Big Conceptual Idea: Operations and Algebraic Thinking, K-5 (pp. 22-28)
Prior to instruction, view the Topic 3 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 105A-105F), the Topic Planner (pp.105l-105K), all 8 lessons, and the Topic Performance Assessment (pp. 163-164A).

Mathematical
Read Topic 3-4 Cluster Overview/Math Background (pp. 105A-105F)

Topic Essential Question:
How can unknown multiplication facts be found using known facts?

Reference Answering the Topic Essential Question (TE, pp. 161-162) for key elements of answers to the Essential Question.


The lesson map for this topic is as follows:

| $3-1$ | $3-2$ | $3-3$ | $3-4$ | $3-5$ | $3-6$ | $3-8$ | $3-7$ | Assessment |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Instructional note:

The Nevada Academic Content Standard (NVACS) cluster 3.OA.B states, "Understand properties of multiplication and the relationship between multiplication and division". In Topic 3, students work with the Properties of Operations to build reasoning strategies using known facts explored in Topic 2. Students do this by multiplying with $3,4,6,7$, and 8 as factors. Students will continue to work with multiplication using contextual problems (3.OA.A.3) and explore patterns that occur with factors and products (3.OA.D.9). Building understanding of multiplication strategies, including using the Distributive and Associative Properties will lead to fluency later in grade 3 (See Topic 5).

The Topic 3 Professional Development Video states, "Using known facts along with the properties of multiplication is a strategy for learning the multiplication facts for $3,4,6,7$, and 8 " (Dr. Schielack, enVisionmath2.0, 2016). This topic introduces the Distributive and Associative Properties of Multiplication with a focus on the standard 3.OA.B.5, "Apply properties of operations as strategies to multiply and divide." (NVACS, p. 23). While grouping symbols are not explicitly stated in the standards until 5 th grade, the use of parentheses is an assumed part of the mathematics in the properties and should be used to communicate the grouping of the expressions and thus the order of the operations. For more information about the use of parentheses in 3 rd grade and the progression of Order of Operations please read page 27 of the K-5, Operations and Algebraic Thinking progression document. The footnote on this standard indicates that students do not need to use the formal terms for these properties; therefore, it is acceptable for students to refer to them as the turn-around, break-apart, and order properties of multiplication. However, you may want to consider restating their informal language with the formal terms to support precise mathematical vocabulary development.

Topic 3 uses the Distributive Property of Multiplication extensively to support student understanding by linking an array with the decomposition of a factor. Students model decomposing a factor into smaller factors to breaking apart larger arrays into smaller arrays. For example, given an array that models $7 \times 5$, students may choose to decompose or break the first factor ( 7 ) into a 5 and 2 as they know their 5 facts and their 2 facts. Thus, students are using the distributive property to solve unknown facts using known facts: $(5 \times 5)+(2 \times 5)$.

In lesson 3-7, the focus is on understanding the Associative Property, which allows factors to be grouped in different ways. When given three or more factors students are able to group the factors differently depending upon what is more efficient for the student. For example, given the factors $2 \times 5 \times 3$ (which can be modeled with two separate $5 \times 3$ arrays), students may group the factors as ( $2 \times 5$ ) $x 3$ or as $2 \times(5 \times 3)$. They should see the equivalence between these groupings and the new facts created by associating different factors together; in this case $10 \times 3$ or $2 \times 15$. Consider using one of the A/D/E days for this topic to spend more time exploring these ideas from the Associative Property of Multiplication.

When students demonstrate readiness, consider replacing the two-colored counters used in arrays with colored tiles to begin building area models. This will begin to lay the foundation for the connection between the array model and the area model explored in Topic 6. Note: If colored tiles are used, be sure that there are no gaps between tiles as you are now connecting to area concepts.

Looking ahead to the Topic Assessment, consider having tools available for students that may need them. Item 11 Part B requires students to generalize their understanding and apply it to a new situation. In the Topic Performance Assessment for item 2 Part B, accept multiple answers for where students draw the line; also accept responses where students have drawn multiple lines to use the Associative Property of Multiplication.

