 (TE, p.F24-F24A).
MP.2 MP.3 MP.4 MP.7	Developing the Big Idea: In this lesson, students are developing understanding of Math Practice 2: Reason Abstractly and Quantitatively through writing and solving word problems.	 MP. 2 Behaviors: Identifies and understands the quantities in the problem. Shows and explains how quantities are related (e.g., bar diagram). Translates real-world contexts correctly to numbers, expressions, equations, or concrete or pictorial representations. Connects numbers, expressions, equations, or concrete or pictorials representations back to real-world contexts.
		Independent Practice/Math Practices and Problem Solving: If students struggle to write number stories, encourage them to tell their stories orally before writing, as suggested in the <i>Coherence</i> note (TE p.422). Modeling with manipulatives is also

References

helpful

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- Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

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▶ Grade 2 Topic 8: Work With Time and Money

Big Conceptual Idea: K-5 Progression on Measurement and Data (Measurement Part) (pp. 2-3)

Prior to instruction, view the Topic 8 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 435A-435E), the Topic Planner (pp. 435I-435K), the Topic Performance Assessments (pp. 501-502A), and all 8 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 435A-435E)	Topic Essential Question: How can you solve problems about counting money or telling time to the nearest 5 minutes?
	Reference Answering the Topic Essential Question (TE, pp. 497-498) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	8-8	Assessment
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5 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 8 is measurement. That is, objects have measurable attributes that can be quantified using specific units. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.MD.C.

Standards listed in **bold** indicate a focus of the lesson.

Number of lessons: 8 A/D/E: 5 days NVACS Focus: MD.C

2nd Grade Curriculum Pacing Framework: Balanced Calendar

2.MD.C Work with time and money.

- 7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
- 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

This work also supports the 2.NBT.A cluster. As indicated in the *Progression Documents*, the concept of *unit* is central to this work (CCSWT, 2012, p.3). Just as students understand ones, tens and hundreds as *units* and *units of units* (one hundred can be thought of as 1 hundred, 10 tens, or 100 ones), they will develop understanding of units in money (ones, fives, tens, twenty-fives, fifties, and hundreds) and time (minutes and hours). This provides teachers opportunity to **facilitate connections to students**' **schema around place value** from Topics 3-5. Students often find the units of time more difficult to understand as they refer to duration and are not tangible (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p.286).

Money. As students work in Topic 8, they will need to know the names and values of coins, which are conventions of our social system. This information is something that students must be told (Van de Walle, et al., 2014, p.289). However, students need an understanding of 5, 10 and 25 for these values to make sense. For example, students may be confused when we point to *one* nickel and say "This is five." if that student still needs to count objects by ones to determine "how much". When working on the values of coins, consider taking an approach that focuses on purchasing power (a quarter can buy the same thing that 25 pennies can buy). Through the study of the attributes of each coin (color, size, heads/tails sides, etc.) students will develop an understanding that a coin's value is not dependent on its size (Van de Walle, et al., 2014).

Coins are non-proportional representations. Reference the *Math and Science Project: Money Matters* and *Extension* activities included in the *Topic Opener* to support students in the study of coins and their attributes when launching the topic (TE p.435). Working with students to create an anchor chart or concept map provides students with a student-created resource for reference throughout the topic. As students identify connections and construct understandings have them add to this chart. For instance, students will develop connections to their place value understanding, use their mental math skills, and skip-count by 5s and 10s (NVACS, 2010, 2.NBT.A.2). Use questioning to help students connect money to the big mathematical idea of measurement. Although money does not explicitly appear in the NVACS for first grade, students use coins in the development of the 5-structure and 10-structure; as well as work with coins as a manipulative for skip counting. These experiences will support students' work with units in money.

Throughout lessons 8-1 to 8-5, students will have the opportunity to consider equivalent representations of a given monetary value. Add these equivalent representations to the class anchor chart or concept map. In lessons 8-1 to 8-4 children are encouraged to order and count coins and bills efficiently by starting with the greatest value and counting on. This leads into an appropriate strategy for creating an organized list (see lesson 8-5) used to find all the different ways to show the same amount of money. Refer to the *Prevent*

Misconceptions and Error Intervention notes included in your teacher's edition on the Step 2 Develop: Visual Learning pages for insight into possible misconceptions that may arise, and how to respond. As a consideration for long-term planning, students will benefit from learning opportunities throughout the school year to support their understanding of 2.MD.C.8.

Time. The concept of measuring time can be challenging for students as it is unlike other commonly measured attributes. Time is not visible or tangible, but rather, is the duration of an event. In first grade, (NVACS, 2010, 1.MD.B.3) students used analog and digital clocks to tell time to the hour and half-hour. In second grade, (NVACS, 2010, 2.MD.C.7) students learn to read analog and digital clocks in 5-minute intervals. Building on schema from first grade, teachers may consider having students time aspects of their daily lives as a fun and useful way to help students build an understanding of the units of time (minutes, hours). This real-world application can occur throughout the year. As with money, work with students to create an anchor chart or concept map to record understandings and connections. Use questioning to help students connect time to the big mathematical idea of measurement.

To help your students understand how to read analog clocks, consider these suggestions from *Teaching Student-Centered Mathematics*:

- Begin with using approximate language to describe the time on a one-handed clock (with the minute hand broken off).
 Encourage children to describe the time with phrases such as: "It's a little past 11 o'clock." "It's halfway between 12:00 and 1:00." "It's about 2 o'clock."
- Encourage students to consider what happens to the big hand as the little hand goes from hour to hour. If the minute hand is pointed at 12, where is the hour hand pointing? When the hour hand is about half way between numbers, where is the minute hand pointing? Focusing on this relationship will help students construct an understanding of the functions of the minute hand and hour hand.
- Use a one-handed clock and a two-handed clock. Cover the two-handed clock. Throughout the day, take a minute to
 discuss the time as shown on the one-handed clock in approximate language. Then, have children predict where the
 minute hand should be. Finally, reveal the two-handed clock and check for reasonableness of student predictions.
- Use counting by fives to help children learn to tell time in 5-minute intervals. Encourage children to move away from "the
 minute hand is pointing at the 4", towards "the minute hand is about 20 minutes after the hour." Continue to work
 towards encouraging students to first look at the hour hand for an approximation of the time, and then use the minute
 hand for precision.
- Encourage students to relate the time after the hour to the time before the hour to help with both telling time and number sense.
 (Van de Walle, et al., 2014, p.287-288)

Math Practice 2: Reason abstractly and quantitatively

Focus opportunities for students to develop MP.2 behaviors. This is the focus of the *Math Practices and Problem Solving* lesson 8-5. Reference the Teacher's Edition (pp. F24-F24A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

	Essential Academic Vocabulary Use these words consistently during instruction.						
New Academic Vocabulary: (First time explicitly taught)		Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)					
half-dollar greatest value least value dollar dollar sign dollar bills	tally marks quarter past half past quarter to a.m. p.m.	dime nickel penny quarter cents					

Additional terminology that students may need support with:

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students able to determine the value of a collection of coins and understand that different coins have different values?"

"Are students able to tell time to the nearest 5 minutes, and do they understand that numbers on an analog clock do not indicate numbers of minutes before or after an hour?"

Lesson	Evidence	Look for
8-4	Solve & Share (student work samples)	Focus CTC around the big idea:
	*Money	Strategies and models to solve problem
		understand that size of coin/bill does not indicate value
8-6	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	*Time	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 497-502
Assessments (summative)	SE pp. 497502	

Standards listed in **bold** indicate a focus of the lesson. **NVACS Mathematical Development** (Content and Instructional Clarifications & Considerations Practices) of the Big Idea Lesson 8-1: Solve Problems With Coins 2.MD.C.8 Access Prior Learning: Possible 2-day Lesson Resources: It is optimal to use real coins, but plastic coins are an alternative. Many teachers In Topics 3-5, second grade 2.NBT.A.2 request families to send in a bag of coins for the child to use in math. students worked with place value and skip counting. MP.1 Day 1: Topic Opener: MP.2 Developing the Big Idea: Although money is not in the first grade standards, students do work with coins in Bridges In this lesson, students are MP.3 Number Corner. Regardless, students will benefit from beginning the topic with the *Topic* developing understanding of coins Opener and a discussion of the Topic Essential Question (TE, p. 435). Follow with the Math and MP.5 as non-proportional units in which Science Project: Money Matters and Extension activity (TE, p.435). This project pairs well with their size does not relate to their the Topic 8 Vocabulary Words Activity (TE, pp.437-438) using the words, cents, penny, nickel, value. They count on to determine dime, quarter, and half-dollar. Also, consider having students include attributes of each coin on the graphic organizer (Teaching Tool 60) and post them as resources on the math focus wall. the value of a collection of coins Introduce the remaining vocabulary words as they appear in instruction. already listed in order from greatest value to least using Finally, use the *Review What You Know* (TE, p. 436) to help identify students who may need concrete objects and drawings. additional support. Consider offering support as indicated in the *Item Analysis for Diagnosis* and Intervention chart (TE, p.436) which connects missed items to corresponding lessons in the MDIS kit. Lastly, enlist the help of families by sending the Home School Connection for Topic 8 (available on Pearson Realize Online) which encourages children to practice counting coins. DAY 2 Solve & Share: Ensure that all students have access to coins. Child-watch for students who demonstrate understanding of coins and their values as they solve this two-step Add To Result Unknown word problem. Students may need support with how to draw coins in an efficient manner. Visual Learning: As students discuss the Visual Learning, look for opportunities to add equivalent representations of coin values (a guarter is equivalent to 2 dimes and 1 nickel) to the vocabulary resources created in the *Topic Opener* above. Focus the conversation on efficient ways to count coins, beginning with the coin of greatest value. Also, highlight the importance of the cent sign in indicating the unit. This is a helpful connection to MP.6 Attend to Precision. -continues on next page-

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their student edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The Independent Practice page offers problems that support procedural skill and fluency. The Math Practices and Problem Solving page offers problems that support application. The Quick Check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Lesson 8-2: Continue to Solve Problems With Coins

2.MD.C.8 2.NBT.A.2

MP.3 MP.4 MP.5

MP.6

Access Prior Learning:

In the prior lesson, second grade students counted on, to determine the value of coins already listed in order from greatest value to least.

Developing the Big Idea:

In this lesson, students are developing understanding of coins as non-proportional units in which their size does not relate to their value. They order and count on to determine the value of a collection of coins using concrete objects, drawings and mental math.

Solve & Share:

Allow students to problem solve before introducing new vocabulary, greatest value and least value. Formatively assess students' understanding of these terms during problem solving, then, use the share to highlight these comparisons. Select students to use the graphic organizer "Picture the Word" (Teaching Tool 58) to create a shared resource defining greatest value and least value. This provides students with an opportunity to represent coins efficiently, as discussed in lesson 8-1 above. Add these graphic organizers to the focus wall.

Visual Learning: When planning, prepare intentional responses to student misconceptions by reviewing Prevent Misconceptions and Error Intervention notes (TE, p.450). Use real coins if available.

Independent Practice/Math Practices and Problem Solving:

Continue to ensure that all students have access to concrete coins. Child-watch for students who struggle to order and count coins beginning with the coin of greatest value. Engage these students in small group instruction using the Intervention Activity (TE, p.453A) during Assess and Differentiate.

Lesson 8-3: Solve Problems With Dollar Bills

2.MD.C.8 2.NBT.A.2

MP.2 MP.4

MP.6

MP.7

Access Prior Learning:

In lesson 8-1 and 8-2, second grade students ordered and counted coins beginning with the coins of greatest value.

Developing the Big Idea:

In this lesson, students are developing understanding of bills as non-proportional units in which their size does not relate to their value. They make collections of coins with a value of 100 cents. They also order and count on to determine the value of a collection of bills.

Resources: Ensure that students are using concrete objects. Teaching Tools 32-33 offer printable bills. These are located in the Teacher's Resource Masters Volume 2.

Solve & Share:

Continue to ensure that all students are encouraged to use concrete coins in addition to drawings. Ask students to show two ways to build 100 cents with coins. Child-watch for students who demonstrate flexibility and efficiency. If students use 100 pennies, ask if there is a more efficient way to build 100 cents. Doing so supports students' understanding of units in money, connecting to their place value understanding.

Visual Learning:

Focus the discussion on making connections between counting coins and counting bills. This will support students when they respond to Do You Understand? Show Me! (TE, p.456). Also, highlight the importance of the dollar sign in indicating the unit. This is a helpful connection to MP.6 Attend to Precision. Students may need support with how to draw bills in an efficient manner.

\$20 \$1 \$5 \$10 \$100

Lesson 8-4: Continue To Solve Problems With Dollar Bills

2.MD.C.8 2.OA.A.1

MP.1 MP.2 MP.4

MP.6

Access Prior Learning:

In the prior lesson, students made collections of coins with a value of 100 cents. They also ordered and counted on to determine the value of a collection of bills.

Developing the Big Idea:

In this lesson, students are developing understanding of addition and subtraction word problems involving money (bills). This lesson offers opportunities to connect to the purpose of the *Problem Solving Record Sheet* (Teaching Tool 1). Encourage students to identify what they know and what they need to find before solving. Continue to encourage all students to use concrete money manipulatives to support their pictorial representations.

Solve & Share:

This problem presents a *Put Together Total Unknown* problem with three addends. Child-watch for students who apply strategies such as skip counting or mental math (e.g., two \$10 bills is \$20). If students use less efficient strategies such as counting all or counting on, ask them if they can use skip counting or known facts to solve the problem more efficiently. All students should show their work with drawings, but some may need to model the problem using concrete bills. This is the case for students who demonstrate a misconception similar to Clay's Work in Analyze Student Work (TE, p.461). As noted in the Coherence section (TE, p.461A), also watch for and encourage students to keep track of their thinking with organized lists. This will connect to their use of organized lists in lesson 8-5. If students use these strategies, select them to share.

Independent Practice/Math Practices and Problem Solving:

Item 9 presents an opportunity for students to explore money as non-proportional representations- the size does not affect the value. All U.S. bills have the same dimensions regardless of value. Also, the count of bills is irrelevant without considering the unit or value. Consider engaging students in a discussion around these ideas.

*CTC: Solve & Share (student work samples)

Lesson 8-5: Math Practices And Problem Solving: Reasoning

2.MD.C.8 2.OA.A.1

MP.1

MP.3

MP.4

MP.8

Access Prior Learning:

In Topic 7, second grade students focused on Math Practice 2: Reason Abstractly and Quantitatively.

Students focused on MP2. Behaviors in Topic 7. Reference the Math Practices and Problem Solving Handbook for suggestions for developing, connecting and assessing MP.2 (TE, pp.F24-F24A). Also, consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.

MP.2

Developing the Big Idea:

solving word problems.

In this lesson, students are developing understanding of Math Practice 2: Reason Abstractly and Quantitatively through writing and

MP. 2 Behaviors:

- Identifies and understands the quantities in the problem.
- Shows and explains how quantities are related (e.g., bar diagram, table).
- Translates real-world contexts correctly to numbers, expressions, equations, or concrete or pictorial representations.
- Connects numbers, expressions, equations, or concrete or pictorials representations back to real-world contexts.

Solve & Share:

During problem solving, child-watch for students who generalize their work with ordering coins and bills from greatest value to least value to complete the table in a systematic manner. Select and sequence the share to progress from less systematic to more systematic student solutions. Facilitate conversation that encourages students to connect peers' strategies and evaluate them for efficiency. Ask, "Which strategy would be most helpful in finding all the possible combinations to make a given value?"

Lesson 8-6: Tell Time To Five Minutes

2.MD.C.7 2.NBT.A.2

Access Prior Learning:

In first grade (1.MD.B.3), students told time to the nearest hour and half-hour using analog and digital clocks.

Possible 2-Day Lesson

Resources: Teaching Tools 34-35 are available for printable analog and digital clocks. These are located in the Teacher's Resource Masters, Volume 2.

MP.5 MP.6 MP.8

MP.2

Developing the Big Idea:

In this lesson, students are developing greater precision with telling time to the nearest 5 minutes.

If students demonstrate confusion around telling time with analog clocks, refer to the suggestions provided in the Instructional Note at the beginning of this document. Also, refer to the suggestion offered in the Prevent Misconceptions note (TE, p.474). This lesson offers students the opportunity to use skip counting by 5s up to 60 when telling time.

Securing the Big Idea:

In this lesson, students are securing understanding of how to tell time using analog and digital

Independent Practice/Math Practices and Problem Solving:

The Item 9 instructional note (TE, pp.475-476) offers an opportunity to engage students in a conversation around 0 as a placeholder in place value (106 is not the same as 16), as well as in telling time.

clocks to the nearest hour and halfhour.

*CTC: Quick Check (digital platform)

Lesson 8-7: Tell Time Before And After The Hour

2.MD.C.7 2.NBT.A.2 MP.3

MP.4

MP.6

MP.8

Access Prior Learning:

In first grade (1.G.A.3), students partitioned circles into halves and fourths, using the words: halves, fourths, quarters, half of, fourth of, and quarter of.

To support students in understanding quarter past, half past and quarter to, intentionally use these phrases when referring to time throughout the school day and year. For example, "We will go to music at half past ten." Also, work with students to construct a vocabulary graphic organizer such as "Picture the Word" (Teaching Tool 58). As students draw a variety of analog and digital clocks that show the term, ask them to identify commonalities. It is also helpful for students to consider non-examples. Understanding of the unit of an hour is critical to connecting these terms to their meaning, and supports fraction development (Van de Walle et al., 2014, p.253).

In lesson 8-6, second grade students told time to the nearest 5 minutes.

Lesson 8-8: A	Developing the Big Idea: In this lesson, students are developing understanding that time can be described in different ways, including before and after the hour using: quarter past, half past and quarter to. M. And P.M.	Visual Learning: Partitioning circles into halves and fourths can help students understand why 30 minutes can be referred to as half past, or 45 minutes can be referred to as a quarter to. Further support is offered in the Intervention Activity, "The Face of Time!" (TE, p.483A). Consider using Teaching Tools 34 and 35.
2.MD.C.7 2.NBT.A.2	Access Prior Learning: In the prior lesson, second grade	As stated in the <i>Coherence</i> note, students' work with a.m. and p.m. deepens their understanding of time as relevant to their daily lives (TE, pp.487-488).
	students described time in different ways, including before and after	and ordering of time as relevant to their daily invest (1 ±, pp. 167-166).
MP.2	the hour using: <i>quarter past, half</i>	
MP.6	past and quarter to.	
MP.8	Doveloning the Dig Idea.	
	Developing the Big Idea: In this lesson, students are	
	developing understanding of a.m.	
	and p.m. They use reasoning to determine which is appropriate to	
	describe the time of the event.	

