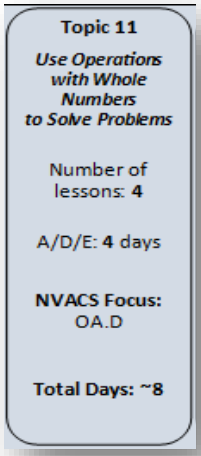
Grade 3 Topic 11: Use Operations with Whole Numbers to Solve Problems



Big Conceptual Idea: Operations and Algebraic Thinking, K-5 (pp. 22-28)

Prior to instruction, view the Topic 11 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 571A-571F), the Topic Planner (pp.57II-571J), all 4 lessons, and the Topic Assessments (pp. 603-604A).

Mathematical Background: Read Topic 11 Cluster Overview/Math Background (TE, pp. 571A-571F)	Topic Essential Question: What are ways to solve two-step problems? *Reference Answering the Topic Essential Question (TE, pp. 601-602) for key elements of answers to the Essential Question.*

The lesson map for this Topic is as follows:

11-1	11-2	11-3	11-4	Assessment

4 A/D/E days used strategically throughout the topic.

3rd Grade Curriculum Pacing Framework: Balanced Calendar

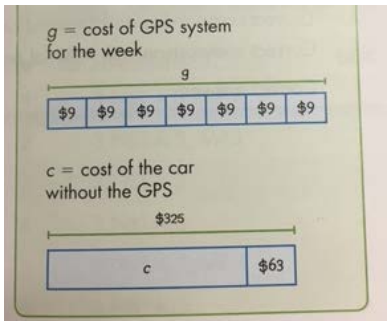
Instructional note:
In Topics 1 through 5, students developed conceptual understanding of multiplication and division. In Topics 6 & 7, students applied this understanding of multiplication to area and data concepts. In Topics 8 & 9, students further developed their understanding of addition and subtraction concepts. All of this work developed operational sense.

In Topic 11 students further develop their operational sense by, "Solving problems involving the four operations…" (NVACS, 3.OA.D). Topic 11 provides students the opportunity to become more secure in this understanding as they solve two-step word problems using the four operations, identify arithmetic patterns and explain them using the properties. The focus in this topic is on algebraic thinking using, "real-world situations that can be represented using variables, operations, and equations" (TE, p. 571C). In supporting students with developing algebraic thinking, it is crucial that the algebraic language of an unknown and what the unknown represents is addressed through classroom discussion.

Third grade is the first time students start representing the unknown with a letter variable, in previous grades the unknown was represented with a box or symbol. Students should make the connection that a letter is equivalent to a question mark or an open box (Small, 2014).

> "Students need to be aware that any letter they choose is acceptable, and no one letter is preferable to another. Many teachers and students advocate using a letter that helps the student remember what the value represents. For example, in the problem "There were 24 students at 4 tables. The same number of students was at each table. How many were at each table?" students might use *s* in the equation 24 ÷ 4 = *s* to represent students in the problem. *Many* students just pick any word in the problem to suggest a letter and might use *t* from table. This, again, is not incorrect, but in the end it is critical that the students understand what their answer represents—the number of students at each table, not the number of tables" (Small, 2014, p. 31).

In this topic, students identify what the unknown is and represent it with a variable in both models and equations. To support students as they reason with two-step word problems **enVision**math2.0 asks students, "What is the hidden question?" referring to the first step that must be answered before the stated problem can be solved for. Students have worked with the idea of a "hidden question" since 2nd grade and have revisited it in previous topics.



This topic extensively uses the bar diagram to model the two-step problems, often encouraging the use of a bar diagram for each step. Encourage students to make the area of each part of the part-part-whole bar diagram representative to the quantity of the numbers that part is representing. **enVision**math2.0 does a great job of modeling this for the students. In doing this, students are supported in developing magnitude of number, estimation skills (especially when the unknown is one of the parts), and being able to discern an additive situation from a multiplicative situation. As shown in the included image (TE, p. 571A), a multiplicative situation will have equal sized parts, whereas, an additive situation may have different size parts.

