Grade 5 Topic 10: Understand Volume Concepts

Topic 10
Understand Volume Concepts
Number of lessons: **6**
A/D/E: **5** days
NVACS Focus: MD.C
Total days: ~11

5th grade Curriculum Pacing Framework: Balanced Calendar

Big Conceptual Idea: Geometric Measurement (pp. 26-28)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 583A-583F), the Topic Planner (pp. 583I-583J), all 6 lessons, and the Topic Assessments (pp. 629-630A).

Mathematical Background: Read Topics 10 Cluster Overview/Math Background (TE, pp. 583A-583F)	Topic Essential Question: What is the meaning of volume of a solid? How can the volume of a rectangular prism be found? *Reference Answering the Topic Essential Questions (TE, pp. 627-628) for key elements of answers to the Essential Question.*

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	10-5	10-6	Assessment

5 A/D/E days used strategically throughout the topic

Instructional Note:

Instruction is focused on Nevada Academic Content Standards (NVACS) cluster 5.MD.C; "Geometric measurement; understand concepts of volume and relate volume to multiplication and to addition," (2010). This cluster is composed of eight individual standards:

- 5.MD.C.3- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- 5.MD.C.3.a- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- 5.MD.C.3.b- A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
- 5.MD.C.4- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.C.5- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 5.MD.C.5.a- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- 5.MD.C.5.b- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- 5.MD.C.5.c- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Volume can be a very challenging concept for fifth graders and much is asked by the eight standards for this cluster. As stated in the Progression Documents for the Common Core Math Standards;

> "Volume not only introduces a third dimension and thus a significant challenge to students' spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container," (Common Core Standards Writing Team (CCSWT), 2012 p. 26).

How can we provide the opportunities students need to form a conceptual understanding of volume that will transfer to real world problem solving? The Progression Documents and researchers agree that students will need experiences with concrete tools that allow them to extend their spatial understanding to include a third dimension. Caldwell claims, "Building prisms with unit cubes is an important step in building an understanding of volume as an attribute of a three dimensional shape that describes the space inside it," *(Topic 10 PD video enVisionmath2.0).* From these experiences, students "learn to mentally decompose and recompose a right rectangular prism built from cubes into layers…given the prism, they have to be able to decompose it, understanding that it can be partitioned into layers, and each layer partitioned into rows, and each row into cubes" (CCSWT, 2012, p.26).

The *Solve and Share* problem in the first lesson of Topic 10 shows a picture of a 4x1x1 rectangular prism. Will students connect this two dimensional representation to a real world three dimensional figure? Building the shape allows students the experience that will push their thinking into three dimensions. The *Visual Learning Bridge* in the same lesson will add a layer to this prism. This can also be modeled with physical tools to help students observe the connection between surface area and volume. Understanding that volume can be decomposed into layers is very important to understanding the additive nature of volume; as well as, applying volume formulas (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Conceptual understanding of volume builds towards understanding volume formulas. This occurs as student reasoning develops and increases in sophistication and efficiency. Students will increasingly apply multiplicative reasoning to determine volumes by looking for and making use of structure such as the number of cubes in a single layer. Then, students will be ready to learn the formulas for computing volume for right rectangular prisms (CCSWT, 2012). Students may view the formulas as efficient methods connected to their previous work and experiences with finding the number of unit cubes that pack into a right rectangular prism. Having developed the understandings behind formulas in meaningful ways, students come to abstract their understandings and thus are no longer required to memorize these as isolated pieces of mathematical fact. Instead, students are able derive formulas from what they already know (Van de Walle, Karp, & Bay-Williams, 2010).

Consider switching the *Solve and Shares* from *Lessons 10-2* and *10-3* so that your students can use manipulatives with the *Solve and Share* for *Lesson 10-3* and build multiple rectangular prisms with the same volume. This hands-on experience allows students to see how the dimensions of length, width, and height combine to create volume using a visual model. They can bring this conceptual understanding to the *Solve and Share* for 10-2 which deals with a liquid and pushes students to apply multiplication to find the volume.

Math Practice 5: Use appropriate tools strategically

Focus on opportunities for students to develop *Mathematical Practice 5* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 10-6. Reference the Teacher's Edition (TE, pp. F25-F25A) and the NVACS (2010, p.7).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	Review Academic Vocabulary: (Vocabulary explicitly taught in prior grades or topics)
volume cubic unit rectangular prism unit cube formula Base	*cube* *area* *perimeter*

Additional terminology that students may need support with: *layers, additive*

Collaborative Team Conversations (CTC)

Consider using *one* of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for <u>evidence of mathematical understanding</u>:

Guiding questions: "Do students recognize the attributes of a solid figure that can be used to find volume? Do they recognize that volume can be decomposed into layers (connecting to understanding of area)?"
"Are students able to find the volume of non-overlapping right rectangular prisms by decomposing the shape into right rectangular prisms and adding the volumes of the parts?"