References

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▶ Grade 2 Topic 9: Numbers to 1,000

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 9 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 503A-503E), the Topic Planner (pp.503I-503L), the Topic Performance Assessments (pp. 581-582A) all 10 lessons.

Mathematical Background:	Topic Essential Question:
Cluster Overview (TE, pp.	How can you count, read, and show numbers to 1,000?
503A-503E)	Reference Answering the Topic Essential Question (TE, pp. 577-578) for key elements of answers to the Essential Question.

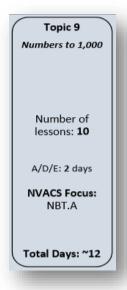
The lesson map for this topic is as follows:

		<u>, , </u>								
9-1	9-2	9-3	9-4	9-5	9-6	9-7	9-8	9-9	9-10	Assessment

2 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of topic 9 focuses on place value understanding through the structure of the base-10 numeration system. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.NBT.A.



2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

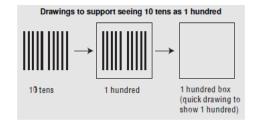
2.NBT.A Understand place value.

- 1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens called a "hundred".
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- 2. Count within 1,000; skip-count by 5s, 10s, and 100s.
- 3. Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.
- 4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. Although this topic focuses on place value, place-value instruction does not need to occur in isolation (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 176). Students have been building their place value understanding through their work with addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both 10 ones and 1 ten. In second grade, students extend these place value understandings to three-digit numbers, understanding one hundred as a bundle of 10 tens and as a "hundred". This lays the foundation for students to understand the repeated structure of our number system. Each unit represents the bundling of ten units to the right.

The use of concrete manipulatives, drawings and layered place-value cards, such as Arrow Cards (found under "Instructional Tools" on the WCSD Curriculum & Instruction website) help students to connect written numbers to their meanings in terms of hundreds, tens and ones, as well as sums of these base-10 units (CCSWT, 2015, p.8). It is important that students construct this understanding and impose their own understanding on the model. On the contrary, telling students that a pre-grouped model, such as a hundreds flat, is worth one hundred is ineffective. When considering language, help students connect standard language, "one hundred thirty-five", to base-ten language, "1 hundred, 3 tens, 5 ones; 1 group of a hundred, 3 groups of ten, 5 ones, etc". It is recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).



Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

In this Topic, students will continue to develop their mental math skills through skip-counting by 5s, 10s, and 100s (2.NBT.A.2). They will also use place value understanding to compare numbers. Students will reason about the value of a digit based on its place in the number. For example, students will reason that 100, the smallest 3-digit number, is larger than any other 2-digit number. As a result, students will learn to compare the digit in the largest place value position first. Students benefit from both examples and counterexamples. By including counterexamples in class discussions, students are afforded the opportunity to explore their misconceptions and deepen their understanding of place-value (Van de Walle, et al., 2014, p.189). One misconception that often arises is that of zero as a placeholder. Engaging students in an examination of numbers such as 405, 45, and 450 can help students understand the importance of zero in our number system (Van de Walle, et al., 2014, p.189).

Math Practice 7: Look for and make use of structure

Focus on opportunities for students to develop MP.7 behaviors. This is the focus of the *Math Practices and Problem Solving* lesson 9-10. Reference the Teacher's Edition (pp. F29-F29A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

	Essential Academic Vocabulary Use these words consistently during instruction.						
New Academic Vocabulary (First time explicitly taught)	y :	Review Academic Voc (Vocabulary explicitly taught in pr					
thousand place-value chart standard form expanded form word form	equals, = decrease increase	compare digit greater than, > hundred less than, <	pattern regroup* (T4) * Do NOT use borrow or carry as these are misleading, however trade and exchange may be used (Van de Walle, 2014, p. 218).				

Additional terminology that students may need support with: bundled, exchange, trade

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students developing conceptual understanding and make connections to the Base-10 system to recognize that digits in each place value represent amounts of hundreds, tens, or ones?"

Lesson	Evidence	Look for
9-5	Solve & Share (student work samples)	 Focus CTC around the big idea: student strategies and models understanding there are multiple ways to group 1s, 10s, & 100s to show the same number use of models/representations to show numbers
9-10	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	Items 1, 2, 4, and 5	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 577—582
Assessments (summative)	SE pp. 577-582	

Standards listed in **bold** indicate a focus of the lesson.

	old indicate a focus of the lesson.	
NVACS		
(Content and	Mathematical Development of	Instructional Clarifications & Considerations
Practices)	the Big Idea	mati deticital cidimodilona di considerationa
Lesson 9-1: U	Jnderstand Hundreds	
2.NBT.A.1a	Access Prior Learning:	A note of CAUTION: When referring to regrouping, do NOT use the terms borrow or carry as
2.NBT.A.1b	In first grade, (1.NBT.B.2) students understood that 2-digit numbers	they are misleading. Instead, <i>trade</i> , <i>regroup</i> and <i>exchange</i> may be used (Van de Walle et al., 2014, p. 218).
	represent an amount of tens and	T 10
MP.2	ones. They also constructed the	Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 503),
MP.4	understanding of ten as 10 ones	Review What You Know (TE, p. 504), and Topic 9 Vocabulary Words Activity (TE, pp. 505-506)
MP.5	and 1 ten.	only. Introduce remaining vocabulary words as they appear in instruction. Post the essential
MP.7	Davidoning the Dig Idea.	question and student strategies on your math focus wall.
	Developing the Big Idea: In this lesson, students are developing understanding of the base-10 number system and the relationships that exist between ones, tens and hundreds to 1,000. Students count by hundreds to 1,000.	Solve & Share: Ensure that <i>all</i> students use concrete place-value blocks to support and connect to their drawings. Child-watch for students who demonstrate place value understanding through the use of tens. Also child-watch for students who organize their drawings to make them easy to count. Strategically conference with a student to plant the idea that place-value blocks can be drawn efficiently (a dot for ones, a line for tens, and a square for hundreds). Have this student share, and engage students in a discussion around why that student's drawings are more efficient. Establish a class norm for representing place-value blocks with these symbols.
		Visual Learning: Ensure that <i>all</i> students use concrete place-value blocks to support conceptual understanding of the bundling of ten units to make one of the next larger unit (10 tens makes 1 hundred, 10 hundreds make 1 thousand). When planning, refer to <i>the Prevent Misconceptions</i> and <i>Error Intervention notes</i> (TE, p.512) to help you anticipate possible misconceptions and plan intentional teacher responses.
		Independent Practice/Math Practices and Problem Solving: As an extension opportunity and formative assessment for item 6, ask students, "How can you represent 300 without using hundreds flats?" Child-watch for students who are able to show equivalent representations (See Group C below) such as 30 tens, 300 ones, or a combination of tens and ones such as 25 tens and 50 ones. Equivalent representations show the number using fewer than the maximum number of tens, or hundreds.
		Group A Unitary or count-by-ones approach "count-by-ones approach "leasted tham(by area)." Group B Base ten or groups-of-dens approach "The base and three mark." "The base and three mark." Group C Equivalent or Notestad and bursean approach "One to and three mark." "One to and three mark."
		Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2 (Vol. 1). Harlow: Pearson Education International.
		As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The <i>Independent Practice</i> page offers problems that support procedural skill and fluency. The <i>Math Practices and Problem Solving</i> page offers problems that support application. The <i>Quick Check</i> items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.
Lesson 9-2: N	Models And 3-Digit Numbers	
2.NBT.A.1	Access Prior Learning:	Place value mats are helpful tools for organizing place-value blocks. Consider adding 2 ten
2.NBT.A.3	In the prior lesson, second grade students worked with the base-10	frames to the ones section of the mat. This will reduce students' need to recount the number of ones. It will also support students in identifying when 10 ones need to be bundled into 1 ten.
MP.1	number system and the	This sample serves to illustrate the use of ten frames on a place
MP.2	relationships that exist between	value mat. Although you will want to use a mat that also includes
MP.4	ones, tens and hundreds to 1,000.	hundreds. A blackline master is included at the end of this
	Developing the Big Idea:	document.
MP.5	In this lesson, students are	
	developing understanding that place-value blocks and drawings	-continues on next page-

can be used to model and write 3digit numbers. Students compose and decompose numbers into ones, tens and hundreds.

Visual Learning:

Before showing the animation, have students solve the problem presented in the animation (2 hundreds flats, 5 tens rods, and 9 ones cubes) using concrete place-value blocks and place-value mats to increase conceptual understanding and engagement. Students can rotely memorize the ones-, tens- and hundreds-digits. As John Van de Walle, et al., caution, "Be aware of how easy it is for a child to show a number on a mat using base-ten blocks and learn to write the number without any understanding of what the number represents." (2014, p.190) To truly assess place-value understanding, focus questioning on having students show what the digit represents. Given the number 259, some students may be able to identify 5 as the digit in the tens place. However, these same students may not be able to show the value of this same digit. Students who still operate on ones may show the value of this digit as 5 ones, rather than 5 tens or 50 ones. For this reason, just identifying the tens-digit is less effective than connecting that digit to the value it represents.

Assess and Differentiate:

In the *Intervention Activity*, "Modeling Numbers" (TE, p.521A), ask them to identify the value of the units they counted. The use of <u>Arrow Cards</u> (found under "Instructional Tools" on the WCSD Curriculum & Instruction website) will support students in understanding the value of the three digits in 3-digit numbers as hundreds, tens and ones.

Lesson 9-3: Name Place Values

2.NBT.A.1 2.NBT.A.3

MP.3 MP.4

MP.5 MP.8

Access Prior Learning:

In lesson 9-1 and 9-2, second grade students counted by hundreds and modeled, read and wrote 3-digit numbers.

Developing the Big Idea:

In this lesson, students are *developing* understanding that the value of a digit depends on its place in the number. They continue to develop understanding of groups of 10 in our number system; that ten of one unit makes one of the next larger unit.

Solve & Share:

Continue to encourage students to use concrete place-value blocks. This problem encourages students to engage in MP.3 Critique Reasoning. A review of MP.3 behaviors developed in Topics 1 and 5 may be helpful.

Visual Learning:

Before showing the Visual Learning, have students solve the problem presented in the animation using concrete place-value blocks and place-value mats to increase conceptual understanding and engagement.

Independent Practice/Math Practices and Problem Solving:

As an extension opportunity and formative assessment for item 4, ask students to complete *Graphic Organizer 4:* Some Ways to Show a Number (Teaching Tool 61, also used for the *Topic 9 Vocabulary Words Activity*, TE, pp.505-506) for the number, 354. Students can either show multiple representations for the digit in the hundreds place, or multiple representations for the full value of the number.

Assess & Differentiate:

As with lesson 9-2, students may benefit from continued use of <u>Arrow Cards</u> (found under "Instructional Tools" on the WCSD Curriculum & Instruction website).

Lesson 9-4: Read And Write 3-Digit Numbers

2.NBT.A.3 2.NBT.A.1

MP.2 MP.4

MP.6 **MP.7**

Access Prior Learning:

In first grade (1.NBT.A.1), students read and wrote numbers to 120 in numeral form. First grade students also may have had exposure to expanded form in the context of days of school.

Developing the Big Idea:

In this lesson, students are developing understanding of three ways to write numbers: standard form, expanded form and word form.

As stated by Van de Walle, et al., "...the ways we say and write numbers are conventions, not concepts. Children must learn these by being told..." (2014, p.187). However, students must understand the value of digits in a number to represent numbers in expanded form.

Solve & Share:

Use the *Solve & Share* to formatively assess student understanding of the value of the digits in 231 as well as conventions for naming numbers. Consider pulling up Jamal's Work (*Analyze Student Work*, TE p. 529 and available online under the *Solve & Share* as "Teacher Resources") to engage students in a discussion that focuses on evaluating his work. This will support students' use of MP.3 Critique the Reasoning of Others.

Develop: Problem-Based Learning



Math Practices & Problem Solving: Construct Arguments: Solve & Share

Assign 🖺 Info **T** Teacher resources

Visual Learning:

After the animation, have students create a shared resource for expanded form, standard form and word form to post on the math focus wall.

Lesson 9-5: Different Ways To Name The Same Number Access Prior Learning: 2.NBT.A.3 In first grade, (1.NBT.B.2) students 2.NBT.A.1a understood that 2-digit numbers represent an amount of tens and MP.2 Solve & Share: ones. They also constructed the MP.3 understanding of ten as 10 ones and 1 ten. MP.4 MP.5 Visual Learning: In this topic, second grade MP.6 students have developed understanding that 100 is equivalent to 10 tens or 100 ones.

Developing the Big Idea:

In this lesson, students are developing understanding of equivalent names for numbers. This lesson lays the foundation for regrouping in addition and subtraction of multi-digit numbers. Avoid using the terms *carrying* or *borrowing* as they are misleading. Instead, students understand the term trading, as a lead into regrouping (Van de Walle, et al., 2014, p.218).

Continue to encourage use of place-value blocks and place-value mats. These mats are also used in the Visual Learning animation.

During the animation, have students build the representations of 123 using concrete blocks and place-value mats. Ask them to prove that the amount did not change during regrouping to further deepen their understanding of equivalence.

Independent Practice/Math Practices and Problem Solving:

For item 2, if students have difficulty showing 418 in two other ways using expanded notation, encourage them to build or draw the representation on a place value chart and connect it to the expanded notation.

*CTC: Solve & Share (student work samples)

Lesson 9-6: Place-Value Patterns With Numbers

2.NBT.A.2 2.NBT.B.8

MP.3 MP.5

MP.7

MP.8

Access Prior Learning:

In lessons 3-1 and 5-1, second grade students used hundred charts to add and subtract. In lesson 9-1, students skip-counted by 100s.

Developing the Big Idea:

In this lesson, students are developing understanding of 1 or 10 more and 1 or 10 less through the use of place value patterns.

The focus of this lesson is on place value patterns when counting by 1s, 10s and 100s.

Solve & Share:

During problem solving, encourage students to connect to prior learning around place value patterns and the hundred chart. Ask them to write about a pattern that helped them find the missing numbers. Focus the discussion of student strategies and the patterns they noticed in regards to which digits change when counting by 1s versus when counting by 10s. Consider displaying Gavin's Work (Analyze Student Work, TE, p.541 and available online under the Solve & Share as "Teacher Resources") to engage students in a conversation around patterns and misconceptions when working with numbers beyond 100.

Develop: Problem-Based Learning



Math Practices & Problem Solving: Construct Arguments: Solve & Share

Assign Info T Teacher resources

Independent Practice/Math Practices and Problem Solving:

For items 4 and 14, ask students to provide a written explanation of the pattern(s) they used to find the missing numbers. This can be done on a sticky note to provide ample space to write.

Lesson 9-7: Skip Count By 5s, 10s, And 100s To 1,000

2.NBT.A.2

MP.2 MP.4

MP.7 MP.8

Access Prior Learning:

In Topics 3 and 5, second grade students used open number lines to add and subtract.

In Topics 2 and 8, second grade students used skip counting.

In lesson 9-6, second grade students skip counted by 10s and counted by ones from 2-digit and 3-digit numbers.

Securing the Big Idea:

In this lesson, students are securing understanding of skip counting using patterns and number lines. Students will skip count by 5s, 10s and 100s from 2digit and 3-digit numbers.

Skip counting and analyzing the resulting patterns supports students with invented strategies for multiplication in third grade. By identifying these patterns, students make sense of the relationships and properties of numbers (Van de Walle, et al., 2014, P.248). Skip counting also develops students' mental math skills and number sense.

Solve and Share:

During problem solving, some students may connect this problem to their work with nickels and telling time to the nearest 5-minutes in Topic 8. If students do not identify a connection, consider asking a guestion such as, "What connections can you make to our learning around time and money?" Facilitating these connections helps students develop relational understanding and the concept of mathematics as a series of interwoven ideas. Some students may find the use of manipulatives such as nickels, a clock or other concrete objects helpful.

Visual Learning:

Consider extending students by asking, "Using the patterns that you see, what three numbers came before the first number on the open number line? Using the patterns that you see, what three numbers come after the last number on the open number line?" Students can record their thinking on whiteboards.

Assess and Differentiate:

In the Intervention Activity "Counting by s, 10s, and 100s!" (TE, p.551A), encourage students to identify and discuss patterns in their skip counts. Also, include opportunities for students to skip count across decades and centuries (e.g., 380, 390, 400, 410...) as these situations are often more challenging for students

Lesson 9-8: Compare Numbers Using Place Value

2.NBT.A.4

MP.1

MP.2 MP.3

MP.5 MP.8 Access Prior Learning: In first grade (1.NBT.B.3), students compared two 2-digit numbers

using place value and the symbols >, =, <.

In this topic, second grade students have worked with 3-digit numbers.

Developing the Big Idea:

In this lesson, students are developing understanding that place value can be used to compare numbers using the greater than (>), equals (=), and less than (<) symbols.

Students have more experiences with "more" than they do with "less". For this reason, students may find identifying "less than" more challenging. Make a conscious effort to ask, "Which is less?" questions as often as, or more frequently, than you ask, "Which is more?" questions. Also, when students identify which is greater, follow up by asking them, which is less. The physical construction of quantities also helps students develop understanding of *greater than* and *less than* relationships (Van de Walle, et al., 2014, p.105).

Symbols and language in mathematics are considered conventions. It is recommended that students construct understanding of concepts *before* introducing the symbols and terminology



(Van de Walle, et al., 2014, p.21). In regards to the greater than (>) and less than (<) symbols, they are conventions that should be explicitly taught *after* students develop the concept of greater than and less than. Rather than using the alligator drawings, which are a gimmick that can derail the focus away from mathematics to animals, help students remember these symbols by drawing one dot and two dots, and then connect them with lines. This representation reinforces the meaning of the symbols, while

helping students remember that the side with two dots is next to the number with a greater value than the side with one dot.