## Focus Math Practice 8: Look for and express regularity in repeated reasoning

Focus on opportunities for students to develop Mathematical Practice 8 behaviors, as this is the focus of the Math Practices and Problem Solving lesson 3-8. Reference the Teacher's Edition (TE, pp. F28-F28A) and the Nevada Academic Content Standards for Mathematical Practice (2010, p. 8).

| Essential Academic Vocabulary <br> Use these words consistently during instruction. |  |
| :--- | :--- |
| New Academic Vocabulary: <br> (First time explicity taught) | Review Academic Vocabulary: <br> (Vocabulary explicitly taught in prior grades or topics) |
| Distributive (break-apart) Property of | factor |
| Multiplication | product |
| Associative (Grouping) Property of | commutative property of <br> Multiplication |
|  | multiplication <br> doubles <br> halving |

Additional terminology that students may need support with: break-apart, addend, sum, compose, decompose, generalization

## *Collaborative Team Conversations (CTC)

Consider using one of the following as part of the formative assessment process at the lesson level to collect student work to analyze for evidence of mathematical understanding:

Guiding question: "Are students using the properties of multiplication and known facts to find products of unknown facts?"

| Lesson | Evidence | Look for |
| :---: | :--- | :--- |
| $3-1$ | Solve \& Share <br> (student work samples) | Focus CTC around the big idea: <br> $\bullet \quad$students understand that arrays can be broken apart (decomposed) into <br> smaller arrays based on the distributive property. <br> $3-3$Math Practices and Problem Solving <br> (student work samples) <br> Items 22 and 23 |
| Focus CTC around the big idea: <br> $\bullet \quad$ students apply the properties of multiplication with 4 as a factor. <br> $\bullet \quad$ students use known facts to find products of unknown facts. |  |  |


| Learning Cycle | Topic Assessments | Use Scoring Guide TE pp. 161-164A |
| :---: | :--- | :--- |
| SE pp. 161-164 |  |  |


| NVACS <br> (Content and Practices) | Mathematical Development of the Big Idea | Instructional Clarifications \& Considerations |
| :---: | :---: | :---: |
| Lesson 3-1: The Distributive Property |  |  |
| 3.OA.B. 5 <br> MP. 2 <br> MP. 3 <br> MP. 4 <br> MP. 6 <br> MP. 7 | Access Prior Learning: Commutative Property of Multiplication, familiarity with facts that include $2,5,0,1,10$, and 9 as a factor. <br> Developing the Big Idea: Students are beginning to understand that the Distributive Property of Multiplication can be used to break a large array into a small array that represents known facts. | Topic Opener: <br> Introduce the Topic Essential Question, "How can unknown multiplication facts be found using known facts?" (TE, p. 105). Consider making this an anchor chart in your classroom where each day new ideas are added so that students can see the development and make connections throughout the topic. If building an anchor chart, this could be an appropriate time to quickly revisit the Commutative Property of Multiplication and add it to the chart. You may want to expand on this idea and include in the chart that since they already know $2 \times 7=14$ they also know that $7 \times 2=14$. <br> You might also consider having students complete the Review What You Know prior to beginning instruction on Topic 3 so that you can respond to student instructional needs using the Item Analysis for Diagnosis and Intervention (TE, p. 106-108). <br> Consider introducing vocabulary as they encounter them in the lessons rather than introducing all terms at the beginning of the lesson. |

$\left.\begin{array}{|l|l|l|l|}\hline & & \begin{array}{l}\text { Solve \& Share: } \\ \text { Consider waiting to distribute the } 25 \text { two-color counters until students suggest them in response } \\ \text { to the question, "What tool can you use to solve this problem?" (TE, p. 109). }\end{array} \\ \text { While students are working on the Solve \& Share consider asking them: } \\ \text { e "What equation does the original array represent?" (e.g. } 5 \times 4)\end{array}\right\}$