As students' algebraic thinking develops, they will discover that some multi-step problems are solved using different operations. As a result of this idea, students will revisit the conventions for order of operations by writing a single equation that represents the multiple steps taken to solve the problem (lesson 11-3). The Operations and Algebraic Thinking progression document provides more insight into the conventions of order of operations in regards to third grade:

Understanding and using the Associative and Distributive Properties (as discussed above) requires students to know two conventions for reading an expression that has more than one operation:

1. Do the operation inside the parentheses before an operation outside the parentheses (the parentheses can be thought of as hands curved around the symbols and grouping them).

2. If a multiplication or division is next to an addition or subtraction, imagine parentheses around the multiplication or division (it is done before these operations). In Grades 3 through 5, parentheses can usually be used for such cases so that fluency with this rule can wait until Grade 6.

These conventions are often called the Order of Operations and can seem to be a central aspect of algebra. But actually they are just simple "rules of the road" that allow expressions involving more than one operation to be interpreted unambiguously and thus are connected with the mathematical practice of communicating precisely (MP.6). Use of parentheses is important in displaying structure and thus is connected with the mathematical practice of making use of structure (MP.7). Parentheses are important in expressing the associative and especially the distributive properties. These properties are at the heart of Grades 3 through 5, because they are used in the Level 3 multiplication and division strategies, in multi-digit and decimal multiplication and division, and in all operations with fractions (pp. 27-28).

While we are not expecting security on these conventions and it is not assessed in our Topic Assessment, lesson 11-3 focuses on bullet 2 of these conventions. Consider modeling how to write equations used in the *Solve & Share* and *Visual Learning* for all the lessons in this topic following these conventions.

Focus Math Practice 3: Construct viable arguments and critique the reasoning of others

The standard state, "They (students) make conjectures and build a logical progression of statements to explore the truth of their conjectures…. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and -if there is a flaw in and argument- explain what it is" (NVACS, 2010, p. 6-7). Behaviors associated with MP.3 are described in the Teacher's Edition (pp. F23 - F23A) and the Nevada Academic Content Standards for Mathematical Practice.

Looking ahead to the Topic Performance Assessment, item 4 Part A requires students to use their understanding of estimation and number sense to assess the reasonableness of a fictitious teacher's argument. To support students' development of the mathematical understandings needed to respond to this question consider frequently asking students if a solution is reasonable and why. For students to be successful with this assessment they will have to attend to **all** the information provided in the questions. In item 3, the word "all" makes a large impact on what students need to do to demonstrate what they know.

Therefore, throughout this topic continue to ask students if they have answered **all** of the problem and how they know. While developing the thinking habits that allow students to engage in this problem type are highly beneficial, you may need to scaffold working with the Topic Performance Assessment to help students develop these thinking habits.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
	equation *product* *quotient* *sum* *difference* *addend* *factor* *dividend* *divisor*

Additional terminology that students may need support with: *unknown, hidden question*

*Collaborative Team Conversations (CTC)

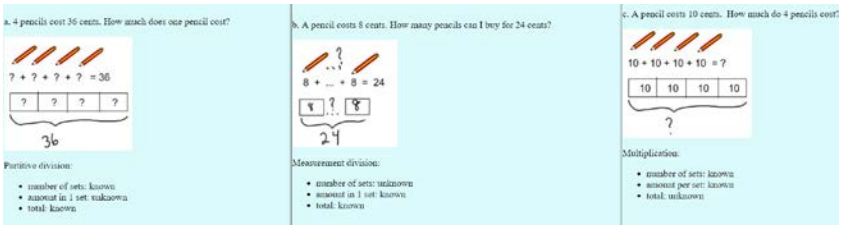
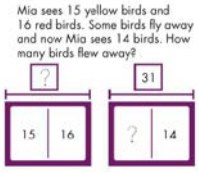
Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for evidence of mathematical understanding:

Guiding question: "Are students making sense of the problem and using appropriate operations to solve multi-step word problems?"