Solve & Share:

When sharing and discussing student strategies, encourage students to identify the value of digits in each number, 501 and 510. As suggested, place value blocks offer an effective visual representation to support student understanding of the value of each digit. This problem offers an opportunity for students to discuss the use of zero as a placeholder in our number system.

Visual Learning:

Have students build or draw the quantities in the animation on place value mats to support understanding of why comparing digits from greatest to least place value works.

Independent Practice/Math Practices and Problem Solving:

As an extension to item 16, and building on the recommended Topic 9 Game, consider having students create their own base-ten riddles. These riddles can be placed in a center for other students to solve, or used as a formative assessment.

Assess and Differentiate:

In the *Intervention Activity*, "Comparison Cards" (TE, p.557A), consider using <u>Arrow Cards</u> (found under "Instructional Tools" on the WCSD Curriculum & Instruction website) to support student understanding of the value of each digit.

Lesson 9-9: Compare Numbers On The Number Line

2.NBT.A.4

MP.2

MP.3 MP.4

MP.7 MP.8

Access Prior Learning:

In first grade (1.NBT.B.3), students compared two 2-digit numbers using place value and the symbols >, =, <. First grade students also used open number lines.

In the prior lesson, second grade students used place value to compare numbers using the greater than (>), equals (=), and less than (<) symbols. Throughout second grade, students have used open number lines.

Developing the Big Idea:

In this lesson, students are developing understanding of greater than and less than relationships with 3-digit numbers using number lines. Students are also developing understanding that number lines go on forever in both directions, so there is always a number greater than and less than a given number.

Solve and Share:

Child-watch for students who use their understanding of place value patterns to identify a number greater than 256 and a number less than 256. These students are able to reason about the value of each digit, connecting to lesson 9-8, when selecting numbers for their response. Some students will benefit from building or drawing 256, and physically, adding to or removing from, to find a number that is greater than and less than. Encourage these students to reflect on the digits of the numbers they build to facilitate movement toward more efficient strategies based on place value understanding. Consider sequencing the share to begin with a student who physically built the numbers, then progress to a student who reasoned about the value of digits. Comparing these strategies will help students develop more efficient methods and deeper place value understanding.

Assess and Differentiate:

In the *Intervention Activity*, "Sticky Numbers" (TE, p.563A), students may benefit from building, drawing or modeling the numbers using <u>Arrow Cards</u> (found under "Instructional Tools" on the WCSD Curriculum & Instruction website).

Lasson 0-10.	Math Practices And Problem Sol	lving: Look For And Use Structure
2.NBT.A.2 2.NBT.B.8 2.NBT.A.4 MP.1 MP.2 MP.3 MP.7	Access Prior Learning: In first grade, students engaged in the Standards for Mathematical Practice including MP. 7 Look For and Make Use of Structure. Developing the Big Idea: In this lesson, students are developing understanding of Math Practice 7: Look For and Make Use of Structure, by looking for patterns to help them solve problems.	Consider using the <i>Math Practice 7 Animation</i> on Pearson Realize Online for an example of MP.7 behaviors. Refer to the <i>Math Practices and Problem Solving Handbook</i> for ideas on developing, connecting and assessing MP.7 (TE, pp.F29-F29A). MP. 7 Behaviors: • Analyze and describe patterns in numbers. • Analyze and describe common attributes and patterns in shapes and solids. • Analyze expressions, equations, procedures, and objects to represent, describe, and work with them in different ways. Visual Learning: During the animation, encourage students to connect to the patterns they identified in the <i>Solve and Share</i> . The use of concrete manipulatives and drawings on place value mats may support students who have difficulty understanding the patterns used to sort the shirts. <i>The Prevent Misconceptions</i> note (TE p.566) also suggests using hundreds charts to support these students. Assess and Differentiate: In the <i>Intervention Activity</i> , "Pattern or No Pattern?" (TE, p.569A), students may benefit from building, drawing or modeling the numbers using Arrow Cards (found under "Instructional Tools" on the WCSD Curriculum & Instruction website). *CTC: <i>Quick Check</i> (digital platform)

References

- Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.
- Washoe County School District (2017, June 6) Resources and approved supplementation. (2017. Retrieved from http://washoeschools.net/Page/1069

Base-Ten Riddles

Base-ten riddles can be presented orally or in written form. In either case, children should use base-ten materials to help solve the riddles. The examples here illustrate a variety of different levels of difficult. After children solve the following riddles, have them write new ones.

- I have 23 ones and 4 tens. Who am I?
- I have 4 hundreds, 12 tens, and 6 ones. Who am I?
- I have 30 ones and 3 hundreds. Who am I?
- I am 45. I have 25 ones. How many tens do I have?
- I am 341. I have 22 tens. How many hundreds do I have?
- I have 13 tens, 2 hundreds, and 21 ones. Who am I?
- If you put 3 more tens with me, I would be 115. Who am I?
- I have 17 ones. I am between 40 and 50. Who am I? How many tens do I have?

(Van de Walle, 2014, p.187)

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (Vol. 1). Harlow: Pearson Education International.

Ones					
Tens					
Hundreds					

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▶ Grade 2 Topic 10: Add Within 1,000 Using Models and Strategies

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 583A-583E), the Topic Planner (pp. 583I-503K), the Topic Performance Assessments (pp. 633-634) all 7 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 583A-583E) **Topic Essential Question:**

What are strategies for adding numbers to 1,000?

Reference Answering the Topic Essential Question (TE, p. 631-632) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	10-5	10-6	10-7	Assessment	

³ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 10 focuses on using models and strategies to add within 100.

...there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, Karp, Lovin, Bay-Williams, 2014, p.176).

Topic 10

Add Within 1,000
Using Models and
Strategies

Number of
lessons: 7

A/D/E: 3 days

NVACS Focus:
NBT.B

Total Days: ~10

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).

Topics 10-11 compose a major cluster focused on the big idea of the base-10 numeration system through addition and subtraction within 1,000. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.NBT.B.

2.NBT.B Use place value understanding and properties of operations to add and subtract.

- 7. Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- 8. Mentally add 10 or 100 to a give number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
- 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

This work builds upon understandings developed in Topics 3-5 and Topic 9. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above and excerpted here, "it is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers" (National Council of Teachers of Mathematics, 2000, p.83). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding 100 as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that allow students to see 100 as 10 groups of ten or 100 singles (connecting cubes, beads in a jar, linked

paper clips, etc.) are essential. Groupable models allow children to move from a count-by-ones approach, to constructing groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a hundreds flat, is worth 100 singles or 10 tens is ineffective. When considering language, help students connect standard language, "one hundred thirty-five", to base-ten language, "1 hundred 3 tens 5 ones; 1 group of a hundred 3 groups of ten 5 ones, etc". Also, it is recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

Topics 10-11 are mirrored topics, as the strategies used for addition in Topic 10 are later used for subtraction in Topic 11. Focus planning conversations to go beyond *what* strategies are used to *why* those strategies are important for students' development of the big idea. Reference the lesson level instructional notes below for content to support these conversations. In both topics, students will work with algorithms. The authors of enVisionmath2.0 placed the algorithms in sequence with other strategies, with the intent that students connect their understanding of place value strategies to construct meaning of the algorithms. They also intended for students to see algorithms as one of many strategies for addition and subtraction, not the pinnacle of addition and subtraction strategies.

As NVACS 2.NBT.B.7 states, "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." The "Progressions for the Common Core State Standards in Mathematics" elaborate on what it means to "relate to a written method", by including the following examples for addition:

The computation shown proceeds from left to right, but could have gone from right to left. Working from left to right has two advantages: Many students prefer it because they read from left to right; working first with the largest units yields a closer approximation earlier.

Addition: Recording newly composed units in the same row +147 425 Add the enes, Add the enes, 8 + 7, and record 7 + 4 + 1, and these 15 ones record these 12 with 1 on the line in the tens column line in the 2+1+1, and record these 4 hundreds below the hundreds and 5 below in the hundreds column column. and 2 below in the tens place. Digits representing newly composed units are placed below the addends, on the line. This placement has several advantages Each two-digit partial sum (e.g., "15") is written with the digits close to each other, suggesting their origin. In "adding from the top down," usually sums of larger digits are computed first, and the easy-to-add "1" is added to that sum, freeing students from holding an altered digit in memory. The original numbers are not changed by adding numbers to the first addend; three multi-digit numbers (the addends and the total) can be seen clearly. It is

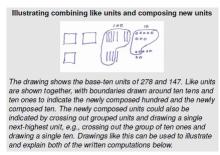
easier to write teen numbers in their usual order (e.g., as 1 then

5) rather than "write the 5 and carry the 1" (write 5, then 1).

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The first written method for addition, Partial Sums, records intermediate steps and is helpful in building toward the second written method for addition, the U.S. Traditional algorithm. The Progression Documents go on to articulate that drawings, such as the one pictured below can be used by students in explaining the written methods above. Knowing that our trajectory is building toward the expectation that students will relate strategies to a written method when adding within 1,000, we can view the lessons in Topic 10 as building onto addition algorithms introduced in Topic 4. However, in regards to transitioning from the first written method (Partial Sums) to the second written method (U.S. Traditional Algorithm), the progression document also states, "Some students might make this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency requires a more compact method."

Based on this, we should offer opportunities for our students to construct meaning of the algorithms, but we should not expect all students to transition to use of the *standard* algorithm in second grade. The progression documents recommend that students' solutions that involve count-on or add-on strategies continue to be discussed. It goes on to state that that the major focus "for addition within 1000 needs to be on methods such as those [pictured above] that are simple for students and lead toward fluency (e.g., recording new units in one row)." (CCSWT, 2015, p.10).



Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Math Practice 8: Look for and express regularity in repeated reasoning

Focus on opportunities for students to develop MP.8 behaviors. This is the focus of the Math Practices and Problem Solving lesson 10-7. Reference the Teacher's Edition (pp. F30-F30A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary (Vocabulary explicitly taught in prior grades	./: s or topics)	
	break apart compensation digit equals, = hundred mental math ones	open number line partial sum place-value chart regroup sum tens thousand	

Additional terminology that students may need support with: algorithm, models, patterns, standard algorithm, unit

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students able to use different addition strategies and explain why they work?"

Lesson	Evidence	Look for
10-4	Do You Understand: Show Me!	Focus CTC around the big idea:
	(student work samples)	student strategies and models
		use of multiple strategies to check work
		explanation of strategy
10-1	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
		(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 631-634
Assessments (summative)	SE pp. 631-634	

Standards listed in **bold** indicate a focus of the lesson.

Standards listed in bo	old indicate a focus of the lesson.	
NVACS		
(Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 10-1:	Add 10 And 100	
2.NBT.B.8 2.NBT.B.9 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: In first grade, (1.NBT.C.5) given a 2-digit number, students found 10 more or 10 less without counting. Developing the Big Idea: In this lesson, students are developing understanding of adding 10 or 100 to 3-digit numbers using place value patterns and mental math.	Students can use basic facts to help them mentally solve problems when adding by 10 and 100. Using place value blocks will reinforce conceptual understanding that the tens digit goes up by 1 when adding ten, and that the hundreds digit goes up by 1 when adding 100. These patterns also build on learning opportunities from Topic 9. Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 583), <i>Review What You Know</i> (TE p. 584), and <i>Vocabulary Review Activity</i> (TE, p. 584) only. Post the essential question and student strategies on your focus wall. Consider engaging students in skip-counting by 10s and 100s using place value blocks, place value charts or hundreds charts. Solve & Share: This <i>Solve and Share</i> builds on students' study of place value patterns when skip counting by 5s, 10s, and 100s in Topic 9. Child-watch for evidence of this understanding in students' mental math strategies and explanations. Visual Learning: Encourage students to generalize their understanding by identifying another equation that demonstrates the pattern in the animation. For example, when the animation shows that adding 10 makes the tens digit go up by 1, ask students to use a whiteboard and marker to write an equation for a different 3-digit number for which this also applies (e.g., 482 + 10 = 492). Students may also be asked to identify a basic fact, which helped them to solve their equation. Additional time may need to be spent on the final frame of Visual Learning, which identifies situations when adding 10 changes the tens and hundreds digits (e.g., 290 + 10 = 300). Independent Practice/Math Practices and Problem Solving: As previously indicated, students do NOT need to do all of the problems in their Student Edition (SE). However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The <i>Independent Practice</i> page offers problems that support procedural skill and fluency. The <i>Math Practices and Problem Solving</i> page offers problems that
		*CTC: Quick Check (digital platform)
Lesson 10-2:	Add On An Open Number Line	
2.NBT.B.7 2.NBT.B.9 MP.2 MP.3 MP.4 MP.5	Access Prior Learning: In Topics 3 and 5, second grade students used the open number line and break apart strategies to model addition and subtraction with 2-digit numbers. Developing the Big Idea: In this lesson, students are developing understanding that when adding 3-digit numbers, the numbers can be broken apart into hundreds, tens and ones and added with jumps on the open number line. This model allows students to keep track of their thinking.	Open number lines help students keep track of their thinking and allow students to add/subtract by groups of hundreds, tens or ones. The use of an open number line supports place value understanding as it involves decomposing and composing numbers. It also supports students' number sense and computational fluency. Solve and Share: Child-watch for students who make jumps of hundreds, tens and ones. Ask all students to solve the problem two different ways and evaluate their strategies for efficiency. If students use inefficient methods to add on the open number line, ask, "How can jumps of hundreds and tens help you solve the problem more efficiently? Does the number you start with affect your efficiency?" Also, child-watch for students who have trouble crossing into a new century, from 598 into the 600s and 700s. These students may need support with connecting their understanding of the repeated structure and patterns in our number system. Visual Learning: Give students time to solve 481 + 122 by drawing an open number line on a whiteboard, before interacting with the animation. Child-watch for evidence of students who increase their level of efficiency from the Solve and Share. Reference the "ways" in the Visual Learning, which progress from less to more efficient. Highlight these reflective students during the discussion. Although the animation presents two ways that both begin with jumps of 100, students may begin with smaller jumps to get to a landmark number when appropriate for the numbers. For example, in Guided Practice, item 2, students may begin with 670, make a jump of 30 to 700, and then jump the remaining 202 in a variety of ways. The use of landmark numbers supports students' work with compensation in lesson 10-3.

Lesson 10-3: Add Using Mental Math 2.NBT.B.7 Access Prior Learning: Solve & Share: In Topic 3, second grade students Ask all students to solve the problem in two ways and to evaluate the efficiency of each used the break apart strategy to strategy. Child-watch for students who use mental math strategies, as that is the focus of the MP.1 add 2-digit numbers. In Topic 9, day's lesson. During problem solving, look for opportunities to ask a student, "Can you solve MP.2 second grade students broke apart this problem another way with mental math?" MP.6 3-digit numbers. In the prior lesson, Visual Learning: second grade students broke apart MP.7 **Prior to interacting with the animation**, have students solve the problem presented in the 3-digit numbers to add with jumps animation using a strategy of their choice. If students have difficulty understanding the on an open number line. strategies presented, refer to the *Prevent Misconceptions* note (TE, p.598) for support suggestions. Securing the Big Idea: In this lesson, students are Independent Practice/Math Practices and Problem Solving: securing understanding of break The numbers in item 6 lend themselves nicely to the compensation strategy. Child-watch for apart as a mental math strategy for students who change 250 + 298 to 250 + 300 = 550, then subtract 2, 550-2 = 548. adding 3-digit numbers. Assess and Differentiate: Developing the Big Idea: The Intervention Activity, "Three-Digit Marathon" (TE, p.601A), provides students with support In this lesson, students are for the break apart strategy, as well as schema that they can draw upon in lesson 10-4 on developing understanding of compensation as a mental math strategy for adding 3-digit numbers. Lesson 10-4: Add Using Partial Sums 2.NBT.B.7 Access Prior Learning: Five A/D/E days have been paced into Topic 10. Consider using two of those days to In Topic 4, second grade students 2.NBT.B.9 differentiate and enrich students as they work with the Partial Sums algorithm. Ensure that all used the partial sums algorithm to students model the partial sums algorithm with concrete place value blocks and place-value mats to ensure conceptual understanding as well as procedural understanding. solve addition problems with 2-digit MP.3 numbers. Solve & Share: MP.5 Child-watch for students who use the break apart strategy to add hundreds and hundreds, tens In the prior lesson, second grade MP.7 and tens, and ones and ones. If students use other strategies, honor those strategies, then ask, students broke apart 3-digit "How can you break apart by place value to solve the problem?" This will support their MP.8 numbers using mental strategies to understanding of Partial Sums, the focus of the lesson, in the Visual Learning. add hundreds and hundreds, tens and tens, ones and ones, and then Visual Learning: Prior to interacting with the animation, have students solve the problem, 518 + 327 using added the partial sums. concrete place value blocks or drawings and a place value mat. Have students work in pairs Developing the Big Idea: during the animation. One student should model with place value blocks, and the other partner should record the step (Van de Walle, et al., 2014, p.219). Trade roles when solving the Guided In this lesson, students are Practice problems. developing understanding of the partial sums algorithm when Independent Practice/Math Practices and Problem Solving: adding two (or more) 3-digit Have students use the Partial Sums algorithm, and a second strategy of choice to check for numbers. accuracy. Connecting strategies will support students in making sense of the Partial Sums algorithm, as a way to relate to a written method. This is stated as an expectation in 2.NBT.B.7, and explained in more depth in the Instructional Note at the beginning of this document. *CTC: Do You Understand: Show Me! (student work samples) Lesson 10-5: Use Models to Add 2.NBT.B.7 Access Prior Learning: As indicated in the Instructional Note at the beginning of this document, the progression In lessons 4-3 and 4-4, second document states the following in regards to the standard algorithm: "Some students might make 2.NBT.B.9 this transition in Grade 2, some in Grade 3, but all need to make it by Grade 4 where fluency grade students used the standard requires a more compact method." Based on this, we should offer opportunities for our addition algorithm to add 2-digit MP.3 students to construct meaning of the algorithms, but we should not expect all students numbers. to transition to use of the standard algorithm in second grade. MP.4 Developing the Big Idea: MP.5 Solve & Share: In this lesson, students are MP.7 Continue to encourage use of place-value blocks and place-value mats. Consider offering a developing understanding of the blank workspace to allow students to strategically select an addition strategy. standard addition algorithm. Students progress from concrete to

-continues on next page-

symbolic work with the algorithm. They also regroup ones and tens.

