# ▶ Grade 3 Topic 2: Multiplication Facts: Use Patterns

# Big Conceptual Idea: Operations and Algebraic Thinking (pp. 22-28)

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 1A-1F), the Topic Planner (pp.57A-57B), all 6 lessons, and the Topic Performance Assessment (pp. 103-104A).

| Mathematical             |
|--------------------------|
| Background:              |
| Read Topic 1-2 Cluster   |
| Overview/Math Background |
| (TE, pp. 1A-1F)          |

# **Topic Essential Question:**

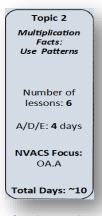
How can unknown multiplication facts be found using patterns and properties?

Reference Answering the Topic Essential Question (TE, pp. 101-102) for key elements of answers to the Essential Question.

# The lesson map for this topic is as follows:

| 2-1 | 2-3 | 2-4 | 2-2 | 2-5 | 2-6 | Assessment |
|-----|-----|-----|-----|-----|-----|------------|
|-----|-----|-----|-----|-----|-----|------------|

4 A/D/E days used strategically throughout the topic



3<sup>rd</sup> Grade Curriculum Pacing Framework: Balanced Calendar

### Instructional note:

This topic focuses on *continuing* to build the meaning of multiplication to meet the 2010 Nevada Academic Content Standards (NVACS) primarily focusing on 3.OA.A cluster heading, "Represent and solve problems involving multiplication and division" and 3.OA.D cluster heading "Solve problems involving the four operations, and identify and explain patterns in arithmetic." This topic also introduces the Zero and Identity properties in lesson 2-3 (3.OA.B.5).

Topic 2 focuses on understanding multiplication through an exploration of patterns. Patterns play an important part of mathematical development and support algebraic reasoning, relationships and lead to mathematical generalizations. Throughout Topic 2, encourage students to reason mathematically, draw conclusions, justify and generalize solutions. For example, in lesson 2-1 the *Solve & Share* provides an opportunity to explore patterns that occur when there are equal groups of 2 (there are 2 legs on *x* amount of chickens). What do students notice about the patterns? Given the conditions of the problem, can there ever be an odd number? Why or why not?

Encourage student reasoning using tools or models that demonstrate the understanding of multiplication as equal groups. Spend time connecting student models to the patterns generated and connect both of these to understanding the meaning of multiplication. Building from Topic 1, ensure that the students' models use represent the given equation. For example, if students build an array to demonstrate 9 x 4, the array must be represented as 9 rows of 4 stated "9 groups of 4". Or if they use a number line there should be 9 "jumps" or "hops" that are in equal increments of 4. Some students may naturally start using the Commutative Property to build more efficient models or to assist their reasoning. For example, given the factors 9 and 2 students may reason about 9 groups of 2 or find it more efficient to think about 2 groups of 9. Highlight and discuss how Properties help us reason more efficiently.

Teachers should use their professional discretion to decide on the placement of Lesson 2-2. The lesson may be kept in the same order as shown in the instructional materials, or moved after lesson 2-4 as shown in this guide. See the lesson note for more information to help with this decision. Regardless of the order taught, ensure that students make the connection of using known facts (factors with 10) to derive unknown facts (factors with 9). See the instructional note in lesson 2-2 for examples.

Looking ahead to the assessment, Part A, item 2 of the Topic Assessment asks students to, "Identify any hidden question" (TE, pp. 101-102). Students in 2<sup>nd</sup> grade worked with the idea of a "hidden question" in 2-step word problems in Topics 8, 13 & 14. They will revisit this idea in Lesson 2-6. Both the Topic Assessment and the Topic Performance Assessment will provide opportunities to work at various DOK levels. Choose the assessment(s) that will provide the most information about student understanding. Consider scaffolding this resource by allowing students to work in groups throughout the topic and by ensuring opportunities for discussion, peer feedback, and revision.