|  |  | Visual Learning: <br> The Visual Learning Animation makes one strategy explicit for solving for a 6 fact. This strategy will also work for 7 facts but the animation does not make this connection. Therefore, you may want to ask students how they could use this strategy to solve for 7's facts. <br> Alternatively, you may consider replacing the Visual Learning Animation with the Another Look! video as it makes the strategy explicit for both 6 \& 7's facts. If choosing to go this route, consider pausing after it displays the equations " $6 \times 4=$ ?" to collect student responses on how they might solve using known facts (e.g. double-double with the four so $(6 \times 2)+(6 \times 2)=24$, double-double with the six so $(3 \times 4)+(3 \times 4)$, or using 2 s facts so $(2 \times 4)+(2 \times 4)+(2 \times 4)=$ 24 , using 3 s facts so $(6 \times 3)+(6 \times 1)=24)$. Consider asking students if doubling will work for 7 (e.g. no because 7 is not a double). You may want to build in additional class practice with counters in modeling and writing the equations for these strategies. <br> Assess and Differentiate/Intervention Activity: <br> If time permits, teach students how to play Teamwork (TE p. 131A). All students should have the opportunity to play this game as this reinforces the idea of using known facts to solve for unknown facts and provides meaningful practice with identifying the known fact and working from that fact. Consider modifying to other factors (e.g. have students role dice to get the factors) for future extended play. |
| :---: | :---: | :---: |
| Lesson 3-5: Apply Properties: 8 as a Factor |  |  |
| 3.OA.B. 5 <br> 3.OA.A. 3 <br> 3.OA.D. 9 <br> MP. 1 <br> MP. 4 <br> MP. 7 <br> MP. 8 | Access Prior Learning: <br> In Topic 2, Grade 3 students identified the patterns in multiplying with 1,2 , and 5 as a factor. In previous lessons within Topic 3, students have developed the understanding that they can use these known facts to solve for facts with $3,4,6$, and 7 as a factor. <br> Developing the Big Idea: <br> This lesson further develops the idea that we can use known facts to solve for unknown facts using the Distributive Property of Multiplication. | Look Back: <br> Consider assigning the Look Back! prompt after the Solve \& Share, and ask groups to take a different factor to share out. These are good ideas to add to the class anchor chart. <br> Visual Learning: <br> Students that are struggling to keep track of all the doubles shown in the Visual Learning Animation may prefer to use the Distributive Property of Multiplication by decomposing the 8 into $5+3$. This also might be a good time to revisit the Commutative Property of Multiplication by posing the question, "For $8 \times 2$, what is the most appropriate/efficient strategy for me to solve (e.g. I know $2 \times 8=16$ so it's 16)?" <br> Convince Me: <br> You might consider discussing the Convince Me! whole group. <br> Independent Practice/Math Practices and Problem Solving: <br> For more information beyond the explanation provided in the Teacher's Edition on Quick Check item 19, watch the Listen and Look For video for this lesson. <br> Assess and Differentiate/Intervention Activity: <br> If time permits, you may consider replacing the Math and Science Activity with the game <br> Teamwork (TE, p. 131A). Please see the comments for this game in Lesson 3-4. |
| Lesson 3-6: Practice Multiplication Facts |  |  |
| 3.OA.B. 5 <br> 3.OA.A. 3 <br> MP. 1 <br> MP. 2 <br> MP. 3 <br> MP. 4 | Access Prior Learning: In Topic 2, Grade 3 students identified the patterns in multiplying with 1,2 , and 5 as a factor. In previous lessons in this topic students have developed the understanding that they can use these known facts to solve for facts with $3,4,6,7$, and 8 as a factor. <br> Developing the Big Idea: <br> This lesson further develops the understanding that we can use known facts to solve for unknown facts by including using a bar diagram as a model for the math. | Solve \& Share: <br> To assess student readiness, you may consider posing the question, "How can we model multiplication using a bar diagram? What other ways can we model multiplication?" <br> Visual Learning: <br> Consider pausing the video after it shows, "Each section is 3 feet long" and posing the question, "What is our multiplication equation for this problem? (e.g. $9 \times 3=$ ?) How could you solve this?" Provide time for students to solve and use this as an opportunity to see what reasoning strategies students are using to solve. <br> Convince Me: <br> If you already provided time for students to solve for $9 \times 3$, you might consider doing the Convince Me! to have students share their strategies and reasoning that wasn't shown in the video. Having exposure to these strategies would be beneficial for the whole group. <br> Independent Practice/Math Practices and Problem Solving: <br> Students have to reason that a week has 7 days in order to solve item 26; therefore, it might be beneficial to assign this item to build problem solving reasoning habits. <br> Assess and Differentiate: <br> If time permits, you may consider replacing the Problem Solving Read Mat with the game Teamwork (TE, p. 131A). Please see the comments for this game in Lesson 3-4. <br> Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.23A). |

Lesson 3-8: Math Practices and Problem Solving- Repeated Reasoning

Access Prior Learning:
3.OA.B. 5

In this topic students have
3.OA.A. 3
developed understanding of how they can use known facts for 1,2 , and 5 to solve for unknown facts
MP. 8 MP. 1
MP. 3
MP. 5
MP. 7 for $3,4,5,6,7,8$, and 9 focusing mostly on the Distributive Property of Multiplication as a justification for why this works.

## Developing the Big Idea:

In this lesson, students develop their understanding of MP. 8 Use repeated reasoning to secure their understanding of using the Distributive Property of Multiplication as a strategy for solving unknown facts.

This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 9 . Refer to the Math Practices and Problem Solving Handbook (TE p. F28-F28A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the student edition (SE, p. F28).

## Solve \& Share:

Consider reintroducing MP. 8 Look for and express regularity in repeated reasoning Thinking Habits (SE, p. F28) before introducing the Solve \& Share. You may want to restate that an equation is an example of MP. 8 Look for and express regularity in repeated reasoning. In this case, the general method we want students to notice is the use of the Distributive Property of Multiplication in solving for unknown facts.