Visual Learning:

Have students solve the problem in the animation using concrete place value blocks or drawings and a place value mat. Have students work in pairs during the animation. One student should model with place value blocks, and the other partner should record the step (Van de Walle, et al., 2014, p.219). Trade roles when solving the Guided Practice problems.

Independent Practice/Math Practices and Problem Solving:

Encourage students to try either the Partial Sums or standard algorithm, and use a second strategy of choice to check for accuracy. Connecting strategies to the algorithm will support students in sense making. Offering students a blank workspace is also helpful.

Assess and Differentiate:

The Intervention Activity, "Regroup to Add" (TE, p. 613A) may be modified to support students with the Partial Sums algorithm, rather than the standard algorithm.

Lesson 10-6: Explain Addition Strategies

2.NBT.B.9 2.NBT.B.7

MP.2

Access Prior Learning: students used several addition

In Topics 3 and 4, second grade strategies and algorithms.

Solve & Share:

Ask students to solve the problem with two different strategies, and then encourage them to evaluate which was more efficient. Select and sequence the share to include a variety of student strategies that increase in efficiency. As students explain their strategies, encourage them to use precise academic vocabulary, referring to the math focus wall as needed.

MP.3 MP.4 MP.5

Securing the Big Idea:

In this lesson, students are securing understanding of addition strategies with 3-digit numbers. Students will select a strategy and explain why it works using place value and properties of operations. Students may not be secure in every strategy, but should demonstrate security with a variety of strategies.

Independent Practice/Math Practices and Problem Solving:

Item 8 offers students an opportunity to engage in MP.3 behaviors. Some students may benefit from making a visual representation of Tommy's strategy before writing an explanation.

Lesson 10-7: Math Practices and Problem Solving: Repeated Reasoning

2.NBT.B.7 2.NBT.B.9

MP.1

MP.2

MP.3

MP.4

MP.8

Access Prior Learning: In first grade, students engaged in

the Standards for Mathematical Practice including MP. 8 Look For and Express the Regularity in Repeated Reasoning.

MP. 8 Behaviors:

Notices and describes when certain calculations or steps in a procedure are

Consider using the Math Practice 8 Animation on Pearson Realize Online for an example of

MP.8 behaviors. Refer to the Math Practices and Problem Solving Handbook for ideas on

Generalizes from examples or repeated observations

developing, connecting and assessing MP.8 (TE, pp. F30-F30A).

- Recognizes and understands appropriate short cuts
- Evaluates the reasonableness of intermediate results

Developing the Big Idea:

In this lesson, students are developing understanding of Math Practice 8: Look For and Express the Regularity in Repeated Reasoning by thinking about things that repeat in a problem, and using one problem to help them solve others.

Visual Learning:

Prior to interacting with the animation, have students solve the problem, 235 + 489 with a strategy of their choice. During the animation, have students model with place value blocks to determine if regrouping is needed to make a ten or a hundred.

Independent Practice/Math Practices and Problem Solving:

Place a sticky note over the workspace for items 7-8 to allow students to choose their own strategy for determining if a problem requires regrouping.

References

- Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

3-Digit Addition Game

Materials:

- Place value mat (Blackline master included on the next page, one per player)
- Playing cards: Digits 0-9 cards only
- Tools to support strategies (Place value blocks, whiteboards, etc.)

Players: 2+

Object of the game: To collect the most cards

Directions:

- 1. Place the card deck face down on the table.
- 2. Each player draws 6 cards and builds two 3-digit addends.
- 3. Players use a strategy of choice to find the sum. Players explain their strategy and check each other's work for accuracy.
- 4. The player with the largest sum takes the cards. In the event of a tie, players draw one more card to add to their sum.
- 5. Play ends when there are not enough cards for both players to make two 3-digit addends.
- 6. The player with the most cards wins.

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▶ Grade 2 Topic 11: Subtract Within 1,000 Using Models and Strategies

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 8-11)

Prior to instruction, view the Topic 11 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 583A-583E), the Topic Planner (pp.635A-635C), the Topic Performance Assessments (pp. 685-686A) all 7 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 583A-583E) **Topic Essential Question:**

What are strategies for subtracting numbers to 1,000?

Reference Answering the Topic Essential Question (TE, pp. 683-684) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

11-1 | 11-2 | 11-3 | 11-4 | 11-5 | 11-6 | 11-7 | Assessment

3 A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 11 focuses on using models and strategies to subtract within 100.

...there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation (Van de Walle, Karp, Lovin, Bay-Williams, 2014, p.176).

Topic 11
Subtract Within
1,000 Using Models
and Strategies

Number of
lessons: 7 over
10 days

A/D/E: 3 days

NVACS Focus:
NBT.B

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

It is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers. When such problems arise in interesting contexts, students can often invent ways to solve them that incorporate and deepen their understanding of place value, especially when students have the opportunities to discuss and explain their invented strategies and approaches (National Council of Teachers of Mathematics, 2000, p.83).

Topics 10-11 compose a major cluster focused on the big idea of the base-10 numeration system through addition and subtraction within 1,000. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.NBT.B.

2.NBT.B Use place value understanding and properties of operations to add and subtract.

- 7. Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- 8. Mentally add 10 or 100 to a give number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
- 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

This work builds upon understandings developed in Topics 3-5 and Topic 9. The structure of the base-10 numeration system uses digits 0-9, groups of 10, and place value- the value of a digit is determined by its place. As noted in the quote above and excerpted here, "it is not necessary to wait for students to fully develop place-value understandings before giving them opportunities to solve problems with two- and three-digit numbers" (National Council of Teachers of Mathematics, 2000, p.83). In fact, when students invent addition and subtraction strategies that require the composition (put together) and decomposition (take apart) of numbers, they are developing place-value understanding while simultaneously developing computational understanding.

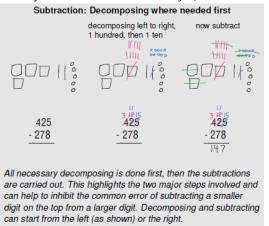
In kindergarten and first grade, students work with patterns in numbers to 100, and begin to understand a group of ten objects as a unit. That is, they understand ten as both ten ones and one ten. In second grade, students extend these place value understandings to three-digit numbers, understanding 100 as a bundle of ten tens and as a "hundred". To foster this development, the use of groupable models, models that allow students to see 100 as 10 groups of ten or 100 singles (connecting cubes, beads in a jar, linked paper clips, etc.) are essential. Groupable models allow children to move from a count-by-ones approach, to constructing

groups/units, thereby imposing their mathematical understandings onto the model. Students' own construction of this knowledge is important and effective. On the contrary, telling students that a pre-grouped model, such as a hundreds flat, is worth 100 singles or 10 tens is ineffective. When considering language, help students connect standard language, "one hundred thirty-five", to base-ten language, "1 hundred 3 tens 5 ones; 1 group of a hundred 3 groups of ten 5 ones, etc". Also, it is recommended that for EL learners, you choose a single variation of base-ten language to use consistently. This will aid students in connecting the base-ten language to standard language (Van de Walle, et al., 2014, p. 178).

Topics 10-11 are mirrored topics, as the strategies used for addition in Topic 10 are later used for subtraction in Topic 11. Focus planning conversations to go beyond *what* strategies are used to *why* those strategies are important for students' development of the big idea. Reference the <u>progression document</u> linked at the top of this document, and lesson level instructional notes for content to support these conversations. In both topics, students will work with algorithms. The authors of **enVision**math**2.0** placed the algorithms in sequence with other strategies, with the intent that students connect their understanding of place value strategies to construct meaning of the algorithms. They also intended for **students to see algorithms as one of many strategies for addition and subtraction, not the pinnacle of addition and subtraction strategies**.

As NVACS 2.NBT.B.7 states, "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." Knowing that our trajectory is building toward the expectation that students will relate strategies to a written method when subtracting within 1,000, we can view the lessons in Topic 11 as building on to subtraction strategies in Topic 5.

Just as with addition within 1,000 in Topic 10, we should offer opportunities for students to construct meaning of the algorithms, but we should not expect all students to transition to use of the U.S. Traditional standard algorithm in second grade. The progression documents recommend that students' solutions that involve subtraction as an unknown-addend problem through count-on or add-on strategies continue to be discussed. It goes on to state that the major focus for subtraction within 1000 "needs to be on methods that lead toward fluency or are sufficient for fluency" (Reference the example below). (CCSWT, 2015, p.10).



Common Core Standards Writing Team. (2015, March 6). Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Math Practice 1: MP.1 Make sense of problems and persevere in solving them

Focus on opportunities for students to develop MP.1 behaviors. This is the focus of the Math Practices and Problem Solving lesson 11-7. Reference the Teacher's Edition (pp. F23-F23A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

	emic Vocabulary stently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabula (Vocabulary explicitly taught in prior grad	
	break apart compensation digit equals, = hundred mental math ones	open number line partial sum place-value chart regroup sum tens thousand

Additional terminology that students may need support with: algorithm, models, patterns, standard algorithm, unit

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students developing conceptual understanding of subtraction through the use of models such as base 10 blocks, number lines, etc.?"

Lesson	Evidence	Look for
11-6	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		understand relationship between different strategies
		explanation of why strategy works
11-7	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	Items 1, 2, and 3	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 683-686
Assessments (summative)	SE pp. 683-686	

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
	Subtract 10 And 100	
2.NBT.B.8 2.NBT.B.9 MP.1	Access Prior Learning: In first grade, (1.NBT.C.5) given a 2-digit number, students found 10 more or 10 less without counting.	Students can use basic facts to help them mentally solve problems when subtracting by 10 and 100. Using place value blocks will reinforce conceptual understanding that the tens digit goes down by 1 when subtracting ten, and that the hundreds digit goes down by 1 when subtracting 100.
MP.2 MP.4 MP.7	In lesson 10-1, second grade students add 10 and 100 to 3-digit numbers using place value patterns and mental math.	Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 635), Review What You Know (TE, p. 636), and Vocabulary Review Activity (TE, p. 636) for the word decrease only. Post the essential question and student strategies on your math focus wall.
	Developing the Big Idea: In this lesson, students are developing understanding of subtracting 10 or 100 from 3-digit	Solve & Share: Some students may need support with the term "harvested" in order to make sense of the problem. Child-watch for students who are able to use place value patterns to answer the questions efficiently. The use of place value blocks and place value mats can provide support for students who have difficulty solving the problem by reasoning about place value patterns.
	numbers using place value patterns and mental math.	Visual Learning: Encourage students to generalize their understanding by identifying another equation that demonstrates the pattern in the animation. For example, when the animation shows that subtracting 10 makes the tens digit go down by 1, ask students to use a whiteboard and marker to write another equation for which this also applies (e.g., 534 – 10 = 524). Students may also
		-continues on next page-

be asked to identify a basic fact, which helped them to solve their equation. Additional time may need to be spent on the final frame of Visual Learning, which identifies situations when subtracting 10 changes the tens and hundreds digits (e.g., 500 - 10 = 490).

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Lesson 11-2: Count Back To Subtract On An Open Number Line

2.NBT.B.7 2.NBT.B.9

MP.4 MP.5 MP.7

Access Prior Learning:

In Topics 3 and 5, second grade students used the open number line to model addition and subtraction with 2-digit numbers. In lessons 5-2 and 5-3, second grade students counted back on the open number line to subtract 2-digit numbers.

Developing the Big Idea:

In this lesson, students are developing understanding that when subtracting 3-digit numbers, the numbers can be broken apart into hundreds, tens and ones and subtracted using jumps on the open number line. Students will count back to subtract.

Open number lines help students keep track of their thinking and allow students to add/subtract by groups of hundreds, tens or ones. The use of an open number line supports place value understanding as it involves decomposing and composing numbers. It also supports students' number sense and computational fluency.

Solve and Share:

Child-watch for students who make jumps of hundreds, tens and ones. Ask all students to solve the problem two different ways and evaluate their strategies for efficiency. If students use inefficient methods to subtract on the open number line, ask, "How can jumps of hundreds and tens help you solve the problem more efficiently?" Also, child-watch for students who have trouble crossing into a new century, from 306 into the 200s. These students may need support with connecting their understanding of the repeated structure and patterns in our number system. Note: Students may make jumps in any order that makes sense with the numbers, thus they may jump down by ones then hundreds then tens.

Visual Learning:

Prior to interacting with the animation, give students time to solve the problem 580-232 by drawing an open number line on a whiteboard. Child-watch for evidence of students who increase their level of efficiency from the *Solve and Share*. Reference the "ways" in the *Visual Learning*, which progress from less efficient to more efficient. Highlight these reflective students during the discussion.

Assess and Differentiate:

Encourage students to use place value blocks to model the jumps on the open number line to strengthen conceptual understanding.

Lesson 11-3: Add Up To Subtract On An Open Number Line

2.NBT.B.7 2.NBT.B.9

MP.4 MP.6 MP.7 MP.8

MP.2

Access Prior Learning:

In lesson 5-4, second grade students used the open number line to model adding up by tens and ones to subtract 2-digit numbers.

Developing the Big Idea:

In this lesson, students are *developing* understanding of adding up by hundreds, tens and ones to subtract 3-digit numbers.

Think-addition strategies such as add up to subtract are powerful ways to solve subtraction problems. This strategy also supports students' use of place value with hundreds and tens. (Van de Walle, et al., 2014, p.215). Certain number combinations lend themselves to think-addition strategies. When the minuend and subtrahend are closer in value (20-17), it is more efficient to add up to subtract (18, 19, 20). Alternatively, when the minuend and subtrahend are further apart in value (20-3), it is more efficient to count back to subtract (19, 18, 17). Look for opportunities to engage students in conversations around the strategic selection of strategies based upon the numbers.

Solve & Share:

This is a *Put Together Addend Unknown* problem. Ask all students to solve the problem in two ways on the open number line, and to evaluate the efficiency of each strategy. Child-watch for students who combine like place values to make bigger jumps (e.g., one jump of 20 instead of two jumps of 10). Also, child-watch for students who count back and for students who add up to subtract. If no students add up to subtract, plant the idea with a student by asking, "How can you add up to subtract on the open number line?" or "Can you start with the other number?" Share this student's strategy as a lead in to the *Visual Learning*.

Visual Learning:

Have students solve the problem, 482 - 247 using an open number line *before* showing and discussing the animation. By revisiting the use of addition to check subtraction in the animation, students' understanding of the inverse relationship between addition and subtraction is reinforced.

Lesson 11-4:	Subtract Using Mental Math	
		Possible 2-day lesson
2.NBT.B.7	Access Prior Learning:	Day 1: Solve & Share:
2.NBT.B.9	In lesson 10-3, second grade students added 3-digit numbers using mental math strategies. In lesson 11-2, second grade students counted back on the open	To encourage the use of mental math strategies, have students think about the problem before picking up their pencil. Encourage them to use their paper to show the process they used in their minds to solve the problem. Child-watch for students who use place value understanding to simplify the problem from 335 – 302 to 35 – 2, or for students who think-addition (302 + = 335).
MP.1	number line to subtract 2-digit	
MP.2	numbers.	Visual Learning: Prior to interacting with the animation, have students solve the problem, 372 – 123 using a strategy of choice. Child-watch for students who try a new or more efficient strategy than the one they used in the Solve and Share.
MP.3 MP.7	Developing the Big Idea:	Independent Practice/Math Practices and Problem Solving: Students solve the <i>Quick Check</i> items marked with pink checkmarks.
	In this lesson, students are developing understanding of mental math strategies for subtracting 3-digit numbers including, count back and friendlier numbers.	Day 2: Facilitate a lesson using a string of numbers intentionally structured to promote student use of mental math strategies. These strings are intended to develop students' use of mental math, but do not require students to only solve the problems <i>in</i> their heads. Instead, focus on their ability to examine the numbers and select an appropriate and efficient way to solve the problem. As students verbally explain their thinking, make a written record so that students can "see" the strategy using an open number line. This becomes a picture for the class to discuss. Relying only on verbal explanations will limit access for children to understand (Fosnot, 2007, p.7). Look for evidence of students' use of a variety of strategies and their ability to examine the numbers before selecting a strategy. For example, child-watch for students who identify think-addition, or add up to subtract as an appropriate strategy for the first two problems, but opt to count back for the third and fourth problems. 2006 - 1999 1999 - 1987 1992 - 8 52 - 6 54 - 29 63 - 38 172 - 45 174 - 89 Consider asking students to select one problem from the string and write an explanation of why their strategy of choice was appropriate for that problem. Independent Practice/Math Practices and Problem Solving Select additional items for students to solve. Assess and Differentiate: In the Intervention Activity, "Subtracting a Step at a Time", encourage students to use place
		value blocks to model.
	Use Models To Subtract	
2.NBT.B.7 2.NBT.B.9 MP.1 MP.4 MP.5 MP.8	Access Prior Learning: In this topic, second grade students used place value understanding to subtract 3-digit numbers. Developing the Big Idea: In this lesson, students are developing understanding of the	Possible 2-day lesson As indicated in the Instructional Note at the beginning of this document, we should offer opportunities for students to construct meaning of algorithms, but we should not expect all students to transition to use of the U.S. Traditional standard algorithm in second grade. The Progression Documents recommend that students' solutions that involve subtraction as an unknown-addend problem through count-on or add-on strategies continue to be discussed. It goes on to state that that the major focus for subtraction within 1000 "needs to be on methods that lead toward fluency or are sufficient for fluency" (Reference the example below). (CCSWT, 2015, p.10).
	standard subtraction algorithm. Students progress from concrete to symbolic work with the algorithm. They also regroup ones, tens and hundreds.	Day 1: Solve & Share: Continue to encourage use of place-value blocks and place-value mats. Also, consider offering a blank workspace to allow students to strategically select a subtraction strategy.
		-continues on next page-

Visual Learning:

Have students solve the problem in the animation using concrete place value blocks or drawings and a place value mat. Have students work in pairs during the animation. One student should model with place value blocks, and the other partner should record the step (Van de Walle, et al., 2014, p.219). Trade roles when solving the *Guided Practice* problems.

Independent Practice/Math Practices and Problem Solving:

Have students solve the *Quick Check* items marked with pink checkmarks. Encourage students to try the standard algorithm, AND use a second strategy of choice to check for accuracy. Connecting strategies to the algorithm will support students in sense making. Offering students a blank workspace is also helpful.