# Focus Math Practice 4: Model with mathematics

Focus on opportunities for students to develop Mathematical Practice 4 behaviors, as this is the focus of the Math Practices and Problem Solving lesson 2-6. Reference the Teacher's Edition (TE, pp. F24 - F24A) and the Nevada Academic Content Standards for Mathematical Practice (NVACS, 2010, p.7).

| Essential Academic Vocabulary Use these words consistently during instruction. |  |  |
|--|--|--|
| New Academic Vocabulary: (First time explicitly taught)                        | Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics) |  |
| multiples  | factor   |  |
| Identity (One) Property of Multiplication                                      | product  |  |
| Zero Property of Multiplication  | array  |  |
|  | multiplication   |  |

Additional terminology that students may need support with: patterns, relationship

# \*Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

**Guiding questions:** 

"Are students identifying patterns in multiplication? Are students using tools, strategies or models to multiply whole numbers?"

| Lesson | Evidence  | Look for  |
|--------|---|---|
| 2-4    | Math Practices and Problem Solving (student work samples) Item 20 | <ul> <li>Focus CTC around the big idea:</li> <li>students are identifying patterns in multiplication and explaining their thinking.</li> <li>students are using various tools, strategies or models to multiply.</li> </ul>   |
| 2-5    | Quick Check (digital platform)                                    | Focus CTC around data analysis and collection of student workspace (scratch paper).  • students understand that multiplication facts can be found by identifying patterns.  • students are using various tools, strategies or models to multiply Printable version available under "Teacher Resources". |

| Learning Cycle          | Topic Assessments | Use Scoring Guide TE pp. 101-104A |
|-------------------------|-------------------|-----------------------------------|
| Assessments (summative) | SE pp. 101-104    |                                   |

Standards listed in **bold** indicate a focus of the lesson.

| NVACS<br>(Content and<br>Practices)                              | Mathematical Development of the Big<br>Idea   | Instructional Clarifications & Considerations  |
|--|---|--|
| 3.OA.A.3<br>3.OA.A.1<br>3.OA.D.9<br>MP.1<br>MP.2<br>MP.3<br>MP.7 | Access Prior Learning: In Topic 1, students used skip counting, the number line, and repeated addition to think about multiplication.  Beginning of the Big Idea: In this lesson, students are beginning to build their understanding and use of strategies for multiplication facts with 2 and 5 as factors with the focus on being able to skip count to find a product.  Students may begin to develop an understanding of why when 2 is a factor the product is always even, and be able to develop their understanding of multiplication by connecting the multiplication equation to have an even number of groups (as in 2 x 6) or an even amount in each group (as in 6 x 2). | (Possible 2-day lesson)  Topic Opener: Introduce the <i>Topic Essential Question</i> , "How can unknown multiplication facts be found using patterns and properties?" (TE, p. 57). Consider making this an anchor chart in your classroom. Each day new ideas are added so that students can see their ideas develop and make new connections throughout the topic.  Also, consider having students complete the <i>Review What You Know</i> prior to beginning instruction on Topic 2 so that you can respond to student instructional needs using <i>the Item Analysis for Diagnosis and Intervention</i> (TE, p. 58-60).  Consider introducing vocabulary as words are encountered in the lessons rather than introducing all terms at the beginning of the lesson.  Solve & Share: Consider asking students to solve this on a blank page, white board, etc. so that the task offers multiple entry points. This will also offer you more information on how your students are understanding and seeing multiplicative situations. |
|  | Students may also <i>begin</i> to understand that when 5 is multiplied by an odd number, the  | -continues on next page-   |

product has a 5 in the ones place. When 5 is multiplied by an even number, it has a 0 in the ones place.

After student solution methods and reasoning have been shared, asking students questions such as, "What do all the products have in common?" (e.g. they're even) and "Why is that?" will help them to make generalizations and reach the understandings stated in *Beginning of the Big Idea* for when 2 is a factor. Have students confirm, clarify, or correct their ideas during the *Visual Learning Animation*.

#### Visual Learning:

The *Visual Learning Animation* for this topic is helpful for reinforcing understanding of meanings of multiplication. If students are still struggling to understand multiplication as the joining of equal groups, view and discuss the ideas modeled as a class. Building in additional pausing points during the *Visual Learning Animation* will allow students to practice the skip counts and represent them on an open number line.