Lesson	Evidence	Look for
11-1	*Solve & Share* (student work samples)	Focus CTC around the big idea: • look for students who explain the properties. • use information from table to accurately solve the problem. • all parts of multi-step problem are answered.
11-3	*Quick Check* (digital platform) Items 1, 4, and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources". • all parts of multi-step problem are answered. • make sense of problems.

Learning Cycle Assessments (summative)	*Topic Assessments* SE pp. 601-604	Use *Scoring Guide* TE pp. 601-604A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 11-1: Solve 2-Step Word Problems- Addition and Subtraction		
3.OA.D.8 MP.1 MP.2 MP.3 MP.4	Access Prior Learning: Throughout this year, students have had the opportunity to solve 2-step word problems with multiple operations and reasoned with the "hidden question". In Topics 8 & 9 students solved addition and subtraction problems. Developing the Big Idea: Students are further *developing* their understanding of 2-step word problems and operational sense by using bar diagrams and equations to model with math. Since bar diagrams represent information in the problem, students discover that bar diagrams can help them better understand what operation(s) to use.	Topic Opener: Introduce the *Topic Essential Question*, "What are ways to solve two-step problems?" (TE p. 571). Consider making this an anchor chart in your classroom. As new ideas are added each day, students can see the development of learning and make connections throughout the topic. You might also consider having students complete the *Review What You Know* prior to beginning instruction on Topic 11 so that you can respond to students' instructional needs using the *Item Analysis for Diagnosis and Intervention*. Consider introducing vocabulary terms as they are encountered in the lessons rather than introducing all terms at the beginning of the lesson. Solve & Share: To assess students' readiness, consider reviewing (students may have done this in lesson 4-1) what is known and unknown in a multiplication bar diagram versus a division bar diagram, as well as, creating bar diagrams for what is known and unknown for addition and subtraction. Multiplication/Division known/unknown bar diagram examples:  Addition/Subtraction known/unknown bar diagram examples:  The first bar diagram models addition, the second bar diagram models subtraction (TE, p. 571C). For more information on bar diagrams with the different addition/subtraction problem types go to: http://langfordmath.com/ECEMath/BasicFacts/PartWholeDiagramsText.html. *-continues on next page-*

		If students do not offer a solution method that is the same as "Diana's Work", then consider discussing "Diana's Work" as a class. Notice that Diana uses a bar diagram to solve for both the hidden question and the final question. Look Back: Consider discussing the *Look Back!* prompt to help students develop habits of estimation to determine reasonableness of solutions. Visual Learning: Read the *Instructional Note* at the beginning of this topic for information on supporting students as they transition from using a box or symbol to represent the unknown to using a letter variable. During the *Visual Learning Animation* consider pausing during step one and step two to discuss how to solve. Also considering pausing and discussing after they pose the question, "Why is the letter *y*, instead of letter *x*, used to represent the unknown quantity in step two?" Convince Me: Consider discussing the *Convince Me!* prompt to help students develop habits of estimation to determine reasonableness of solutions. Assess and Differentiate: Child-watch to identify students who need additional support and consider the *Intervention Activity* provided. *CTC: *Solve & Share* (student work samples)
Lesson 11-2: Solve 2-Step Word Problems- Multiplication and Division		
3.OA.D.8 MP.1 MP.2 MP.3 MP.4 MP.6 MP.8	Access Prior Learning: Throughout this year, students have had the opportunity to solve 2-step word problems with multiple operations and reasoned with the "hidden question". In Topic 5, students solved multiplication and division problems. In the previous lesson students student discovered that bar diagrams can help them better understand what operation(s) to use. Developing the Big Idea: Students are further *developing* their understanding of 2-step word problems and operational sense by using bar diagrams and equations to model with math.	Solve & Share: Consider reviewing the differences in the known/unknown bar diagrams charts prior to introducing the *Solve & Share*. Since bar diagrams represent information in the problem, students discover that bar diagrams can help them better understand what operation(s) to use. Look Back: Consider discussing the *Look Back!* prompt to continue to support students' development of operational sense. Visual Learning: Considering pausing after being asked, "If each region didn't have the same number of teams, could you use division to find the number of teams? Why or why not?" This will allow time for students to discuss before the answer is given. Assess and Differentiate: If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the Fluency Practice Activity (TE, p. 597). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided.
Lesson 11-3: Solve 2-Step Word Problems- All Operations		
3.OA.D.8 MP.1 MP.2 MP.3 MP.4 MP.7	Access Prior Learning: In lesson 11-1, students solved 2-step word problems with addition and subtraction, in 11-2 students solved 2-step word problems with multiplication and division, in both lessons students reasoned with the "hidden question". Developing the Big Idea: Students are further *developing* their understanding of 2-step word problems and operational sense with all 4 operations.	Instructional Note: Read the *Instructional note* at the beginning of this document regarding the conventions for order of operations. Solve & Share: After introducing the *Solve & Share*, consider asking the questions provided in *Build Understanding* to make sure students are able to accurately read and interpret the picture graph. Look Back: Consider asking the *Look Back!* prompt to continue to build students' operational sense and MP.3. Visual Learning: Consider pausing after the first question, to allow students to make an estimation. *-continues on next page-*