Assess and Differentiate:

Support and extend students through the *Intervention Activity* and *On-Level* and *Advanced Activity Centers*.

Day 2:

Solve & Share:

Select a problem, such as item 6 (438 – 162 = ___) from *Independent Practice* and structure it as a *Solve & Share*. Placing these numbers in a word problem will provide context and support student understanding. Encourage students to model with place value blocks and try the standard algorithm. Continue to allow and encourage the use of other strategies as well.

Independent Practice/Math Practices and Problem Solving:

Select additional items for students to solve. Encourage students to try the standard algorithm, AND use a second strategy of choice to check for accuracy. Connecting strategies to the algorithm will support students in sense making. Offering students a blank workspace is also helpful.

Assess and Differentiate:

Continue to support and extend students through the *Intervention Activity* and *On-Level* and *Advanced Activity Centers*.

Lesson 11-6: Explain Subtraction Strategies

2.NBT.B.9 2.NBT.B.7

MP.2

MP.3 MP.4

Access Prior Learning:

In Topic 5, second grade students used several subtraction strategies to subtract 2-digit numbers. Throughout this topic, second grade students have used subtraction strategies and algorithms to subtract 3-digit numbers.

Securing the Big Idea:

In this lesson, students are securing understanding of subtraction strategies with 3-digit numbers. Students will select a strategy and explain why it works using place value and properties of operations.

Possible 2-day lesson

Day 1:

Solve & Share:

Ask students to solve the problem with two different strategies, and then encourage them to evaluate which was more efficient. As students explain their strategies, encourage them to use precise academic vocabulary, referring to the math focus wall as needed. Select and sequence the share to include a variety of student strategies that increase in efficiency.

Visual Learning:

Have students solve the problem, 437 - 245 =____, before viewing and discussing the animation. Child-watch for students who choose a more efficient strategy than the one they used in the *Solve and Share*.

Independent Practice/Math Practices and Problem Solving:

Have students solve the *Quick Check* items marked with pink checkmarks.

Day 2:

Solve & Share:

Select a problem, such as item 3, (464 – 155= ____) from *Independent Practice* and structure it as a *Solve & Share*, placing emphasis on students' explanation of *why* their strategy works. Placing these numbers in a word problem will provide context and support student understanding. See the note above from Day 1 regarding the Solve & Share. Child-watch for student growth in their strategic selection of efficient strategies, as well as their flexibility with multiple strategies.

Independent Practice/Math Practices and Problem Solving:

Select additional items for students to solve.

Assess and Differentiate:

Use the Intervention Activity and Topic 11 game to support and extend students. Ensure that all students have the opportunity to play the game.

*CTC: Solve & Share (student work samples)

E033011 11 7. WIG	atti i lactices / tila i lobicili son	ving: Make Sense And Persevere
2.NBT.B.7 2.NBT.B.9 In the property of the pro	Access Prior Learning: In first grade, students engaged in he Standards for Mathematical Practice including MP. 1 Make Sense of Problems and Persevere in Solving Them. Developing the Big Idea: In this lesson, students are developing understanding of Math Practice 1: Make Sense of Problems and Persevere in Solving Them by making sense of the problem, making a plan, and continuing to try when they get stuck.	Consider using the Math Practice 1 Animation on Pearson Realize Online for an example of MP.1 behaviors. Refer to the Math Practices and Problem Solving Handbook for ideas on developing, connecting and assessing MP.1 (TE, pp.F23-F23A). MP. 1 Behaviors:

References

Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Fosnot, C. T. (2007). Ages and timelines: subtraction on the open number line. Portsmouth, NH: Firsthand/Heinemann.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

3-Digit Subtraction Game

Materials:

- Place value mat (Blackline master included on the next page, one per player)
- Playing cards: Digits 0-9 cards only
- Tools to support strategies (Place value blocks, whiteboards, etc.)

Players: 2+

Object of the game: To collect the most cards

Directions:

- 1. Place the card deck face down on the table.
- 2. Each player draws 6 cards and builds two 3-digit numbers: a minuend (larger number) and a subtrahend (smaller number).
- 3. Players use a strategy of choice to find the difference by subtracting the smaller number from the larger number. Players explain their strategy and check each other's work for accuracy.
- 4. The player with the largest difference takes the cards. In the event of a tie, players draw one more card to subtract from their difference.
- 5. Play ends when there are not enough cards for both players to make two 3-digit numbers.
- 6. The player with the most cards wins.

▶ Grade 2 Topic 12: Measuring Length

Big Conceptual Idea: K-5 Progression on Measurement and Data (Measurement Part) (pp. 12-15)

Prior to instruction, view the Topic 12 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 687A-687E), the Topic Planner pp.687I-687K), the Topic Performance Assessments (pp. 757-758A), and all 9 lessons

Mathematical Background:	Topic Essential Question:
Read Cluster Overview (TE, pp.	What are ways to measure length?
687A-687E)	Reference Answering the Topic Essential Question (TE, pp. 753-754) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

12-1	12-2	12-3	12-4	12-5	12-6	12-7	12-8	12-9	Assessment

⁵ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 12 focuses on measurement. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.MD.A.

2.MD.A Measure and estimate lengths in standard units.

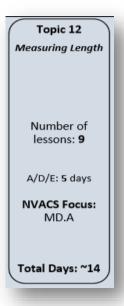
- 1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- 3. Estimate lengths using units of inches, feet, centimeters, and meters.
- 4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Students construct understanding of length as a measurable attribute along an object from end-to-end, expressed in a number of same-sized units. In second grade, students will measure length with both customary (inches, feet, yards) and metric units (centimeters, meters).

Only after children understand and can use single units of measurement should they move to working with common measuring tools. On the 2003 NAEP exam (Blume, Galindo, & Walcott, 2007), only 20 percent of fourth graders could give the correct measure of an object not aligned with the end of a ruler... Even at the middle school level, only 56 percent of eighth graders answered the same situation correctly (Kloosterman, Rutledge, & Kenney, 2009). Students on the same exam also experienced difficulty when the increments on a measuring tool were not one unit. These results point to the difference between using a measuring tool and understanding how it works (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 280).

In first grade, students constructed measurement concepts of length as a number of same-size units that span the object being measured with no gaps or overlaps. They iterated (lay end to end) non-standard units of measurement, ordered up to three objects by length and compared the lengths of two objects using a third object as a reference. In second grade, students build an understanding of the need for standard units (e.g., inches and centimeters) of measurement. They use tools of measurement (e.g., rulers, yardsticks, meter sticks, and measuring tapes) to reinforce their understanding of the iteration of units (NVACS, 2010, 2.MD.A.1). In addition, students connect the size of the unit to the amount of iterations needed to measure a given length by measuring objects twice (NVACS, 2010, 2.MD.A.2). For example, a smaller unit such as centimeters requires more iterations than a larger unit, such as inches, to measure the same length. Thus, there is an inverse relationship between unit size and number of units needed for a given length. This understanding helps students choose appropriate measurement tools given the context and item to be measured (NVACS, 2010, 2.MD.A.1). Finally, students estimate lengths using inches, feet, centimeters, and meters and measure to compare the length difference of two objects (NVACS, 2010, 2.MD.A.3 and 2.MD.A.4).

As you child-watch, look for evidence of student understanding of *how* a measuring tool works. It is helpful to have an awareness of these understandings and common misconceptions, listed below. (Van de Walle, et al., 2014, p.280).



2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

- Leaving gaps between units
- Overlapping units
- Using units that are not of equal size
- When using a ruler, beginning at "1" rather than "0"
- Measuring from the wrong end of the ruler
- Counting the marks on a ruler rather than the spaces in between
- Comparing lengths of two objects at one end only

One recommendation for supporting students in constructing understanding of measurement of length as the spaces between, rather than the number of marks on a ruler, is to have students construct their own ruler (Van de Walle, et al., 2014, p.285). Students can use physical objects to mark off length-units on a strip of paper. Doing so helps students connect measurement as the iteration of a length-unit, such as one-inch or one-centimeter cubes to measurement with a tool such as a ruler (CCSWT, 2012, p.13). In addition, students will benefit from discussions around what they

Make Your Own Ruler Strips of two colors of construction paper glued onto cardstock Discuss how the strips can be used to measure: laying them end to end Use discrepancies to discuss: units must have no gaps or overlaps units must be equal length units must be placed along path being measured Compare to a standard ruler: The numbers are at the end of the units; Notice where 0 is.

(A., V. D., Lovin, L. H., Karp, K. S., & Bay-Williams, J. M. (2014). Teaching student-centered mathematics. Boston: Pearson.)

are counting. By focusing on the length-unit, students will develop understanding that in measurement, the unit is critical. For example, linking together varying size paper clips to measure is not an accurate form of measurement. Five small paper clips and two large paper clips cannot be used to articulate the length of an object as "seven paper clips long". In summation, instruction with measurement should provide students with opportunities to work with manipulative length-units (e.g., 1-inch tiles, 1-centimeter cubes), connect to tools such as rulers, and participate in discussion around their experiences.

Math Practice 6: Attend to precision

Focus on opportunities for students to develop MP.6 behaviors. This is the focus of the Math Practices and Problem Solving lesson 12-9. Reference the Teacher's Edition (TE, pp. F28-F28A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.				
New Academic Vocabulary: (First time explicitly taught)		Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
estimate inch, in. foot, ft. yard, yd. height	nearest inch centimeter, cm nearest centimeter meter, m			

Additional terminology that students may need support with: ruler, measuring tape, meter stick, yardstick

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students developing conceptual understanding of the inverse relationship between the size of the unit and the number of units that are needed to equal the length of an object?"

Lesson	Evidence	Look for
12-4	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		accurate measurements of items
		smaller units = more iterations
12-7	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	items 3, 4 and 5	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 753-758
Assessments (summative)	SE pp. 753-758	

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations				
Lesson 12-1: Estimating Length						
2.MD.A.3 MP.2 MP.5 MP.6	Access Prior Learning: In first grade, (1.MD.A.1) students indirectly compared the lengths of two objects by using a third object. Developing the Big Idea: In this lesson, students are developing understanding of how to estimate the length of an object using the length of a known object. Students will use objects as models, but will express final measurements as the nearest inch, foot, or yard.	Estimation in measurement is often needed in real-world applications. To build students' estimation competencies, the Progression Documents indicate that "research suggests explicit teaching of estimation strategies (such as iteration of a mental image of the unit or comparison with a known measurement) and prompting students to learn reference or benchmark lengths (e.g., an inch-long piece of gum, a 6-inch dollar bill), order points along a continuum, and build up mental rulers." (CCSWT, 2012, p.15). Topic Opener: Consider using one A/D/E (Assessment/Differentiation/Enrichment) day to begin this topic with a "Make Your Own Ruler" experience as suggested in the Instructional Note at the beginning of this document. Also, consider limiting the Topic Opener to discussion of the Topic Essential Question (TE, p. 687), Review What You Know (TE, p. 688), Vocabulary Review Activity (TE, p. 688), and Topic 12 Vocabulary Words Activity (TE, p. 688), Vocabulary Review Activity (TE, p. 688), and Topic 12 Vocabulary Words Activity (TE, p. 688) for the words estimate, inch, fool and yard only. Introduce remaining vocabulary words as they appear in instruction. Post the essential question and student strategies on your math focus wall. Solve & Share: During problem solving, child-watch for students who demonstrate understanding of length as measuring from end-to-end. These students should align both ends of their thumb or elbow/ fingers to both ends of the object they are measuring to accurately find objects that are about 1 inch or 1 fool long. If students are only attending to one end when they align their thumb or end, ask them to compare the object they are measuring to accurately find objects that are about 1 inch or 1 fool long. If students are only attending to one end when they align their thumb or end, ask them to compare the object hey estimate to be 1 inch or 1 fool long to those of a peer. Ask, "Does it make sense that this object is about the same length as?" During the share, draw upon students' la				

Lesson 12-2: Measure With Inches 2.MD.A.3 Access Prior Learning: Focus child-watching in this lesson on students' ability to use a ruler correctly, and to measure In the prior lesson, second grade an object from end-to-end. Students will benefit from first measuring by iterating physical 2.MD.A.1 objects such as 1-inch tiles or with the ruler they constructed in the Topic Opener before using a students used a known object to ruler. Teaching Tool 43 offers printable 1-inch squares. estimate the length of another MP.1 object. Solve and Share: MP.3 Developing the Big Idea: Ensure that all children have access to physical 1-inch objects to support the concept of length-MP.5 unit iteration (not leaving spaces between units). This concept is illustrated by contrasting In this lesson, students are MP.6 Melissa's Work with Henry's Work in the Analyze Student Work samples (TE p.699). Childdeveloping understanding of the watch for evidence of student understanding that measurement requires no gaps or overlaps. measurement of length and height Continue to watch for students who measure the object from end-to-end. During the share, be to the nearest inch, using a ruler. sure to highlight misconceptions that arise so they may be explored and clarified by students. Examples may include gaps, overlaps, only aligning to one end of the line, or using units that are not the same size. Visual Learning: Consider providing students with rulers that only contain inches for this lesson (Reference Teaching Tool 42 for a printable version). Give students time to explore rulers and discuss what they notice before engaging in the Visual Learning animation. As suggested in the Coherence note (TE p.700), point out that there are no gaps or overlaps on a ruler. During the animation, students may need support with understanding the halfway mark. Ask, "Where is the halfway mark between 1 inch and 2 inches? How do you know? Why is it called the halfway mark? Does During the Do You Understand? Show Me! (TE, p.700), consider keeping track of childwatching on a checklist of important measurement ideas, similar to the list of misconceptions in the Instructional Note at the beginning of this document. Independent Practice/Math Practices and Problem Solving: Encourage students to work in pairs. For item 3, have them explain how they estimated before they measure to support the understandings developed in lesson 12-1. Lesson 12-3: Inches, Feet, And Yards 2.MD.A.1 **Access Prior Learning:** Draw upon students' understanding of equivalence in other domains, such as place value when In this topic, students have working with measurement equivalencies such as 12 inches in 1 foot. 2.MD.A.3 estimated and measured length in Solve & Share: inches, feet and yards. MP.2 In addition to what is asked, also require students to explain *how* they estimated and *why* they MP.5 Developing the Big Idea: think the object identified is about 1 inch, 1 foot or 1 yard, respectively. Child-watch for students In this lesson, students are who use a known object to estimate the lengths of unknown objects. MP.6 developing understanding of MP.8 measurement to the nearest inch. Visual Learning: Give students time to explore measuring tools including yardsticks and measuring tapes prior to foot and yard. the Visual Learning animation. Engage students in a discussion of what they notice about each tool and encourage them to compare and contrast these tools to a ruler. Also, ask students to identify scenarios when one tool would be more appropriate than another. Consider capturing this information and student ideas on an anchor chart to add to the math focus wall. Continue to add to this chart through the remainder of the topic. Lesson 12-4: Measure Length Using Different Customary Units 2.MD.A.2 Access Prior Learning: In this lesson, students connect the size of the unit to the amount of iterations needed to measure a given length by measuring objects twice (2.MD.A.2). For example, a smaller unit 2.MD.A.1 In first grade (1.MD.A.1), students such as inches requires more iterations than a larger unit, such as feet, to measure the same understood measurement of length length. Thus, there is an inverse relationship between unit size and number of units needed for as the number of same-size length MP.2 a given length. This understanding helps students choose appropriate measurement tools units that span with no gaps or (2.MD.A.1) given the context and item to be measured. MP.3 overlaps. MP.5 Solve & Share: In this topic, second grade During problem solving, continue to child-watch for understanding of measurement concepts. MP.6 students have estimated and Again, consider keeping record of these understandings through the use of a checklist. If your MP 8 measured with customary units students are showing misconceptions with measurement or the use of measurement tools, consider having two students measure the same object and discuss the discrepancy in their including inches, feet and yards. results. Also, during the share, have students model how they measured their object of choice to clarify existing misconceptions, doing so without losing focus on the essential understanding of the inverse relationship of unit size to measurement. -continues on next page-

* CTC: Solve & Share (student work samples)

Developing the Big Idea:

In this lesson, students are developing understanding of the inverse relationship between the size of the length-unit and the number of units needed to measure a given length. They construct this understanding by measuring objects twice, with two different units

Lesson 12-5: Measure With Centimeters

2.MD.A.3 2.MD.A.1

Access Prior Learning: In this topic, students have estimated and measured with customary units including inches, feet and yards.

As indicated in the NVACS (2.MD.A.1 and 2.MD.A.3) second grade students are expected to estimate and measure lengths using both customary and metric units. Consider having students repeat the "Make Your Own Ruler" activity by iterating physical 1-centimeter units on a strip of paper before the lesson.

MP.2 MP.3 MP.5

MP.6

MP.7

Developing the Big Idea: In this lesson, students are *developing* understanding of estimating and measuring length and height using centimeters.

Solve & Share:

Ensure that all children have access to physical 1-centimeter objects, such as base-ten unit cubes, to support the concept of length-unit iteration (not leaving spaces between units). Continue to child-watch for evidence that student understanding of measurement is generalized from their work with customary units to their work with metric units.

Visual Learning:

Provide students with rulers that show only centimeters (Reference Teaching Tool 44 for a printable version) or rulers that show both inches and centimeters. Give students time to explore the rulers and discuss what they notice before engaging in the *Visual Learning* animation. Encourage students to connect their experience and understanding with rulers and inches to centimeters. Record new thinking to the anchor chart started in lesson 12-3.

Independent Practice/Math Practices and Problem Solving:

Encourage students to work in pairs. For item 5, have them explain how they estimated before they measure to support the understandings developed in lesson 12-1.

Lesson 12-6: Centimeters And Meters

2.MD.A.1 2.MD.A.3

MP.2

MP.8

Access Prior Learning: In lesson 12-5, students estimated and measured length and height with centimeters.

Solve & Share:

In addition to what is asked, also require students to explain *how* they estimated and *why* they think the object identified is about 3 centimeters or 1 meter long, respectively. Child-watch for students who use a known object to estimate the lengths of unknown objects. Reference the instructional note in Lesson 12-1 regarding estimation in measurement.

MP.3 MP.5 **MP.6**

Developing the Big Idea: In this lesson, students are developing understanding of measuring length and height with centimeters and meters.