Lesson	Evidence	Look for
10-1	*Math Practices and Problem Solving* (student work samples) Items 14 through 18	Focus CTC around the big idea: • student strategies used to find volume. • understanding that volume can be composed/decomposed as layers.
10-1	*Quick Check* (digital platform) Items 1, 2 and 5	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".
10-4	*Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10	Focus CTC around the big idea: • student strategies and models used to find the volume of the figure. • understanding that volume of individual right rectangular prisms can be added together to find volume of a larger shape.

Learning Cycle Assessments (summative)	*Topic Performance Assessments* SE pp. 627-630	Use *Scoring Guide* TE pp. 627-630A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 10-1: Model Volume		
5.MD.C.3a 5.MD.C.3b 5.MD.C.4 MP.2 MP.5 MP.7	Access Prior Learning: Students worked with measurement concepts including perimeter and area in previous grades. This is the first time they will work with the concept of volume. Beginning the Big Idea: Students work with rectangular prisms composed of unit cubes to form a conceptual understanding of volume.	Solve and Share: Students may not realize the two-dimensional drawing represents a three-dimensional figure. **Building the prism with unit cubes or similar tools** will help push spatial thinking to include a third dimension. How are students figuring out the dimensions of the prism such as height, width and length? Ask students to look at their rectangular prisms from different viewpoints such as side view, top view, and front view. Can they connect these viewpoints to dimensions such as length, width, and height? Students may need clarification that the volume of single unit cube is 1. How many unit cubes compose the larger rectangular prism? What does this mean about its volume? Watch for students answering 24 cubes. Why do we not count all the cubes shown in the picture (the same cube may be shown more than once from multiple perspectives)? Visual Learning: A layer is added to the rectangular prism. Students may realize it is possible to find the new volume using multiplication. They can test their thinking by counting the unit cubes. Consider using the essential question to draw out student thinking about what volume means and how it can be determined. The *Guided and Independent Practice* items show pictures of rectangular prisms built with unit cubes. Some students may have trouble visualizing the unit cubes on the unseen side of the prisms. Using tools to build the prisms will allow students to view the prisms from all sides. Assess and Differentiate: The *Homework and Practice* page includes two questions, 8 and 9, composed of more than one rectangular prism. Have students use tools to build these shapes and explore the additive nature of volume. The table shown for questions 10-12, ask students to find different configurations for a rectangular prism with a volume of 12. Have students use the given numbers to build the shapes and observe how the sides change. What relationships can they discover between volume and the dimensions of length, width and height? *CTC: *Math Practices and Problem Solving* (student work samples) Items 14 through 18 *CTC: *Quick Check* (digital platform) Items 1, 2 and 5
Lesson 10-2: Develop a Volume Formula		
5.MD.C.5b 5.MD.C.4 5.MD.C.5a MP.1 MP.2 MP.3 MP.4 MP.6	Access Prior Learning: Students learned about the connection between area and multiplication in previous grades and topics. In the previous lesson students explored the concept of volume. Developing the Big Idea: Students build conceptual understanding of volume and begin to build procedural skill through exploring the connection between multiplication and volume.	Solve and Share: The context of this problem may need clarification. In the previous lesson students worked with unit cubes (solid) while this problem asks about capacity. What are we trying to find out about the fish tank? How can this be modeled using tools? If the water is emptied out of the tank, could unit cubes be used to find the tank's volume? Consider using student strategies to facilitate a discussion about meanings of volume and how it can be measured. Why do we record volume measurements as cubic units? Visual Learning: The formula for volume *l x w x h* is demonstrated. The first representation shown has the unit cubes drawn. How would the volume of this shape change if a layer were taken off? If one was added? Such questions might help students see that volume is composed of layers and connected to area. The *Guided and Independent Practice* items move to larger numbers making use of tools difficult. Question 7, uses smaller numbers and can be built with unit cubes. This will allow students to test the formula with a hands on experience. Assess and Differentiate: Consider providing additional opportunities to build rectangular prisms with unit cubes and identify the dimensions used to find volume.