#### Convince Me:

Consider having a whole class discussion around the *Convince Me!* so students can continue their discovery of patterns in products when 2 and/or 5 is a factor.

### Assess and Differentiate/Intervention Activity:

If time permits, teach students how to play "Quick Questions" (TE, p. 65A). All students should have the opportunity to play games that provide opportunities for practicing strategies for facts with 2 and 5 as factors. Students may also continue to play any of the games from topic 1.

# Lesson 2-3: Apply Properties: Multiply by 0 and 1

# 3.OA.A.3 3.OA.A.1 3.OA.D.9

MP.1

MP.2

MP.3

MP.7

Access Prior Learning: In Topic 1, students developed an understanding of multiplication as joining equal groups to find the total number of objects in groups. In 2<sup>nd</sup> grade, students were secure in their understanding of the Zero Property of Addition.

### Developing the Big Idea:

Students *develop* the understanding of multiplication as joining equal groups of objects to *begin* to build an understanding of the Zero Property of Multiplication and Identity (One) Property of Multiplication.

# Instructional note:

See the Instruction note at the top of this document for an explanation of *moving lesson 2-2*.

#### Solve & Share:

For concrete learners or students grappling with the misconception that multiplication always makes numbers bigger, you may consider having paper plates or bags available so that students can model having 6 groups of 0 objects. To support students' development of MP. 4 Model with mathematics, you might consider asking students how they could model or show this problem before letting them work on the *Solve & Share*.

Before moving onto the *Visual Learning*, ask students "If Carlos had 6 bags with 1 apple in each bag, how many apples would he have?" Continue to question students to develop a class conjecture about multiplying by 1. Confirm, clarify, and correct this conjecture during the *Visual Learning Animation*.

### Visual Learning:

Consider pausing the *Visual Learning Animation* after they introduce each property to have students test the property using counters and groups of other factors to confirm the stated property.

### Independent Practice/Math Practices and Problem Solving:

Consider assigning item 26 as this problem allows for multiple entry points and answers. When a student solves the problem, you can extend thinking by asking if that is the only answer. Students may give a generalized rule for the answer (any number greater than 4, as 4 is the minimum number needed to have a greater number of bikes than Barb's class).

# Assess and Differentiate/Intervention Activity:

If time permits, you may consider replacing *Problem Solving Reading Mat* with either the games from previous topics, the game *Quick Questions* (TE, p. 65A), or the *Fluency Practice Activity* (TE, p. 97).

Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.77A).

# Lesson 2-4: Multiply by 10

#### Students have been skip counting by 10 3.OA.A.3 since Kindergarten. In Topic 1 students 3.OA.A.1 used skip counting on the open number line 3.OA.D.9 to represent multiplication. MP.2 Developing the Big Idea: Students are *developing* their MP.3 understanding of patterns in multiplication MP.4 by identifying a pattern when multiplying by MP.7 10. MP.8

Access Prior Learning:

# Solve & Share:

Consider providing students with tools, such as two-colored counters or place-value blocks to solve the problem. Students may only write the products for each week which can make identifying a pattern for multiplying by 10 difficult to see. In this event, consider asking students to write out the equations they used to solve each week and to looks for patterns in the factors and products.

During the class discussion of students' solution methods and reasoning, push to get students to use place value reasoning to support their explanations of the justification offered in the *Transition to the Visual Learning Bridge* (TE, p. 79). For example, students may explain, "Since 6 x 10 means we have 6 groups of 10 miles we do not have any ones because 6 tens is 60 resulting in a 0 in the ones place for all products when 10 is a factor."

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Referring to 60, 70, and 80 as multiples of 10 will support students' understanding of the term multiples.

### Visual Learning:

Consider pausing the video after it asks, "How many miles will Greg run to train for the race?" Discuss as a class what operation is needed to solve this question. The video will ask students this question as it's showing the 10's times tables; however, at this point connect student responses to the 10s fact table. This will support student understanding of multiplication being 'groups of'. Avoid teaching students the "zero trick" of just adding a zero to the right and instead maintain the focus on patterns that appear when multiplying by 10.