Visual Learning:

Give students time to explore measuring tools including meter sticks and measuring tapes prior to the *Visual Learning* animation. Engage them in a discussion of what they notice about each tool and encourage them to compare and contrast these tools to a ruler. Also, ask students to identify scenarios when one tool would be more appropriate than another would.

Lesson 12-7: Measure Length Using Different Metric Units

2.MD.A.2 2.MD.A.1	
MP.1	
MP.2	

MP.3

MP.5

MP.6

Access Prior Learning: In this topic, students have estimated and measured with metric units including centimeters and meters. In this lesson, students connect the size of the unit to the amount of iterations needed to measure a given length by measuring objects twice (2.MD.A.2). For example, a smaller unit such as centimeters requires more iterations than a larger unit, such as meters, to measure the same length. Thus, there is an inverse relationship between unit size and number of units needed for a given length. This understanding helps students choose appropriate measurement tools (2.MD.A.1, MP.5) given the context and item to be measured.

Securing the Big Idea:

In this lesson, students are securing understanding of the inverse relationship between the size of the length-unit and the number of units needed to measure a given length. They construct this understanding by measuring objects twice, with two different units.

Solve & Share:

In addition to what is asked, have students write an explanation of why they think one unit needs more units to measure the pencil, inches or centimeters, supporting MP.3 behaviors.

*CTC: Quick Check (digital platform)

Lesson 12-8: Compare Lengths				
2.MD.A.4 2.MD.B.5 MP.2 MP.3 MP.4 MP.5 MP.6	Access Prior Learning: In this topic, second grade students have estimated and measured length and height in both customary and metric units. Developing the Big Idea: In this lesson, students are developing understanding that the difference in the lengths of two objects can be found using subtraction.	Students' experiences with (a) measuring two parts of a path and adding them together, and (b) using subtraction to compare lengths will lay a foundation for addition and subtraction with lengths in Topic 13. Solve & Share: During child-watching, look and listen for evidence of estimation strategies such as those identified in the lesson 12-1 instructional note. Independent Practice/Math Practices and Problem Solving: Items 8-11 and 14 offer students an opportunity to apply fluency with basic addition and subtraction facts. Child-watch for students' flexible use of strategies such as making ten, think addition and doubles.		
Lesson 12-9:	Math Practices And Problem Sol	ving: Precision		
2.MD.A.1 2.MD.A.3 MP.1 MP.2 MP.3 MP.5 MP.6	Access Prior Learning: In first grade, students engaged in the Standards for Mathematical Practice including MP. 6 Attend to Precision Developing the Big Idea: In this lesson, students are developing understanding of Math Practice 6: Attend to Precision by selecting tools, units and methods to measure precisely.	Consider using the Math Practice 6 Animation on Pearson Realize Online for an example of MP.6 behaviors. Refer to the Math Practices and Problem Solving Handbook for ideas on developing, connecting and assessing MP.6 (TE, p.F28-F28A). MP. 6 Behaviors: Computes accurately Uses symbols appropriately Accurately uses problem-solving strategies Specifies and uses units of measure appropriately Decides whether an exact answer or estimate is needed Calculates efficiently, accurately, and fluently		

References

Common Core Standards Writing Team. (2012, June 23). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Geometric Measurement.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

Topic 14

Graphs and Data

Number of lessons: 6

A/D/E: 4 days

NVACS Focus:

MD.D

▶ Grade 2 Topic 14: Graphs and Data

Big Conceptual Idea: K-5 Progression on Measurement and Data (Data Part) (pp. 6-7, 9-10)

Prior to instruction, view the Topic 14 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background pages (pp. 799A-799E), the Topic Planner (pp.799I-799J), the Topic Performance Assessments (pp. 849-850A), and all 6 lessons.

Mathematical Background:	Topic Essential Question:
Read Cluster Overview (TE, pp.	How can line plots, bar graphs, and picture graphs be
799A-799E)	used to show data and answer questions?
	Deference Angularing the Tonic Federatics Outside /TF nn 045
	Reference Answering the Topic Essential Question (TE, pp. 845-
	846) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

14-1	14-2	14-3	14-4	14-5	14-6	Assessment

⁴ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 14 focuses on collecting and analyzing data that can be used to answer questions. Where do you see graphs and hear about data in your daily world? Maybe you've heard statistics about the weather and predicted water levels on the morning news, or perhaps the latest pharmaceutical commercial cited statistics from a recent study. Have you ever engaged in a conversation about home prices with your

2nd Grade Curriculum
Pacing Framework:
Balanced Calendar

Total Days: ~10

neighbors? Statistical literacy helps us make sense of the world around us, engage as citizens and question information that bombards us constantly. Development of numerical literacy begins as children. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.MD.D.

2.MD.D Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

As identified in the NVACS, second grade students represent and interpret both categorical (2.MD.D.10) and measurement data (2.MD.D.9). These skills enable our students to think critically about statistics and graphs presented to them through print and digital media. Although Topic 14: Graphs and Data is labeled as a "Supporting Cluster", we must consider the real-life implications this learning will have on our students and its importance in their learning trajectory.

The NVAC K-5 data standards are comprised of work with two forms of data: categorical data and measurement data.

Categorical Data. In second grade, categorical data reflects data that is sorted into categories, such as birthdays by season, and is presented in picture graphs and bar graphs (reference lesson 14-3, 14-4 and 14-5). As stated in the Progression Documents, work with categorical data in the primary grades "will support their later work with bivariate categorical data and two-way tables in eighth grade." (CCSWT, 2011, p.2). The progression also identifies notable connections including the content of Topic 7, word problems involving addition and subtraction in *Put Together, Take Apart* and *Compare* situations as identified in Table 1 below (CCSWT, 2011, p.4).

Measurement Data. Measurement data refers to data collected through taking measurements. In second grade, this includes students measuring the length of their shoes and representing the data on a line plot (reference lesson 14-1 and 14-2). This work builds upon length measurement concepts in Topic 12. The Progression Documents also identify notable connections between measurement data and the focus of Topic 13, use of the number line diagram to add and subtract lengths within 100 as shown in Table 1 below (CCSWT, 2011, p.4).

Table 1: Some notable connections to K-5 data work

Grade	Standard	Notable Connections
	Categorical data	
2	2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	 2.OA. Problems involving addition and subtraction put-together, take-apart, compare
	Measurement data	
2	2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	1.MD.2. Length measurement2.MD.6. Number line

Common Core Standards Writing Team. (2011, June 20). *Progressions for the Common Core State Standards in Mathematics (draft). K-3 Categorical Data; Grades 2-5, Measurement Data.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The process of doing statistics focuses on numbers in context, called data, and includes formulating questions, collecting data, analyzing data and interpreting results. In statistics, the context is important, as it gives numbers meaning (CCSWT, 2011, p.3). The full process is needed for meaningful engagement. As you plan for Topic 14, look for extension opportunities that align to this process.

Step 1: Formulate Questions

Clarify the problem at hand.

Formulate one (or more) questions that can be answered with data.

Step 2: Collect Data

Design a plan to collect appropriate data.

Employ the plan to collect the data.

Step 3: Analyze Data

Select appropriate graphical and numerical methods.

Use these methods to analyze the data.

Step 4: Interpret Results

Interpret the analysis.

Relate the interpretation to the original question.

Source: Franklin, C.A., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2005). Guidelines for Assessment and Instruction in Statistics Education: A Pre-K-12 Curriculum Framework. Alexandria, VA: American Statistical Association.

When working with data and graphs our questions may be mathematical or statistical in nature. Let's consider two examples, listed below, presented in *Teaching Student-Centered Mathematics*. Which question is mathematical in nature and which is statistical in nature?

- 1. The average weight of 50 prize-winning tomatoes is 2.36 pounds. What is the combined weight, in pounds, of these 50 tomatoes? (NAEP sample question)
- 2. Table 17.1 gives the times each girl has recorded for seven trials of the 100-meter dash this year. Only one girl may compete in the upcoming track meet. Which girl would you select for the meet and why? (Van de Walle, 2014, p. 334)

As you navigate through Topic 14, look for opportunities to extend questioning to attend to the statistical nature of the data as seen in example 2 above.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop MP.2 behaviors. This is the focus of the Math Practices and Problem Solving lesson 14-6. Reference Teacher's Edition (pp.F24-F24A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
line plot symbol	data bar graph picture graph		

Additional terminology that students may need support with: formulate questions, collect data, analyze data, interpret results

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students applying their understanding that data can be represented visually by creating picture graphs?"

Lesson	Evidence	Look for
14-6	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		picture graphs can be used to write and solve problems
		use 1 symbol to represent different items
14-5	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
		(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 845-850
Assessments (summative)	SE pp. 845-850	

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations			
Lesson 14-1: Line Plots					
2.MD.D.9 2.MD.A.1	Access Prior Learning: In Topic 12, second grade students focused on the big idea of measurement in length.	Line plots are used to represent numerical data along a number line. Students draw a number line and mark each data point with a dot above the corresponding value. One benefit of line plots is that each data point is visible (Van de Walle et al., 2014, p.348). Reference Teaching Tool 46 (<i>Teacher's Resource Masters, Volume 2</i>) for a blank table and line plot.			
MP.2	G	Tonic Opener			
MP.4 MP.5 MP.6	Developing the Big Idea: In this lesson, students are developing understanding that measurement data can be collected and displayed in a line plot. Students will collect and represent data of up to four length measurements.	Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 799), <i>Review What You Know</i> (TE, pp. 800-802), and Topic 14 Vocabulary Words Activity (TE, p.800-802) for the words <i>data</i> and <i>line plot</i> only. Introduce remaining vocabulary words as they appear in instruction. Post the essential question and student strategies on your math focus wall. Solve & Share: During problem solving, child-watch for students who are able to estimate 9 inches, when selecting objects smaller than 9 inches to measure. In addition, child-watch for student understanding of measurement concepts learned in Topic 12. Reference the note below regarding estimation in measurement. Estimation in measurement is often needed in real-world applications. To build students' estimation competencies, the Progression Documents indicate that "research suggests explicit teaching of estimation strategies (such as iteration of a mental image of the unit or comparison with a known measurement) and prompting students to learn reference or benchmark lengths (e.g., an inch-long piece of gum, a 6-inch dollar bill), order points along a continuum, and build up mental rulers." (CCSWT, 2012, p.15).			
		Visual Learning: During the animation, ask questions to extend students in <i>Step 1: Formulate Questions</i> (see statistical process in the Instructional Note at the beginning of this document). For example, you might explain that data is collected to answer questions. Then ask, "What question(s) do you have that could be answered by measuring objects and organizing data in a table like this one?"			
		-continues on next page-			

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The Independent Practice page offers problems that support procedural skill and fluency. The Math Practices and Problem Solving page offers problems that support application. The Quick Check items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Lesson 14-2: More Line Plots

2.MD.D.9 2.MD.A.1

MP.2 MP.4 MP.5

MP.6

Access Prior Learning:

In the prior lesson, second grade students collected and represented up to four length-measurement data points on a line plot.

Developing the Big Idea:

In this lesson, students are developing understanding that measurement data can be collected and displayed in a line plot. Students will collect and represent more than four lengthmeasurement data points.

This lesson offers students an opportunity to "see themselves" in the data by measuring the length of their shoe. This personal context brings meaning to the abstract nature of the line plot (Van de Walle et al., 2014, p. 348).

Extending Step 1: Formulate Questions

Just as in the real world, we want students to collect data with the purpose of answering a question. When students formulate their own questions, it adds meaning to the learning experience. As such, begin lesson 14-2 by having students generate questions they have about each other. This will help them them connect their questions to the data collection in the Solve & Share, thus extending step 1 of the statistical process.

Solve and Share:

* Record class show lengths on the board using a data table, similar to that shown in *Analyze* Student Work (TE, p.809).

During problem solving, child-watch for students who only plot one dot for each numerical value. Encourage these students to think about the context of the problem and reflect on whether or not it makes sense to have fewer dots on the line plot than students in the class. Ask, "How can our class data table help you check your line plot for accuracy?" Labeling the data points and/or crossing off data in the table, may be helpful for some students.

Extending Step 3: Analyze Data

Although the text provides a line plot for the shoe data, engage students in a discussion around the selection of a graphical method (line plot, bar graph, picture graph, etc.) and its appropriateness for analyzing the data to answer the question formulated in "Extending Step 1: Formulate Questions" above. Ask students why the use of a line plot and categorization by shoe length is more helpful than a bar diagram with categories that represent each student. This discussion can also lead to greater depth when students engage in Step 4: Interpret Results.

Conclude the share by asking students to interpret the line plot by asking, "What do you notice? What does the data tell you?"

Assess and Differentiate:

Rather than using the *Intervention* and *On-Level and Advanced* Activities, consider engaging students in the statistical process by generating their own questions, collecting data and representing it on a line plot (Teaching Tool 46). An example of a student guestion may be, "How many minutes do second graders in our class read each night?" Students should then analyze the data in their line plot by writing an "I notice..." sentence. Finally, have students interpret the data by answering their original question.

Lesson 14-3: Bar Graphs

2.MD.D.10

MP.1 MP.2 MP.4

Access Prior Learning:

In first grade (1.MD.C.4), students organized, represented and interpreted data with up to three categories using picture graphs and bar graphs. Students also compared how many more or less in each category.

Developing the Big Idea:

In this lesson, students are developing understanding of bar graphs for representing and analyzing data. The height of the bars make comparing data easier. Bar graphs make the largest and smallest categories clearly visible. They also provide data that lends itself to Put Together, Take Apart and Compare word problems (NVACS, 2010, p. 88). To extend students, use the data in the lesson to write word problems for students to solve that involve these three problem types. If students have difficulty reading bar graphs, consider modifying the graph to make the parts countable. This can be done by using sticky notes to construct the bars of the graph, so students can count the sticky notes. After offering this support, return to full rectangular bars (Van de Walle, 2014, p.345-346).

Solve & Share:

In addition to what is asked, extend Step 1: Formulate Questions of the statistical process by asking students to generate a question that could be asked and answered with the "Birthdays by Season" data provided. Step 3: Analyze Data and Step 4: Interpret Results can also be supported by asking students to write something they notice about the data and connect it back to answer their question.

-continues on next page-

Visual Learning:

As suggested in the Coherence note (TE, p.816), engage students in discussion that compares and contrasts bar graphs and line plots.

Assess and Differentiate:

Rather than doing the On-Level and Advanced Activity Centers, "Center Games" (TE, p.819A), engage all students in the statistical process (see the Instructional Note at the beginning of this document) so that they can "see themselves" in the data. Reference the *Intervention Activity*, "Getting to School" (TE, p.819A) for one such example, but allow students to ask their own question(s) and collect their own data. Teaching Tools 47 and 48 offer blank data tables and bar graphs.

Lesson 14-4: Picture Graphs

2.MD.D.10

MP.2 MP.3

MP.4 MP.8

Access Prior Learning:

In first grade (1.MD.C.4), students organized, represented and interpreted data with up to three categories using picture graphs and bar graphs. Students also compared how many more or less in each category.

Developing the Big Idea:

In this lesson, students are developing understanding of picture graphs for representing and analyzing data. The use of single symbols to represent data makes comparing data easier.

Picture graphs, also called pictographs, use single symbols to represent data. These symbols can represent *one* or a designated quantity. They also provide opportunities to practice skip counting and make connections to early multiplication (Van de Walle et al., 2014, p.344). In third grade, students will work with data sets that have multiple categories and represent that data with scaled picture graphs, where each symbol represents more than one piece of data. Teaching Tool 49 offers a blank tally chart and picture graph.

A note of CAUTION: Students may need support with how to draw symbols that all look the same and are aligned, so that data is more accurately represented and easily compared.

Solve & Share:

In addition to what is asked, extend Step 3: Analyze Data and Step 4: Interpret Results by asking students to write an "I notice... which is evidence of [or means]..." statement about the data. If your students do not demonstrate understanding of data, as illustrated in Ehrin's Work found in Analyze Student Work (TE, 821), select a student to share and use class discussion to clarify the meaning of data. If you do not see this misunderstanding in your classroom, do not introduce it.

Assess and Differentiate:

Rather than doing the On-Level and Advanced Activity Centers, "Center Games" (TE, p.825A), engage all students in the statistical process (see the Instructional Note at the beginning of this document) so that they can "see themselves" in the data. Reference the Intervention Activity, "Let's Vote!" (TE, p.825A) for one such example, but allow students to ask their own question(s) and collect their own data. Teaching Tool 49 offers a blank tally chart and picture graph.

Consider posting just the picture graphs from the *On-Level* and *Advanced Activity Centers*, "Look and See" (TE, p. 825A). Encourage students to write guestions they have or things they notice about the data on sticky notes and place them under the graphs.

Lesson 14-5: Draw Conclusions From Graphs

2.MD.D.10 2.OA.A.1

MP.1 MP.3

MP.4

MP.7

Access Prior Learning:

In first grade (1.MD.C.4), students organized, represented and interpreted data with up to three categories using picture graphs and bar graphs. Students also compared how many more or less in each category.

In lesson 14-3, second grade students learned about bar graphs. In lesson 14-4, second grade students learned about picture graphs.

Developing the Big Idea:

In this lesson, students are developing understanding of how to draw conclusions from picture graphs and bar graphs.

The focus of this lesson supports Step 3: Analyze Data and Step 4: Interpret Results of the statistical process. This focus helps students to understand that graphs provide information. When students create their own graphs, they are more invested, and gain a better understanding of how information is presented (Van de Walle, et al., 2014, p.343). It is helpful to ask students, "What does the graph tell us about _____?" Students will come to understand that graphs convey information (e.g., Most students like math more than any other subject.), and that we can make inferences about the data (e.g., Math is fun and engaging for those students) (Van de Walle, et al., 2014, p.349).

Solve & Share:

Reference the *Analyze Student Work* (TE, p. 827) for possible student solutions. Although the picture graphs in Mike's Work and Leah's work is correct, the lack of alignment of picture symbols makes the data difficult to interpret. Students may benefit from support with how to draw symbols that are aligned to make the data easier to count and compare. To extend early finishers, ask them to write a response to, "What new questions arise from these data?" or "What does the graph not tell us?"