Lesson 10-3: Volume of Prisms		
5.MD.C.5a 5.MD.C.5b MP.1 MP.2 MP.4 MP.7 MP.8	Access Prior Learning: Students learned about the connection between area and multiplication in previous grades and topics. In the previous lesson students explored the connection between volume and multiplication. Developing the Big Idea: Students build procedural skill through exploring the connection between surface area and volume.	Solve and Share: Students are asked to find 5 possible configurations of a rectangular prism with a volume of 24 cubic units. Consider asking students to build the different models. How can shapes have the same volume if they look different and have different measurements? Facilitate a discussion to draw out observations that connect finding volume to surface area and multiplication. The *Look Back!* fixes the height at 2 and asks students to determine possible lengths and widths. Use this question to draw out generalizations connecting area to volume. Visual Learning: The formula for volume *V= B x h* is shown. How does this formula connect to generalizations made by students during the *Solve and Share*? When a layer is removed or added, which of the dimensions of a rectangular prism are changed? Item 15 on the *Math Practices and Problem Solving* page can be used as part of the formative assessment process to check for understanding of the volume formula *V= B x h.* Assess and Differentiate: The *Homework and Practice* page models using the *V= B x h* formula using a representations showing the unit cubes. The *Intervention Activity* gives students a hands on experience with unit cubes to explore why the volume formula works.
Lesson 10-4: Combine Volume of Prisms		
5.MD.C.5c MP.1 MP.2 MP.4 MP.7	Access Prior Learning: Students learned that area is additive in previous grades. Students worked to find the volume of rectangular prisms in previous lessons. Developing the Big Idea: Students apply understanding to find the volume of non-overlapping rectangular prisms.	Solve and Share: Consider providing Teaching Tool 19 (combining volumes) for student use. Students are asked to find the volume of two non-overlapping rectangular prisms. The word "combining" used in the problem may help students to create a plan. Concrete tools can also be used to model two smaller rectangular prisms. What happens to the volume of the individual prisms when they are moved together and then pulled apart? How will students decide which dimensions can be used to create two individual prisms from this shape? Does it make sense to use all of the measurements given? Why or why not? Is there more than one way to find the volume of these two combined prisms? The *Look Back!* asks students to give the dimensions they used for each of the shapes. Use of geometric language to describe attributes of the rectangular prisms will help students identify the appropriate dimensions needed to find volume. Visual Learning: An example of splitting two non-overlapping rectangular prisms into two separate rectangular prisms is shown. The *Convince Me!* asks students if they can find a different way to separate the shape and calculate volume. *Guided Practice* and *Independent Practice*: The items give more measurements than students need to find the volume one way. Consider asking students to make a plan and justify their thinking for one or more shapes before calculating. Assess and Differentiate: *Another Look!* on the *Homework and Practice* models pulling two non-overlapping rectangular prisms apart calculating a volume for each before adding the volumes together. *CTC: *Math Practices and Problem Solving* (student work samples) Items 8, 9 and 10
Lesson 10-5: Solve Word Problems Using Volume		
5.MD.C.5c MP.2 MP.3 MP.4 MP.8	Access Prior Learning: Students found the volume of rectangular prisms in previous lessons. Securing the Big Idea: Students build procedural skill through application to real world problems.	Solve and Share: Look for students modeling the context of the problem and applying knowledge of volume to compute a solution. A class discussion using student ideas and strategies may draw out the idea that modeling problems and making a plan helps us to apply our knowledge to new problems. Visual Learning: A problem with a similar context to the *Solve and Share* is modeled. Is there more than one way to solve this type of problem? Encourage students to try various strategies when working on the questions that are similar to this on the *Independent Practice* page. Question 7 shows three non-overlapping rectangular prisms pushed together. Why is the remainder ignored? Assess and Differentiate: *Another Look!* gives a volume problem with a missing dimension. Students must use knowledge of the volume formula to find the height. However, the context uses "depth" instead of height so clarification may be needed.

Lesson 10-6: Math Practices and Problem Solving- Use Appropriate Tools		
5.MD.C.3a 5.MD.C.3b 5.MD.C.4 MP.1 MP.2 MP.3 MP.4 MP.5 MP.7	Access Prior Learning: Students have used tools throughout Topic 10. Students found volume measurements for rectangular prisms in previous lessons. Securing the Big Idea: Students use knowledge of volume concepts to create a plan and choose an appropriate tool to solve a real world problem.	Solve and Share: Students can be very creative with this problem and test their knowledge of volumes additive nature. There are many possible solutions. Consider asking students to share not only their solutions but their choice of mathematical tools and how it helped them solve this problem. Visual Learning: Do students agree with the tool choices modeled to solve the problem in the *Visual Learning Bridge*? Are there other possibilities that could work? The *Guided Practice* problem only shows two steps but asks students to find the volume for 5 steps. How will students find the volume of the missing steps? Look for a wide range of strategies including use of concrete tools, representations, additive/multiplicative thinking and algebraic expressions. Assess and Differentiate: The *Intervention Activity* poses a design challenge and assigns tools to groups. This task could be modified (consider reducing the cubic units needed based on available tools) and used whole class as well.

References

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Common Core State Standards Writing Team. (2012). *Progressions for the Common Core State Standards in Mathematics (draft). K-5, Geometric Measurement.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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