Consider continuing to support students' understanding of the term "multiple(s)" by asking them to identify the multiples of 10 in the 10's times table.

# Independent Practice/Math Practices and Problem Solving:

Consider including item 15 so that students have the opportunity to revisit reasoning with repeated subtraction situations.

### Assess and Differentiate:

The *On-Level* and *Advanced Activity Centers* for this lesson includes 9 as a factor. Consider allowing students to play the game, but first challenge them to use what they know about multiplication with factors of 10 to develop a strategy for solving problems with 9 as a factor. Alternatively, you may wish to have students play a game from previous topics or lessons.

Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.83A).

\*CTC: Math Practices and Problem Solving (student work samples)

# Lesson 2-2: 9 as a Factor

3.OA.A.3 3.OA.A.1 3.OA.D.9

> MP.1 MP.2

MP.3 MP.7 Access Prior Learning:

In Topic 1 students came to understand multiplication as the joining together of equal groups. In *Lesson 2-4, Multiples of 10*, students identified a pattern for solving for multiplication problems with 10 as a factor.

# Developing the Big Idea:

In this lesson, students are *developing* their understanding of multiplication as joining equal groups and of multiples of 10 to generate a derived fact strategy for multiplication problems with 9 as a factor.

The purpose of this lesson is to explore the patterns that occur in multiplication and build mathematics curiosity and wonder at why these patterns occur. 9 as a factor reveals fascinating and unique opportunities to recognize and explore patterns beyond those that are found when looking at other factors, which may just reveal multiples or simple and easily recognizable patterns. Exploring 9 as a factor can push toward arithmetic patterns beyond those explored in earlier grades.

The pattern explorations should not be seen as tricks to help students memorize the 9's facts.

If using 2-2 after 2-4 then build opportunities for students to use **known** facts such as 10 x 4 (40) to **derive** a 9 fact such as 9 x 4 (I know that 10 groups of 4 is 40 so *one less group* of 4 will be 36). Moving from known to derived facts will be explored further in topic 3.

### Solve & Share:

Prior to the *Solve & Share* assess student readiness by asking students to state the meaning of the equation 6 x 10 from yesterday's *Solve & Share* (e.g. 6 x 10 means we have 6 groups of 10 for a total of 60) and asking what if Duke ran 9 miles. How many will he run in 6 weeks?

Consider asking a student that has direct-modeled the *Solve & Share* either with counters or pictorial representations *and* a student that used a 10 as a factor and completed a derived fact strategy to share.

Orchestrate a class discussion around these two solution methods and the reasoning used. As a class, consider how to model the math (MP.4) when using 10 as a factor. Connect the reasoning between the student's model who use a derived fact to model and that of the student that chose to direct model the  $4 \times 9 = 36$ .

#### Look Back:

Consider discussing the *Look Back!* problem revisit ideas about the Commutative (Order) Property of Multiplication.

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### Visual Learning:

Since the Visual Learning Animation is more procedural than conceptual, consider replacing the animation by writing the 10's and 9's times tables next to each other. Facilitate a discussion to help students identify a pattern for using multiples of 10 to solve problems with 9 as a factor. Students can generalize to discover a derived fact strategy for multiplication problems with 9 as a factor. For example:

| $1 \times 10 = 10$ | $1 \times 9 = 9$   |
|--------------------|--------------------|
| $2 \times 10 = 20$ | $2 \times 9 = 18$  |
| $3 \times 10 = 30$ | $3 \times 9 = 27$  |
| $4 \times 10 = 40$ | $4 \times 9 = 36$  |
| $5 \times 10 = 50$ | $5 \times 9 = 45$  |
| $6 \times 10 = 60$ | $6 \times 9 = 54$  |
| $7 \times 10 = 70$ | $7 \times 9 = 63$  |
| $8 \times 10 = 80$ | $8 \times 9 = 72$  |
| $9 \times 10 = 90$ | $9 \times 9 = 81$  |
| 10 x 10 = 100      | $10 \times 9 = 90$ |

See the *Instructional Note* at the beginning of this topic for an explanation of student reasoning of this strategy.