Visual Learning:

Extend the Visual Learning by looking for opportunities to ask students Put Together, Take Apart and Compare word problems using the data presented (NVACS, 2010, p. 88). For example, a *Put Together* problem might read: How many tickets did Kim and Neil sell together?

-continues on next page-

		A <i>Take Apart</i> problem could read: Two children sold a total of 6 tickets. Who sold the tickets, and how many did each child sell? [This problem offers two solutions: Leah (2) and Neil (4) OR Tino (1) and Kim (5).] Finally, a <i>Compare</i> problem may read: How many fewer tickets did Leah sell than Kim?
		Assess and Differentiate:
		Rather than doing the <i>On-Level</i> and <i>Advanced Activity Centers</i> , "Problem-Solving Reading Mat" (TE, p.831A), engage all students in the statistical process (see the Instructional Note at the beginning of this document) so that they can "see themselves" in the data. Reference the Intervention Activity, "Analyzing Graphs" (TE, p.831A) for one such example, but allow students to ask their own question(s) and collect their own data. Teaching Tools 47, 48, and 49 offer blank data tables, bar graphs, tally charts and picture graphs.
		*CTC: Quick Check (digital platform)
Lesson 14-6:	Math Practices And Problem Sol	
2.MD.D.10	Access Prior Learning:	Students focused on MP2. Behaviors in Topics 7 and 8. Reference the Math Practices and
2.OA.A.1	Second grade students focused on Math Practice 2: Reason abstractly	Problem Solving Handbook for suggestions for developing, connecting and assessing MP.2 (TE p.F24-F24A). Also, consider having students self-reflect on their understanding of this math
MP.1	and quantitatively in Topics 7 and	practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.
MP.2	8.	instaboginasi and shouldges a grown minuset in maintinaires.
MP.3	Securing the Big Idea:	MP. 2 Behaviors:
MP.4	In this lesson, students are	Identifies and understands the quantities in the problem. Shows and explains how quantities are related (e.g., her diagram).
MP.6	securing understanding of Math Practice 2: Reason abstractly and	 Shows and explains how quantities are related (e.g., bar diagram). Translates real-world contexts correctly to numbers, expressions, equations, or
MP.8	quantitatively behaviors in the context of second grade.	concrete or pictorial representations. • Connects numbers, expressions, equations, or concrete or pictorials representations

References

*CTC: Solve & Share (student work samples)

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▶ Grade 2 Topic 15: Shapes and Their Attributes

Big Conceptual Idea: K-6 Progression on Geometry (pp. 2-5, 10-12)

Prior to instruction, view the Topic 15 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 851A-851E), the Topic Planner (pp. 851I-851K), the Topic Performance Assessment (pp. 917-918A), and all 8 lessons.

Mathematical Background: Read Cluster Overview (TE, pp. 851A-851E)	Topic Essential Question: How can shapes be described, compared, and broken into parts?
	Reference Answering the Topic Essential Question (TE, pp. 913-914) for key elements of answers to the Essential Question.

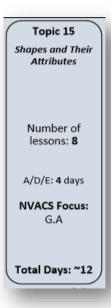
The lesson map for this topic is as follows:

15-1	15-2	15-3	15-4	15-5	15-6	15-7	15-8	Assessment

⁴ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 15 focuses on how 2-D and 3-D shapes can be described, classified and analyzed by their attributes. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) standard cluster 2.G.A.



2nd Grade Curriculum Pacing Framework: Balanced Calendar

2.G.A Reason with shapes and their attributes.

- 1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
- 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
- 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

The NVACS Critical Areas for 2nd Grade, identifies geometry as one of four critical areas for instruction. The National Research Council of the National Academies also articulates this importance:

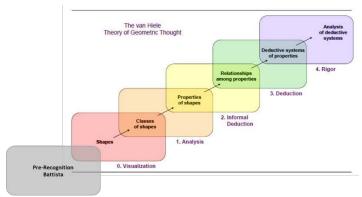
There is expert consensus that two areas of mathematics are particularly important for young children to learn:

- (1) Number, which includes whole number, operations, and relations,
- (2) Geometry, spatial thinking, and measurement.

National Research Council of the National Academies. (2009). Mathematics Learning in Early Childhood. Washington D.C.: The National Academies Press.

Although labeled as an "Additional Cluster", the content of Topic 15 is necessary to build foundations for 3rd grade fractional concepts and MUST NOT BE SKIPPED. Specifically, students work with partitioning shapes in lessons 15-5 to 15-8. Refer to the Conceptual Understanding section of the Math Background pages (TE, p. 851E) for clarification on how Topic 15 concepts will become necessary background for future third grade math learning.

Geometry is often taught with an emphasis on terminology for naming shapes. This approach can limit student access to learning opportunities that build their spatial sense and geometric reasoning. It is often said that some people are "spatial thinkers" while others are not. This is not true. As Pierre van Hiele and Dina van Hiele-Geldof found in their research, we all have the capability of developing spatial reasoning and this geometric thought evolves through a hierarchy of levels. These levels, referred to as "The van Hiele Theory of Geometric Thought" are not age dependent. They are, however, sequential and require geometric *experiences* to advance from one level to the next (Van de Walle, Karp, Lovin, Bay-Williams, 2014).



Students who are at Van Hiele's *Level 0: Visualization* refer to shapes by what they "look like" (e.g., it looks pointy, or it looks like a house). These students are able to notice that shapes are alike or different, but need experiences to help them build an understanding that shapes have properties; and that properties can be used to classify shapes (Van de Walle et al., 2014, p. 302). Reference the list below for Van de Walle's suggestions for supporting student movement from Level 0 to Level 1:

- Challenge children to test ideas about shapes using a variety of examples from a particular category. Say to them, "Let's see
 if that is true for other rectangles," or "Can you draw a triangle that does not have a right angle?" In general, encourage
 children to see whether observations made about a particular shape apply to other shapes of a similar kind.
- Provide ample opportunities for children to draw, build, make, put together (compose), and take apart (decompose) shapes in both two and three dimensions. These activities should be built around understanding and using specific characteristics or properties. (2014, p. 305)
- Emphasize the properties of figures rather than simple identification. As new geometric concepts are learned, the number of properties that figures have can be expanded.

Students who are at Van Hiele's *Level 1: Analysis*, are learning to look at classes of shapes, rather than just individual shapes. According to Van de Walle, et al.,

...in describing a shape, level 1 thinkers are likely to list as many properties of a shape as they know. They do not see relationships between these properties and so cannot determine which properties are sufficient in describing a shape. They are able to consider all shapes within a class rather than just the single shape on their desk. Instead of talking about this rectangle, they can talk about all rectangles. By focusing on a class of shapes, children are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides of the same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background and children begin to appreciate that a collection of shapes goes together because of properties. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class (2014, pp. 302-303).

Now that we've explored two of the levels, let's consider the implications for instruction. As van Hiele found, students must have geometric experiences to advance in the levels of geometric thought. Therefore, we should strive to incorporate four features of effective geometry instruction for young children as identified in *Teaching Student-Centered Mathematics* (2014):

- Show and compare diverse examples and nonexamples.
- Facilitate discussions about shapes and their attributes.
- Examine a wider variety of shape classes.
- Challenge children with a wide range of geometric tasks.

As you navigate through Topic 15, look for opportunities to emphasize these features in your geometry instruction. Examples are included in the lesson level instructional notes below.

Math Practice 2: Reason abstractly and quantitatively

Focus on opportunities for students to develop MP.2 behaviors. This is the focus of the Math Practices and Problem Solving lesson 14-6. Reference the Teacher's Edition (TE, pp. F24-F24A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary (First time explicitly taught)	y :	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)	
vertices, vertex quadrilaterals pentagons hexagons angle	right angle equal shares halves thirds fourths	polygon cube face edge	

Additional terminology that students may need support with: alike, attributes, different, fraction, partition, plane shape, properties, quarter, solid figure

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students able demonstrate their understanding by identifying and drawing plane shapes that have specified attributes?" "Are students able to analyze the attributes of different shapes to find similarities and differences?"

Lesson	Evidence	Look for
15-2	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		identifying attributes that define a polygon
15-5	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
		(scratch paper). Printable version available under "Teacher Resources".
		understand rows and columns
		counting or adding unit squares to find the total square units

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 913-918
Assessments (summative)	SE pp. 913-918	

Standards	iistea in b e	ola indicate	a rocus or	the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 15-1:	2-Dimensional Shapes	
2.G.A.1 MP.3 MP.4 MP.6	Access Prior Learning: In first grade (1.G.A.1), students distinguished between defining attributes and non-defining attributes, and built and drew shapes with those defining attributes. First grade students (1.G.A.2), also composed 2-dimensional shapes including rectangles, squares, trapezoids, triangles, half-circles, and quartercircles). Developing the Big Idea: In this lesson, students are developing understanding that the number of sides and vertices can be used to classify and sort shapes based on their attributes. Students will work with polygons including triangles, quadrilaterals, pentagons and hexagons.	Provide families with the Home-School Connection Letter for Topic 15 (Teacher's Resource Masters, Volume 2) Prior to the Topic Opener, engage students in shape sorts. Provide small groups of students with cut outs of various shapes including regular polygons, irregular polygons, and non-examples of polygons. It is helpful to pull shapes from the Lesson 15-1 Visual Learning, and Lesson 15-2 Visual Learning and Independent Practice. Then follow the suggestions from Teaching Student-Centered Mathematics: • Each student selects a shape and tells something they find interesting about it. • Each student selects two shapes and finds something that is alike and something that is different about their two shapes. • The group selects a target shape and makes a rule. Then, they find all other shapes that fit their rule. For example, the rule may be, "This shape has a straight side and a curved side". Repeat using the same target shape, but with a different rule. • Do a "secret sort". One student selects 3-5 shapes that fit a rule. The other students choose from the shapes left in the pile. They try to find shapes that fit in the set and guess the secret rule. (Van de Walle, et al., 2014, p. 309) In this lesson, students draw shapes from a given set of attributes while engaging in MP.1 behaviors (Van de Walle et al., 2014, p. 320). This process helps students develop understanding of the defining attributes of shapes, and how they can be used to describe, classify and analyze 2-dimensional geometric figures. -continues on next page-

Topic Opener:

Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE, p. 851), *Review What You Know* (TE, p. 852), *Vocabulary Review Activity* (TE, p. 852) for the term *plane shape*, and the *Topic 15 Vocabulary Words Activity* (TE, pp. 853-854) for the word *vertices (vertex)* only. Introduce remaining vocabulary words as they appear in instruction. Post the essential question and student strategies on your math focus wall.

Solve & Share:

During the share, look for opportunities to ask students to identify examples and non-examples in each other's work. If you have difficulty finding non-examples in student work, consider displaying Shawn's Work in *Analyze Student Work* (TE, p.859 and available online under the *Solve & Share* as "Teacher Resources). By comparing examples and non-examples, students are able to engage in a conversation grounded in MP.3 behaviors and the defining attributes of shapes.

Develop: Problem-Based Learning



Math Practices & Problem Solving: Construct Arguments: Solve & Share

Assign Info T Teacher resources

Return to the Topic Essential Question posted on the math focus wall to add new student thinking.

Visual Learning:

During the animation, encourage students to write and draw responses on individual whiteboards to increase engagement and understanding.

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

After students solve item 12, engage the class in a conversation about their thinking. This problem provides an opportunity for students to consider the classification of geometric figures.

Lesson 15-2: Polygons and Angles

2.G.A.1

MP.2 MP.6 MP.7

Access Prior Learning:

In first grade (1.G.A.1), students distinguished between defining attributes and non-defining attributes, and built and drew shapes with those defining attributes.

In lesson 15-1, second grade students identified plane shapes by the number of sides and vertices.

Developing the Big Idea:

In this lesson, students are developing understanding that plane shapes can be described by their defining attributes including sides and angles.

This lesson focuses on reasoning at Van Hiele's *Level 1: Analysis*. Students consider the attributes of polygons through analysis of examples and non-examples. Students are also explicitly taught the words *polygon*, *angle* and *right angle* for the first time. Use the Frayer Model (Teaching Tool 62) to capture students' new understanding of these terms after *Visual Learning*.

Solve and Share:

During problem solving, child-watch for students who demonstrate *Level 0: Visualization* understanding by describing what the shapes "look like" without using attributes. For example, a student may say the shapes look like windows. Refer to the Instructional Note at the beginning of this document for clarification on *Level 0* and appropriate supports to offer students. Also, child-watch for students who demonstrate *Level 1: Analysis* understanding. These students will attend to the attributes of the shapes. Select and sequence students to share solutions to progress from less sophisticated to more sophisticated geometric reasoning.

Independent Practice/Math Practices and Problem Solving:

If students identify item 5 as having four angles, they may not be counting the concave angle. Engage students in a discussion that clarifies that concave, "dented in", angles are counted as angles. Students will also discover that polygons have the same number of sides, vertices and angles.

Extend student thinking on item 5 by asking students to draw another pentagon and give a written explanation that proves that both shapes are pentagons. This can be done on a sticky note or plank piece of paper and used as a formative assessment.

*CTC: Solve & Share (student work samples)

Lesson 15-3: Draw 2-Dimensional Shapes 2.G.A.1 Access Prior Learning: NOTE: As suggested in "Pose the Solve-and-Share Problem" (TE, p. 871), have all students In first grade (1.G.A.1), students construct polygons using toothpicks, crayons or straws before drawing them. distinguished between defining MP.1 attributes and non-defining Solve & Share: MP.6 attributes, and built and drew As students problem-solve, remind them of the resources on the math focus wall that may help MP.7 them compare the shapes they've constructed. These resources include the frayer model shapes with those defining graphic organizers from lesson 15-2. attributes. First grade students (1.G.A.2) also composed 2-Visual Learning: dimensional shapes including Prior to interacting with the animation, have students draw a polygon with 5 vertices on their rectangles, squares, trapezoids, whiteboards. At the conclusion of the animation, engage students in a discussion of the Lesson triangles, half-circles, and quarter-Essential Question: What information should you give to others if you want them to draw a circles). particular polygon? (TE, p. 872) Record students' thoughts on the math focus wall for future In the prior lessons, second grade students identified polygons by the number of sides, vertices and Independent Practice/Math Practices and Problem Solving: angles. Students also learned If students respond to item 7 by drawing a triangle, ask them, "Does your shape have 5 about right angles. vertices? How does this affect the total number of sides?" to support MP.6 Attend to Precision. Use item 11 as a formative assessment of students' understanding of polygons as closed plane Developing the Big Idea: shapes with three or more straight sides. As an extension, consider allowing students to write In this lesson, students are their own geometry riddles, similar to item 7. These can be recorded on cards for peers to developing understanding of how solve. to define and differentiate shapes based upon attributes. They will also draw 2-dimensional shapes using specific attributes. Lesson 15-4: Cubes 2.G.A.1 Access Prior Learning: The only 3-dimensional figure included in second grade standards is the cube. Reference In first grade (1.G.A.2), students NVACS, 2010, 2.G.A.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, composed 3-dimensional shapes MP.2 pentagons, hexagons, and cubes. In this lesson, students consider how a solid figure (cube) is including cubes, right rectangular different from a plane shape (square). MP.3 prisms, right circular cones, and MP.6 right circular cylinders, to create a Solve & Share: composite shape, and compose MP.7 Ensure that all students have access to concrete cubes to explore while problem-solving. new shapes from the composite shape. Visual Learning: As student understandings emerge, record their thinking around solid figures (3-D) vs. plane In lesson 15-3, second grade shapes (2-D). Add this information to the math focus wall. students learned about right angles. Developing the Big Idea: In this lesson, students are developing understanding of how to describe a cube by telling about its faces, edges and vertices. Students also draw a cube using these attributes. Lesson 15-5: Divide Rectangles Into Equal Squares Access Prior Learning: 2.G.A.2 The concept of equal sharing comes intuitively for students. Their experiences of sharing with 2.OA.C.4 In first grade (1.G.A.3), students siblings or friends allow fractional concepts to emerge. It is important to capitalize on these partitioned rectangles into two and meaningful connections to equal shares. As Van de Walle et al. states, "One of the most significant ideas for children to develop about fractions is the sense that fractions are numbersfour equal shares and described MP.1 quantities that have values...Researchers have acknowledged for some time the importance of the shares using halves, fourths, these two actions [partitioning (splitting equally] and iterating (counting a repeated amount)] to MP.3 quarters, half of, fourth of, and meaningfully working with the numerical nature of fractions" (2014, p. 253). quarter of. Students also described MP.4

Child-watch for student understanding of repeated addition, introduced in Topic 2. Select and

-continues on next page-

sequence students to share so that both equations (5 + 5 + 5 + 5 and 4 + 4 + 4 + 4 + 4) are

Solve & Share:

MP.5

MP.7

the whole as two of or four of the

shares.

Securing the Big Idea:

In this lesson, students are *securing* understanding of repeated addition.

Developing the Big Idea:

In this lesson, students are *developing* understanding of how to divide rectangles into rows and columns of equal squares.

presented. Ask students to compare both solutions for accuracy and equivalence. Also, ask them which unit each equation utilizes- rows (5 + 5 + 5 + 5) or columns (4 + 4 + 4 + 4 + 4). This conversation will offer an entry point for students into the *Visual Learning*.

Visual Learning:

Look for opportunities to help students connect their understanding of measurement concepts, such as no gaps or overlaps, to tiling of squares over a rectangle. Students should use the foam square tiles from their student manipulative kits.

*CTC: Quick Check (digital platform)

Lesson 15-6: Partition Shapes

2.G.A.3 Access Prior Learning:

MP.2 MP.4 MP.6 **MP.8** In first grade (1.G.A.3), students partitioned rectangles into two and four equal shares and described the shares using *halves*, *fourths*, *quarters*, *half of*, *fourth of*, and *quarter of*. Students also described the whole as two of two shares or four of four shares.

Developing the Big Idea:

In this lesson, students are developing understanding of how to divide circles and rectangles into halves, thirds and fourths; and that you can show halves, thirds and fourths of the same whole in different ways.

"Fraction symbolism represents a fairly complex convention that can be misleading to children. That is why it is important in grades pre-K-2 to use fraction words and postpone introducing fraction symbolism. Let children first focus on making sense of fractions without the complication of also trying to make sense of the symbolism." (Van de Walle, et al, 2014, p. 256) In other words, do not use fraction symbols. As indicated by the text, expect students to use fraction words instead.