		After viewing the *Visual Learning Animation,* consider asking students how they could write a single equation for the *Solve & Share* (e.g. 9 x 5 + 75). Independent Practice/Math Practices and Problem Solving: Consider assigning item 9 *Number Sense* to practice estimation using compatible numbers. Assess & Differentiate: Child-watch to identify students who need additional support and consider the *Intervention Activity* provided (TE, p. 589A). *CTC: *Quick Check* (digital platform)
Lesson 11-4: Math Practices and Problem Solving- Critique Reasoning		
3.OA.D.8 MP.3 MP.1 MP.2 MP.5 MP.6	Access Prior Learning In lesson 11-3 students continued to develop their understanding of 2-step word problems and operational sense with all 4 operations and MP.3 as they justified the operations they used. Developing the Big Idea In this lesson, students further *develop* their understanding of MP. 3 *"Construct viable arguments and critique the reasoning of others"* using all 4 operations to justify conjectures.	This lesson provides an opportunity to focus on the Thinking Habits and display the behaviors associated with Math Practice 3, *"Construct viable arguments and critique the reason of others*." Refer to the *Math Practices and Problem Solving Handbook* (TE, pp. F23-F23A, F29) for suggestions on how to develop, connect and assess this Math Practice. Also reference the handbook in the Student Edition (SE, p. F23). Solve & Share: Consider reintroducing MP. 3 Thinking Habits (SE, p. F23) before introducing the *Solve & Share*. Watch for students that agree with Skip's reasoning and support by asking the questions provided in *Ask Guided Questions as Needed*. Also consider using the time where students are working on the *Solve & Share* as an opportunity to child-watch for behaviors associated with MP.3 that are listed in the *Math Practices and Problem Solving Handbook* (p. F23A), and after discussing student solution methods and reasoning, have students self-score for the behaviors associated with this math practice. Look Back: Consider facilitating a whole class discussion using the *Look Back!* question to support students' mathematical reasoning skills and place value understandings. Visual Learning: Consider pausing the animation to discuss Danielle's reasoning. Convince Me: Consider assigning the *Convince Me!* as it provides an opportunity for students to reason more with MP.3 by supporting a conjecture. Assess & Differentiate: If time permits, you may consider replacing the *Math and Science Activity* with games from previous topics or the *Fluency Practice Activity* (TE, p. 597). Child-watch to identify students who need additional support and consider the *Intervention Activity* provided.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf

Langford, L. (n.d.). Multiplication Divison Models. Retrieved from http://langfordmath.com/ECEMath/Multiplication/MultDivModelsText.html

Langford, L. (n.d.). Part Whole Diagrams. Retrieved from http://langfordmath.com/ECEMath/BasicFacts/PartWholeDiagramsText.html

Small, M. (2014). *Uncomplicating algebra to meet common core standards in math, K-8.* New York, NY: Teachers College Press, Nelson Education.