Consider modeling the multiple by 10, and subtract the extra group using Base-10 blocks or counters and connect to the array model to support student understanding.

If the Visual Learning Animation is replaced with the above activity, item 2 in the Guided Practice will need to be skipped or reworded to have students describe using a multiple of 10 to solve a 9s fact.

# Independent Practice/Math Practices and Problem Solving:

Notice that item 15 is the same equation students used in today's Solve & Share. This is an opportunity to see if students will recognize that they have worked with this problem already and it will therefore have the same product.

### Assess and Differentiate/Intervention Activity:

If time permits, teach students how to play Toss and Talk (TE, p. 83A). All students should have the opportunity to play this game.

Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.71A). For this activity, you can still do the Intervention Activity Modeling 9s Facts; however, consider replacing the worksheet with relating the models made to the finger strategy for finding multiples of 9.

# Lesson 2-5: Multiplication Facts: 0, 1, 2, 5, 9, and 10

#### Access Prior Learning: In previous lessons in Topic 2 students 3.OA.A.3 have identified patterns that can be used as 3.OA.A.1 strategies for solving for facts with 0, 1, 2, 3.OA.D.9 5, 9, and 10. MP.3

# Developing the Big Idea:

This lesson further develops the MP.4 understandings students began to MP.5 understand in the previous lessons to apply MP.6 the strategies for solving for facts with 0, 1, MP.7 2, 5, 9, and 10 as a factor.

# Solve & Share:

For the whole class discussion consider sequencing solution strategies so that the first student to share has a solution method that is similar to Stephanie's work (TE, p. 85). Consider having the second student to share be one that used solutions from having solved for the previous numbers of boxes to determine the products for the other boxes.

# Visual Learning:

As students likely did not use a bar diagram to find the solution to the Solve & Share, the Try It! provides the opportunity to work with modeling the math (MP.4) using a bar diagram. It may be helpful to discuss how the bar diagram is able to represent the joining of the equal groups.

# Assess and Differentiate:

If time permits, consider replacing *Problem Solving Reading Mat* with either the games from previous topics, the game Quick Questions, Toss and Talk, or the Fluency Practice Activity (TE, p. 97).

Child-watch to identify students who need additional support and pull them into a small group to complete the Intervention Activity (TE, p.23A).

\*CTC: Quick Check (digital platform)

# Lesson 2-6: Math Practices and Problem Solving- Model with Math

#### Access Prior Learning: In this topic students have identified 3.OA.A.3 patterns that can be used as strategies for solving for facts with 0, 1, 2, 5, 9, and 10. Lesson 1-7 involved students solving for a MP.4 2-step word problem. Students also worked MP.1 with 2-step word problems involving MP.2 addition and subtraction and the idea of a

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 4. Refer to the Math Practices and Problem Solving Handbook (TE p. F24-F24A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also, reference the handbook in the student edition (SE, p. F24).

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MP.3 MP.5 "hidden problem" in 2<sup>nd</sup> grade Topics 8, 13 & 14.

# Developing the Big Idea:

In this lesson, students are *developing* their understanding of multiplication as joining equal groups and the use of patterns for multiplying with 0, 1, 2, 5, 9, and 10.

#### Solve & Share:

Consider reintroducing MP. 4 Thinking Habits (SE, p. F24) before introducing the *Solve & Share*. Restating that an equation is an example of MP.4 Modeling the Math can be a good reminder. Many students are under the misconception that MP. 4 means they must show a drawing or concrete representation of the math. While having a drawn or concrete representation of the math can make for a stronger argument (MP.3), it is not necessary for modeling mathematical situations.

Consider using the time when students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.4 that are listed in the *Math Practices and Problem Solving Handbook* (p. F24A). After discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice.

#### Convince Me:

Consider assigning the *Convince Me!* as it offers another opportunity to work with MP.4 and assess for behaviors attributed to this math practice.

### Assess and Differentiate:

If time permits, consider assigning the *Math and Science Activity* (TE, p. 95A) as this relates the mathematics in this topic to a real world context.

Child-watch to identify students who need additional support and pull them into a small group to complete the *Intervention Activity* (TE, p.95A).

# References

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