As Van de Walle et al. suggests, introduce fraction words, *halves, thirds, fourths*, during discussion when students are sharing their solution strategies. He goes on to clarify, "Children need to be aware of two aspects of fractional parts: (1) the number of parts and (2) the equality of the parts (in size, not necessarily in shape). Emphasize that the number of equal parts or fair shares that make up a whole determines the name of the fractional parts or shares." (2014, p. 256)

Solve & Share:

During problem-solving, child-watch for students who demonstrate understanding of lesson 15-5 by partitioning the rectangle with no gaps or overlaps. Select students who demonstrate a misconception to share first. This will allow other students to help clarify the error and move forward with accurate partitioning. During discussion of accurate solutions, introduce fraction words as appropriate to the work shown.

Visual Learning:

Prior to interacting with the animation, ask students to draw two circles on their whiteboard. Ask them to split one into equal shares, and the other so that it does NOT show equal shares. Continue to ask students to draw examples and non-examples throughout the animation.

Independent Practice/Math Practices and Problem Solving:

For item 10, formatively assess student understanding of equal shares by asking for a written explanation of how/why the drawings show fourths. Offering a sticky note or blank piece of paper will encourage responses that are more thoughtful as they provide more room to write.

Lesson 15-7: Equal Shares, Different Shapes

2.G.A.3

MP.1

MP.2

MP.3

MP.4

MP.7

Access Prior Learning:

In first grade (1.G.A.3), students partitioned rectangles into two and four equal shares and described the shares using *halves*, *fourths*, *quarters*, *half of*, *fourth of*, and *quarter of*. Students also described the whole as two of two shares or four of four shares.

In lessons 15-5 and 15-6, second grade students partition rectangles and other shapes into equal shares and described the shares.

Developing the Big Idea:

In this lesson, students are developing understanding that equal shares of the same whole do not have to have the same shape.

Van de Walle et al., suggests, "In addition to helping children use the words *halves, thirds, fourths*, and *quarters*, be sure to make regular comparisons of fractional parts to the whole. Make it a point to use the term *whole*, *one whole*, or simply *one* so that children have a language they can use regardless of the model involved" (2014, p. 256).

Solve & Share:

During problem solving, extend everyone's thinking by using the "Extension for Early Finishers" on TE p. 895.

Visual Learning:

Prior to interacting with the animation, have students use their whiteboards to show different ways to divide a square into 3 equal shares.

Independent Practice/Math Practices and Problem Solving:

For items 7 and 11, child-watch for students who do not use the whole when dividing into equal shares. Ask, "How can you share the *whole* rectangle equally?" Using a context such as brownies, students will understand that we do not want to waste the extras. Due to similarity in the sound of the terms, some students may confuse *shares* with *squares*.

Lesson 15-8:	Math Practices And Problem Sol	ving: Repeated Reasoning
2.G.A.2 2.G.A.3 2.OA.C.4 MP.1 MP.2 MP.3 MP.4 MP.7 MP.8	Access Prior Learning: Second grade students focused on Math Practice 8: Repeated Reasoning in Topic 10. Securing the Big Idea: In this lesson, students are securing understanding of Math Practice 8: Reason abstractly and quantitatively behaviors in the context of second grade. Students will use repeated reasoning to find different ways to divide rectangles into rows and columns and then into equal shares. Students will also write equations using repeated addition.	Students focused on Math Practice 8: Repeated Reasoning in Topic 10. Reference the Math Practices and Problem Solving Handbook for suggestions for developing, connecting and assessing MP.8 (TE p.F30-F30A). Also, consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics. MP. 8 Behaviors: • Notices and describes when certain calculations or steps in a procedure are repeated • Generalizes from examples or repeated observations • Recognizes and understands appropriate short cuts • Evaluates the reasonableness of intermediate results Solve & Share: As recommended in the "Pose the Solve-and-Share Problem" (TE p. 901), provide students with ¾ inch squares (Teaching Tool 52) to help them problem solve. Visual Learning: Prior to interacting with the animation, have students use their whiteboards to show different ways Sam could design his quilt square. Reference Error Intervention: Item 1 (TE p. 902) to support students who have difficulty coming up with a second design.

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▶ Grade 2 Topic 13: More Addition, Subtraction, and Length

Big Conceptual Idea: K-5 Progression on Measurement and Data (Measurement Part) (pp. 2-4, 12-15) Prior to instruction, view the Topic 13 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 759A-759E), the Topic Planner (pp. 759I-759J), the Topic Performance Assessments (pp. 797-798A), and all 5 lessons.

Mathematical Background:	Topic Essential Question:
Read Cluster Overview (TE,	How can you add and subtract lengths?
pp. 759A-759E)	Reference Answering the Topic Essential Question (TE, pp. 795-796) for key elements of answers to the Essential Question.

The lesson map for this topic is as follows:

13-1 13-2 13-3 13-4	13-5	Assessment
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³ A/D/E days used strategically throughout the topic.

Instructional note:

The big idea of Topic 13 focuses on using measurement in addition and subtraction situations involving lengths. Focus instruction on Nevada Academic Content Standards (NVACS, 2010) cluster 2.MD.B.

Pacing Framework: **Balanced Calendar** 2.MD.B Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

In this topic, students will develop understanding of addition situations as totals and subtraction situations as comparisons of lengths. For example, students will measure the distance around objects, laying the foundation for perimeter. The work of this topic applies learning from throughout second grade. In Topics 3-5, second grade students developed understanding of addition and subtraction strategies within 100, including use of open number lines. In Topic 7, students developed understanding of addition and subtraction situations in word problems, applying the addition and subtraction strategies from earlier topics. These situations included Add To, Take From, Put Together/Take Apart, and Compare problem types (Reference CCSS, p. 88, Table 1. Common addition and subtraction situations.) Finally, in Topic 12, students developed measurement concepts in length using customary and metric units.

As students work with operations involving lengths, they will use a number line diagram to represent their thinking. These diagrams are different from open number lines, as they contain consecutive whole units that are already marked. Important understandings when working with number line diagrams, as articulated in the Progression Documents are below:

to use a number line diagram to understand number and number operations, students need to understand that number line diagrams have specific conventions: the use of a single position to represent a whole number and the use of marks to indicate those positions. They need to understand that a number line diagram is like a ruler in that consecutive whole numbers are 1 unit apart, thus they need to consider the distances between positions and segments when identifying missing numbers. These understandings underlie students' successful use of number line diagrams. Students think of a number line diagram as a measurement model and use strategies relating to distance, proximity of numbers, and reference points" (CCSWT, 2012, p. 14).

In this topic, students use estimation as an effective way to think about the numbers and their meaning. However, estimation is also difficult for students. Focus estimation opportunities on building student understanding of the concept of about. Rather than asking students to provide a specific number, start working with estimation by using these prompts from *Teaching Student-Centered* Mathematics:

- *More or less than* ____? *Will it be more or less than 10 footprints?*
- Closer to _____ or to _____? Is the bar closer to 10 cubes or closer to 50 cubes?
- About _____? Use one of these numbers: 5, 10, 15, 20, 25, 30, 35, 40, About how many footprints wide is the hallway? (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p.121)

2nd Grade Curriculum

Math Practice 5: Use appropriate tools

Focus on opportunities for students to develop MP.5 behaviors. This is the focus of the Math Practices and Problem Solving lesson 13-5. Reference the Teacher's Edition (pp. F27-F27A) and the *Nevada Academic Content Standards for Mathematical Practice*.

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.			
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)		
number line diagram	estimate inch, in. foot, ft. yard, yd. height	nearest inch centimeter, cm nearest centimeter meter, m	

Additional terminology that students may need support with:

*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding question: "Are students developing conceptual understanding that measurements can be added and subtracted in the same way as other whole numbers?"

Lesson	Evidence	Look for
13-2	Solve & Share (student work samples)	Focus CTC around the big idea:
		student strategies and models
		making sense of word problems involving length
		accurate measurements
13-2	Quick Check (digital platform)	Focus CTC around data analysis and collection of student workspace
	items 3, 4 and 5	(scratch paper). Printable version available under "Teacher Resources".

Learning Cycle	Topic Assessments	Use Scoring Guide TE pp. 795-798
Assessments (summative)	SE pp. 795-798	

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 13-1:	Add and Subtract With Measurer	ments
2.MD.B.5 MP.2 MP.4 MP.6	Access Prior Learning: Throughout second grade, students have developed addition and subtraction strategies. In Topic 12, second grade students measured lengths of objects, added to find total lengths and subtracted to compare lengths of objects. Developing the Big Idea: In this lesson, students are developing understanding of adding and subtracting length measurements and that the answer should include the unit of measurement.	Topic Opener: Consider limiting the <i>Topic Opener</i> to discussion of the <i>Topic Essential Question</i> (TE, p. 759), <i>Review What You Know</i> (TE, p. 760), and <i>Vocabulary Review Activity</i> (TE, p. 760) only. Post the essential question and student strategies on your math focus wall. Solve & Share: In this lesson, students explore the distance around shapes. Look for opportunities for students to make real-world connections to the concept of perimeter. During problem solving, child-watch for students who measure all four sides of the rectangle, and those who measure only two sides because they understand that opposite sides of a rectangle are equal lengths. Reference Jess's Work in <i>Analyze Student Work</i> (TE, p. 761) for an example of this understanding. Measurement of the perimeter of rectangles also offers students practice with their doubles facts. During the share, focus the conversation on students' choice of operation (how they knew this was an addition situation) as this directly connects to the question presented in the Visual Learning animation: <i>How do you know when to add or subtract when solving problems involving measurements?</i> -continues on next page-

Visual Learning:

Prior to interacting with the animation, ask students to solve the problem presented: *The book is 9 inches long and 6 inches wide. What is the distance around the front cover of the book?*Child-watch for students who understand that only length and width are needed to solve the problem. Also, provide students an opportunity to solve the second problem presented (*How much longer is the teacher's arm than the child's arm? Will you add or subtract to solve this problem?*) by stopping the animation once the data table is displayed.

Independent Practice/Math Practices and Problem Solving:

As previously indicated, students do NOT need to do all of the problems in their Student Edition. However, ALL students NEED to have opportunities to solve problems at varying DOK levels. The *Independent Practice* page offers problems that support procedural skill and fluency. The *Math Practices and Problem Solving* page offers problems that support application. The *Quick Check* items (marked with a pink check) offer both opportunities. Have students complete these items first and continue on to other items as appropriate.

Lesson 13-2: Find Unknown Measurements

Access Prior Learning:

2.MD.B.5 2.OA.A.1

MP.1

MP.2

MP.3

MP.4

Throughout second grade, students have developed addition and subtraction strategies. In Topic 12, second grade students measured lengths of objects, added to find total lengths and subtracted to compare lengths of objects.

Developing the Big Idea:

In this lesson, students are developing understanding of adding and subtracting length measurements using pictures and equations to solve word problems.

In the *Visual Learning* animation, students are asked to estimate, "Will the number of inches Tim jumped be more or less than 24?" Estimation is an effective way to get students to think about the numbers and their meaning. Reference the Instructional Note at the beginning of this document for additional information regarding estimation in measurement.

Visual Learning:

The *Visual Learning* animation presents a *Compare Smaller Unknown* problem (NVACS, 2010, p.88). Common addition and subtraction situations.) Have students use their whiteboards and markers to record estimates and their strategy for solving the problem as they interact with the animation and engage in class discussion. After completing the animation, have students compare their answer to their estimate and respond to, "Does your answer make sense?" Engage students in a discussion of *why* estimates are helpful when problem solving.

*CTC: Solve & Share (student work samples)
*CTC: Quick Check (digital platform)

Lesson 13-3: Continue To Find Unknown Measurements

2.MD.B.5 2.OA.A.1

MP.1

MP.3 MP.4

MP.6

Access Prior Learning:

In lessons 13-1 and 13-2, second grade students added and subtracted measurements to solve problems.

Developing the Big Idea:

In this lesson, students are developing understanding of adding and subtracting length measurements using pictures and equations to solve word problems.

Solve & Share:

This problem is a *Take From Change Unknown* problem (NVACS, 2010, p. 88). The context of a piece of ribbon lends itself to representation through a number line. Child-watch for students who use a subtraction equation and count back to solve the problem; and for students who write an addition equation and add up to solve the problem. Selecting and sequencing the share to include these different approaches will support student understanding of the inverse relationship between addition and subtraction. It will also offer students an entry point into the strategies presented in the Visual Learning.

Visual Learning:

The problem in the Visual Learning mirrors the *Solve & Share*. Prior to interacting with the animation, ask students to solve the problem and challenge them to write both an addition and subtraction equation to represent the situation.

In guided practice, continue to emphasize the importance of MP.1 in making sense of problems. Reference Error Intervention Note: Item 2 for how to respond to students who scan the problem for numbers and solve the problem inaccurately.

Independent Practice/Math Practices and Problem Solving:

Item 8 presents students with a problem involving two units of measurement: feet and inches. After students have had the opportunity to solve this problem independently, engage them in a discussion around how MP.1 behaviors were important to their success in accurately solving the problem.

Lesson 13-4: Add And Subtract On A Number Line

2.MD.B.6 Ac

Access Prior Learning:

In lessons 13-2 and 13-3, second grade students added and subtracted measurements to solve problems.

MP.4 **MP.5** MP.7

MP.2

MP.3

Developing the Big Idea:

In this lesson, students are developing understanding of adding and subtracting length measurements using a number line diagram. Students represent addition or subtraction as line segments above the number line.

According to the Progression Documents, students need to understand the following number line conventions:

"the use of a single position to represent a whole number and the use of marks to indicate those positions. They need to understand that a number line diagram is like a ruler in that consecutive whole numbers are 1 unit apart, thus they need to consider the distances between positions and segments when identifying missing numbers. These understandings underlie students' successful use of number line diagrams." (CCSWT, 2012, p. 14)

Solve & Share:

This problem is an *Add To Result Unknown* problem (NVACS, 2010, p. 88). The context of walking blocks helps students as they represent addition and subtraction of lengths on a number line diagram. Engage students in a brief discussion of how this number line looks different from an *open* number line. Listen for students who are able to connect their understanding of measurement tools, such as rulers, to the number line diagram in the *Solve & Share*.

Visual Learning:

Prior to interaction with the animation, have students solve the *Compare Smaller Unknown* problem presented in the animation on their whiteboards (NVACS, 2010, p. 88). Students should begin with an estimation. Reference the estimation note in Lesson 13-2. Child-watch for students who successfully make sense of the relationships between the numbers and accurately estimate that Tim's jump will be less than 24 inches. The process of estimation engages students in MP.1 Make Sense of Problems and Persevere In Solving Them behaviors. After completing the animation, have students compare their answer to their estimate and respond to, "Does your answer make sense?" Engage students in a discussion of *why* estimates are helpful when problem solving.

The use of a question mark (?) to represent the unknown supports students' algebraic thinking.

Assess and Differentiate:

The Intervention Activity, "Yardstick Number Line" utilizes a yardstick as a number line model for students to use when solving problems (TE, p. 783A). This tool may be helpful for all students and will facilitate connections between number line diagrams and tools of measurement.

Lesson 13-5: Math Practices And Problem Solving: Use Appropriate Tools

2.MD.B.5 2.MD.B.6 2.OA.A.1

Access Prior Learning:

In Topic 3, second grade students focused on MP. 5 Use Appropriate Tools Strategically behaviors.

- MP.1 MP.3
- MP.4
- MP.5 MP.6

MP.8

Securing the Big Idea: In this lesson, students are

securing understanding of MP.5
Use Appropriate Tools Strategically behaviors in the context of second grade.

Students focused on Math Practice 5: Use Appropriate Tools Strategically in Topic 3. Reference the *Math Practices and Problem Solving Handbook* for suggestions for developing, connecting and assessing MP.5 (TE. pp. F27-F27A). Also, consider having students self-reflect on their understanding of this math practice using the Self-Assessment Tool (Teaching Tool 65). Self-reflection engages students in metacognition and encourages a growth mindset in mathematics.

MP. 5 Behaviors:

- Identifies available tools
- Thinks about correct tools to use without prompting
- Uses tools correctly and accurately
- Knows when to use a particular tool
- Decides if the results obtained using a tool make sense

Solve & Share:

Child-watch for MP.5 behaviors, specifically looking for students who select an appropriate measuring tool and unit; and for students who use the tool correctly. Students may apply their ability to estimate when they reflect on whether the results of their measurements make sense. Also, child-watch for students who connect their understanding of part-part-whole relationships when solving the problem. These students will correctly identify the difference between the two lines. Conversely, some students may show a misconception or misunderstanding of the problem, as seen in Keri's Work as shown in *Analyze Student Work* (TE, p. 785). During the share, focus the conversation on students' application of the MP.5 behaviors listed above.

Visual Learning:

Prior to interaction with the animation, give students time to make an estimate (*Will Sara be more or less than 56 feet from the goal?*), select a tool and solve the *Take From Result Unknown* problem presented (NVACS, 2010, p. 88). Child-watch for students who misinterpret the situation, and estimate that Sara will be more than 56 feet from the goal. Unprompted, these students are likely to add 56 + 24 to solve the problem. The use of drawings or role-playing may support these students in clarifying their understanding of the context of the problem.

References

- Common Core Standards Writing Team. (2011, June 20). *Progressions for the Common Core State Standards in Mathematics (draft). K-3 Categorical Data; Grades 2-5, Measurement Data.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.
- Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

Number Line Difference Game

Materials:

- Number Line Diagram (Teaching Tool 38, one per player laminated or in a page protector)
- Wet erase pen (one per player)
- Number Tiles in a cup or bag: Digits 0-9

Players: 2+

Object of the game: To find the largest difference in each round.

Directions:

- 1. Place the number tiles, digits 0-9 in a cup or bag.
- 2. Each player draws 4 tiles and builds two 2-digit numbers: a minuend (larger number) and a subtrahend (smaller number).
- 3. Players use the number line diagram (Teaching Tools 38) to represent and solve the subtraction problem. Players explain their strategy and check each other's work for accuracy.
- 4. The player with the largest difference earns a point. In the event of a tie, players draw one more tile to subtract from their difference.
- 5. Players return the tiles to the cup or bag and repeat. Play ends after 3 rounds.
- 6. The player with the most points wins.