You may also consider using the time where students are working on the Solve \& Share as an opportunity to child-watch for behaviors associated with MP. 8 that are listed in the Math Practices and Problem Solving Handbook (F28A), and afterwards discussing student solution methods and reasoning. Ask students to self-score for the behaviors associated with this math practice.

Finally, during the whole group discussion on students' solution strategies and reasoning, ensure that the generalization recognizes that when decomposing a factor, it's being broken into addends. For further ideas on how to facilitate the conversation so that students recognize this, preview the Listen and Look For video prior to teaching the lesson so that you can use their questions. This understanding is important as students learn to decompose factors and make known facts from unknown facts.

## Visual Learning:

During the Visual Learning Animation, consider asking the students what they decomposed the factors for each equation into to make a known fact (e.g. A 3 was decomposed into the addends 2 and 1, B 4 was decomposed into the addends 2 \& 2, C 6 was decomposed into the addends 5 and 1, D 7 was decomposed into the addends 5 and 2).

## Independent Practice/Math Practices and Problem Solving:

Watch for students that do not recognize the " 35 minutes to bake the pizzas" is extraneous information and try to use it to solve the problem. These students are not reasoning with the context to make sense with the mathematics and need additional support on how to problem solve.

## Assess and Differentiate/Intervention Activity:

If time permits, teach students how to play Clip and Cover (TE, p. 155A). All students should have the opportunity to play the games.

Child-watch to identify students who need additional support and pull them into a small group to do the Intervention Activity (TE, p.155A).

## Lesson 3-7: The Associative Property: Multiply by 3 Factors

## 3.OA.B. 5 <br> 3.OA.A. 3

In Topic 2, Grade 3 students identified the patterns in multiplying with 1,2 , and 5 as a factor. In previous lessons in this topic,
MP. 1
MP. 2
MP. 3
MP. 4
MP. 8

Watching the Topic Professional Development Video will help to clarity the ideas around the Associative Property of Multiplication and strengthen facilitation of the Visual Learning Animation during instruction.

## Solve \& Share:

The Listen and Look For video for this lesson goes into detail about how to use this problem to facilitate understanding of the Associative Property of Multiplication.

While students work to solve this problem, watch for those that do not include parentheses as a grouping symbol. These students might be working with the misconception that parentheses are limited to the Distributive Property of Multiplication. Offer the clarification that parentheses are used anytime we want to communicate that we have grouped expressions together.

Watch for students that represent the problem as $(5 \times 3)+(5 \times 3)$ or $15+15$. Help them connect these forms of repeated addition to multiplication; ask students to write a number sentence for this situation using only multiplication. $2 \times 15$ could be a first step. Where did the 15 come from? Do students notice it represents the total number of squares in one quilt? Can they replace the 15 with ( $5 \times 3$ ) to create the number sentence $2 \times(5 \times 3)$ ? Help students to see the equivalence between these different forms. What will happen now if we decide to use the 2 $x(5 \times 3)$ model but instead associate the factors 2 and 5 ? We have created a new number sentence, $(2 \times 5) \times 3$ and a different number sentence of $10 \times 3$. Do students notice that there are 10 rows between the two quilts with 3 squares in each row? The associative property allows us to represent the same situation using different but equivalent number sentence models.

|  |  | Visual Learning: <br> The Visual Learning Animation uses a very similar problem to model use of the Associative <br> Property of Multiplication. Help students make connections between their reasoning and <br> strategies used during the Solve \& Share and those seen in the Visual Learning Animation. |
| :--- | :--- | :--- |
| In this lesson, students have seen that when working with 3 factors, the order they multiply <br> them does not change the final product although it does require an extra step. Connect to our <br> Topic Essential Question, "How can unknown multiplication facts be found using known facts?" <br> by asking, "How can we use the Associative Property of Multiplication to decompose an <br> unknown multiplication fact into known facts? Can we solve $4 \times 2 \times 5$ as $8 \times 5$ or $4 \times 10$ ? How <br> can we use this property to make simpler problems for solving?" |  |  |
| Assess and Differentiate: <br> If time permits, you may consider returning to the game Teamwork (TE p.131A) with a variety of <br> factors (see notes in Lesson 3-4) and have students decomposing the larger arrays into 2 or <br> more small arrays while writing the multiplication equation modeled. |  |  |

## References

Common Core Standards Writing Team. (2011, May 29). Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards Instructional Support/Nevada Academic Standards/Math Doc uments/mathstandards.pdf.

Fosnot, C. T., \& Dolk, M. (2001). Young mathematicians at work: Constructing multiplication and division. Portsmouth, N.H.: Heinemann.
Van de Wall, J., Karp, K., Lovin, L., \& Bay-Williams, J. (2014). Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5 (2nd ed.). New York, NY: Pearson.

This page is intentionally left blank

